



PHI202

Introduction to Logic

Course Manual

ODL Edition

Francis Ofor, Ph.D.

Introduction to Logic

PHI202



University of Ibadan Distance Learning Centre
Open and Distance Learning Course Series Development
Version 1.0 ev1

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Vice-Chancellor's Message

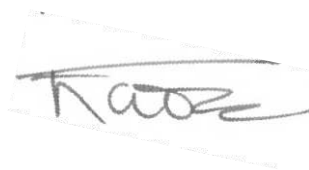
The Distance Learning Centre is building on a solid tradition of over two decades of service in the provision of External Studies Programme and now Distance Learning Education in Nigeria and beyond. The Distance Learning mode to which we are committed is providing access to many deserving Nigerians in having access to higher education especially those who by the nature of their engagement do not have the luxury of full time education. Recently, it is contributing in no small measure to providing places for teeming Nigerian youths who for one reason or the other could not get admission into the conventional universities.

These course materials have been written by writers specially trained in ODL course delivery. The writers have made great efforts to provide up to date information, knowledge and skills in the different disciplines and ensure that the materials are user-friendly.

In addition to provision of course materials in print and e-format, a lot of Information Technology input has also gone into the deployment of course materials. Most of them can be downloaded from the DLC website and are available in audio format which you can also download into your mobile phones, IPod, MP3 among other devices to allow you listen to the audio study sessions. Some of the study session materials have been scripted and are being broadcast on the university's Diamond Radio FM 101.1, while others have been delivered and captured in audio-visual format in a classroom environment for use by our students. Detailed information on availability and access is available on the website. We will continue in our efforts to provide and review course materials for our courses.

However, for you to take advantage of these formats, you will need to improve on your I.T. skills and develop requisite distance learning Culture. It is well known that, for efficient and effective provision of Distance learning education, availability of appropriate and relevant course materials is a *sine qua non*. So also, is the availability of multiple platform for the convenience of our students. It is in fulfillment of this, that series of course materials are being written to enable our students study at their own pace and convenience.

It is our hope that you will put these course materials to the best use.

A handwritten signature in black ink, appearing to read 'Isaac Adewole', is enclosed within a faint rectangular border.

Prof. Isaac Adewole

Vice-Chancellor

Foreword

As part of its vision of providing education for “Liberty and Development” for Nigerians and the International Community, the University of Ibadan, Distance Learning Centre has recently embarked on a vigorous repositioning agenda which aimed at embracing a holistic and all encompassing approach to the delivery of its Open Distance Learning (ODL) programmes. Thus we are committed to global best practices in distance learning provision. Apart from providing an efficient administrative and academic support for our students, we are committed to providing educational resource materials for the use of our students. We are convinced that, without an up-to-date, learner-friendly and distance learning compliant course materials, there cannot be any basis to lay claim to being a provider of distance learning education. Indeed, availability of appropriate course materials in multiple formats is the hub of any distance learning provision worldwide.

In view of the above, we are vigorously pursuing as a matter of priority, the provision of credible, learner-friendly and interactive course materials for all our courses. We commissioned the authoring of, and review of course materials to teams of experts and their outputs were subjected to rigorous peer review to ensure standard. The approach not only emphasizes cognitive knowledge, but also skills and humane values which are at the core of education, even in an ICT age.

The development of the materials which is on-going also had input from experienced editors and illustrators who have ensured that they are accurate, current and learner-friendly. They are specially written with distance learners in mind. This is very important because, distance learning involves non-residential students who can often feel isolated from the community of learners.

It is important to note that, for a distance learner to excel there is the need to source and read relevant materials apart from this course material. Therefore, adequate supplementary reading materials as well as other information sources are suggested in the course materials.

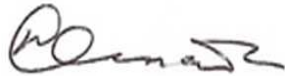
Apart from the responsibility for you to read this course material with others, you are also advised to seek assistance from your course facilitators especially academic advisors during your study even before the interactive session which is by design for revision. Your academic advisors will assist you using convenient technology including Google Hang Out, You Tube, Talk Fusion, etc. but you have to take advantage of these. It is also going to be of immense advantage if you complete assignments as at when due so as to have necessary feedbacks as a guide.

The implication of the above is that, a distance learner has a responsibility to develop requisite distance learning culture which includes diligent and disciplined self-study, seeking available administrative and academic support and acquisition of basic information technology skills. This is why you are encouraged to develop your computer skills by availing yourself the opportunity of training that the Centre’s provide and put these into use.

In conclusion, it is envisaged that the course materials would also be useful for the regular students of tertiary institutions in Nigeria who are faced with a dearth of high quality textbooks. We are therefore, delighted to present these titles to both our distance learning students and the university's regular students. We are confident that the materials will be an invaluable resource to all.

We would like to thank all our authors, reviewers and production staff for the high quality of work.

Best wishes.

A handwritten signature in dark ink, appearing to read 'Okunade', with a stylized flourish at the end.

Professor Bayo Okunade

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About this course manual

Introduction to LogicPHI202 has been produced by University of Ibadan Distance Learning Centre. All course manuals produced by University of Ibadan Distance Learning Centre are structured in the same way, as outlined below.

How this course manual is structured

The course overview

The course overview gives you a general introduction to the course. Information contained in the course overview will help you determine:

- If the course is suitable for you.
- What you will already need to know.
- What you can expect from the course.
- How much time you will need to invest to complete the course.

The overview also provides guidance on:

- Study skills.
- Where to get help.
- Course assignments and assessments.
- Margin icons.

We strongly recommend that you read the overview *carefully* before starting your study.

The course content

The course is broken down into Study Sessions. Each Study Session comprises:

- An introduction to the Study Session content.
- Study Session outcomes.
- Core content of the Study Session with a variety of learning activities.
- A Study Session summary.
- Assignments and/or assessments, as applicable.
- Bibliography

Your comments

After completing Introduction to Logic we would appreciate it if you would take a few moments to give us your feedback on any aspect of this course. Your feedback might include comments on:

- Course content and structure.
- Course reading materials and resources.
- Course assignments.
- Course assessments.
- Course duration.
- Course support (assigned tutors, technical help, etc.)

Your constructive feedback will help us to improve and enhance this course.

Course Overview

Welcome to Introduction to LogicPHI202

In this course, we shall look at the meaning, nature and value of logic. Logic, as we shall explain, is essentially about arguments. Consequently, we shall also look at the meaning, structure and types of arguments. A major part of our effort will be geared towards developing skills and techniques for distinguishing good arguments from bad ones. Here, we shall concentrate on both formal and informal arguments. We shall conclude this course by learning how to analyse arguments in artificial or symbolic language.

Course outcomes



Upon completion of Introduction to LogicPHI202 you will be able to:

- *distinguish* between good and bad arguments.
- *analyse* arguments both in natural and symbolic language using various techniques.
- *engage* in critical reasoning without errors and fallacies.
- *develop* a clear analytical mind and a good reasoning faculty.
- *highlight* the relevance of logic to the concerns of daily life.

Timeframe



How long?

This is a 15 week course. It requires a formal study time of 45 hours. The formal study times are scheduled around online discussions / chats with your course facilitator / academic advisor to facilitate your learning. Kindly see course calendar on your course website for scheduled dates. You will still require independent/personal study time particularly in studying your course materials.

How to be successful in this course



As an open and distance learner your approach to learning will be different to that from your school days, where you had onsite education. You will now choose what you want to study, you will have professional and/or personal motivation for doing so and you will most likely be fitting your study activities around other professional or domestic responsibilities.

Essentially you will be taking control of your learning environment. As a consequence, you will need to consider performance issues related to time management, goal setting, stress management, etc. Perhaps you will also need to reacquaint yourself in areas such as essay planning, coping with exams and using the web as a learning resource.

We recommend that you take time now—before starting your self-study—to familiarize yourself with these issues. There are a number of excellent resources on the web. A few suggested links are:

- <http://www.dlc.ui.edu.ng/resources/studyskill.pdf>

This is a resource of the UIDLC pilot course module. You will find sections on building study skills, time scheduling, basic concentration techniques, control of the study environment, note taking, how to read essays for analysis and memory skills (“remembering”).

- http://www.ivywise.com/newsletter_march13_how_to_self_study.html

This site provides how to master self-studying, with bias to emerging technologies.

- <http://www.howtostudy.org/resources.php>

Another “How to study” web site with useful links to time management, efficient reading, questioning/listening/observing skills, getting the most out of doing (“hands-on” learning), memory building, tips for staying motivated, developing a learning plan.

The above links are our suggestions to start you on your way. At the time of writing these web links were active. If you want to look for more, go to www.google.com and type “self-study basics”, “self-study tips”, “self-study skills” or similar phrases.

Need help?



As earlier noted, this course manual complements and supplements PHI202at UI Mobile Class as an online course, which is domiciled at www.dlc.ui.edu.ng/mc.

You may contact any of the following units for information, learning resources and library services.

Distance Learning Centre (DLC)

University of Ibadan, Nigeria
Tel: (+234) 08077593551 – 55
(Student Support Officers)
Email: ssu@dlc.ui.edu.ng

Head Office

Morohundiya Complex, Ibadan-Ilorin Expressway, Idi-Ose, Ibadan.

Information Centre

20 Awolowo Road, Bodija, Ibadan.

Lagos Office

Speedwriting House, No. 16 Ajanaku Street, Off Salvation Bus Stop, Awuse Estate, Opebi, Ikeja, Lagos.

For technical issues (computer problems, web access, and etcetera), please visit: www.learnersupport.dlc.ui.edu.ng for live support; or send mail to webmaster@dlc.ui.edu.ng.

Academic Support



A course facilitator is commissioned for this course. You have also been assigned an academic advisor to provide learning support. The contacts of your course facilitator and academic advisor for this course are available at the course website: www.dlc.ui.edu.ng/mc

Activities



This manual features “Activities,” which may present material that is NOT extensively covered in the Study Sessions. When completing these activities, you will demonstrate your understanding of basic material (by answering questions) before you learn more advanced concepts. You will be provided with answers to every activity question. Therefore, your emphasis when working the activities should be on understanding your answers. It is more important that you understand why every answer is correct.

Assignment



Assignment

This manual also comes with tutor marked assignments (TMA). Assignments are expected to be turned-in on course website. You may also receive TMAs as part of online class activities. Feedbacks to TMAs will be provided by your tutor in not more than 2-week expected duration.

Schedule dates for submitting assignments and engaging in course / class activities is available on the course website. Kindly visit your course website often for updates.

Assessments



Assessments

There are two basic forms of self assessment in this course: in-text questions (ITQs) and self assessment questions (SAQs). Feedbacks to the ITQs are placed immediately after the questions, while the feedbacks to SAQs are at the back of manual.

Bibliography



Reading







For those interested in learning more on this subject, we provide you with a list of additional resources at the end of this course manual; these may be books, articles or websites.

Getting around this course manual

Margin icons

While working through this course manual you will notice the frequent use of margin icons. These icons serve to “signpost” a particular piece of text, a new task or change in activity; they have been included to help you to find your way around this course manual.

A complete icon set is shown below. We suggest that you familiarize yourself with the icons and their meaning before starting your study.

			
Activity	Assessment	Assignment	Case study
			
Discussion	Group Activity	Help	Outcomes
			
Note	Reflection	Reading	Study skills
			
Summary	Terminology	Time	Tip

Study Session 1

The Meaning and Value of Logic

Introduction

In this Study Session, we will examine the meaning of logic in the strict, technical and professional sense. We will also be looking at the relevance of logic to the concerns of daily life.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

- 1.1 *discuss* the fundamental principles of logic.
- 1.2 *present* the value and relevance of logic to the concerns of daily life.

1.1 Laws of Logic

Logic The branch of philosophy that deals with the study of the basic principles, techniques, or methods for distinguishing between good and bad arguments, valid and invalid arguments, deductive and inductive arguments, as well as sound and unsound arguments

Let us start on a frank note! The word '**logic**' is not new to many of us. We have heard of it before and many of us have used it severally in our speeches and writings. But despite its familiarity, it is a word that most people find difficult to define in clear terms. This is because the word 'logic' can be used in at least three different, yet, equally correct senses. In the first sense, the term logic is used to describe the totality of all laws guiding the human thought (Wallace, 1974).

It is a truism that humans are rational beings whose thinking processes are based on certain principles. The totality of these principles has been described by many, using the word 'logic'.

In another sense, the word 'logic' can be used to describe the principles guiding the operation of a mechanism. Every garget or thing has its own inner logic, which describes the way the garget ought to operate. For instance, when we operate our GSM handset, it follows a particular procedure. If a call comes in, we press the *receive button* and the call is received. To end the call, we press the *end button*. If we press the *end button* and the handset starts sending messages indiscriminately, then something is wrong, and the set will be said not to be operating the way it ought to operate, that is, according to its inner logic. The operation of a mechanism is therefore guided by certain principles which can be referred to as the inner 'logic' of that mechanism.

The foregoing conceptions of logic are all correct in their own right, but, these are not the only senses in which we seek to define logic in this course. Here, we are interested in the meaning of logic in the strict, technical and professional sense as an academic discipline. In this sense, logic is that branch of philosophy that deals with the study of the basic

principles, techniques, or methods for evaluating arguments. Evaluation here involves making a distinction between good and bad, valid and invalid, deductive and inductive as well as sound and unsound arguments. Understood in this sense, logic reflects upon the nature of thinking itself (Popkin and Stroll, 1993). It attempts to answer such questions as, what is correct reasoning? What distinguishes a good argument from a bad one? Are there methods to detect fallacies in reasoning and, if so, what are they? These preoccupations of logic distinguish it from psychology, which concerns itself with the mental processes of the thinker (see Azenabor, 2001). Although, reasoning is a form of thinking, the fact is that not all thinking is reasoning. One may think about a number of issues without doing any reasoning about them. In other words, there are many mental processes in which the psychologists may be interested, which are nevertheless different from reasoning. Reasoning is a special kind of thinking in which inferences take place. The psychologist merely examines the thinking process, while the logician concerns himself with the formulation of rules that will enable us to test whether the particular piece of reasoning is correct, coherent and consistent. This distinction between correct and incorrect reasoning is the central problem with which logic deals and all the principles and techniques of logic have all been developed primarily for the purposes of making this distinction clear.

However, the principles, processes and techniques of logic are not arbitrary! This is because there are certain fundamental laws that every thinking process must follow for it to be correct. We have three of such laws. These are ‘The Law of Identity’, ‘The Law of Contradiction’ (also known as The Law of Non-contradiction) and ‘The Law of Excluded Middle’.

The Law of Identity states that if any statement is true, then it is true. In other words, every single statement is identical with itself. The Law of Contradiction states that no statement can both be true and false at the same time. For instance, my statement ‘Professor Francis Egbokhare is a man’ is true. I cannot claim that it is false at the same time. If I insist that the statement is both true and false at the same time, then, it is either I am completely delirious or I am under a serious attack of amnesia. Finally, ‘The Law of Excluded Middle states that any statement is either true or false. We cannot say that a statement is neither true nor false; there is no such middle ground! The law is to the effect that if a statement is not true, then it is false, and if it is not false, then it is true. These are the laws, which all statements or reasoning must conform to, for them to be taken as correct. All the branches of philosophy, and indeed other areas of knowledge, employ thinking and reasoning. But, whether the reasoning is correct or not will depend upon whether it is in accord with the laws of logic just described (Joyce, 1936). This makes logic the most fundamental of all branches of knowledge. How do we then explain the relevance of such a discipline to human endeavours generally?

1.2 Value of Logic

The question of the value of logic deserves as much attention as that devoted to the issue of the meaning of logic. What benefit, one may ask,

can anyone derive from the study of logic? Or, put more generally, of what benefit is logic to human endeavours generally? This question is apposite because as Ludwig Wittgenstein once remarked, what is the use of a discipline:

If all it does is to enable you talk about some abstruse questions...and if it does not improve your thinking about important questions of everyday life? (Kahane, 1978)

As a discipline, logic is relevant to the human person in several important ways. In the first place, in real life situations, we encounter arguments in our everyday activities. These arguments are usually more complex than and not as organised as those we find in logic textbooks. They also pose a lot of problems to the human mind in the same degree that long and complex mathematical problems do. As an act, logic induces in us certain abilities that enhance our capacity for the development and construction of good arguments. A person who has some training in logic will therefore be in a better position to analyse issues, with a view to differentiating the essentials from the inessentials than a person without any training in logic. In fact, a critical analysis and examination of whatever we read in books, watch on the television or even discuss in our everyday conversation, will be of great help in the development of human knowledge; such reflective thinking can lead to fruitful deductions or inferences.

Thinking is an essential ingredient of life, but thought, like all potent weapons, is exceedingly dangerous if mishandled. Clear thinking, which logic enhances, is therefore desirable not only in order to develop the full potentialities of the mind, but also as a means of avoiding disaster. As Blaise Pascal once remarked, “all our dignity lies in thought” (Barry and Soccio, 1988) and “logic is the anatomy of thought” (Pospesel and Marans, 1978). With the tools of logic therefore, people can easily think through popular opinions and dangerous beliefs and arrive at some knowledge that will be of relevance to the promotion of peaceful co-existence among the people.

ITQ

1. How would you define logic in the technical sense?

Feedback

- From what you have learnt so far, we believe that your definition would regard logic as the branch of philosophy that is concerned with studying the basic methods or techniques for evaluating arguments.

Again training in logic can affect a person’s character and attitude. Someone with a good knowledge of logic will be rational and more intellectually alert. He will be slow to accept other people’s ideas without proper scrutiny, and he will be more likely to question his own prejudices and rationalisations.

Once more, an individual with a tint of logic is less likely to be influenced by political demagogues or advertiser’s pleadings. Such a

person can easily spot or detect fallacies and inconsistencies in different lines of argument. He knows how to make a distinction between “persuasions through various psychological techniques and those based on rational arguments and supporting evidence” (Aja, 1992).

Finally, the value of logic is better appreciated in the law court. Court proceedings and dispute settlements testify to the relevance of training in logical reasoning. Cases are often won in the court given the force of the arguments and evidences presented in support of those cases. Even in ordinary communal disputes settlement, people look forward to convincing evidences and persuasive arguments. What is explicit or characteristic of training in logic is that it enables us to provide good reasons as evidence for whatever claim we wish to establish. Training in logic will help enhance the capacity for well structured and convincing arguments.

We need to emphasize that what we have celebrated as the benefits of logical training can only be made possible where people already possess some natural abilities. These abilities include some form of native intelligence, fertile imagination, curiosity and ingenuity amongst others. These are the needed raw materials for the effective learning of logic in the formal sense.

One main point which we have been able to deduce from our discussion so far is that a training in logic will help to bring the relation of the discipline to the concerns of our daily lives into clearer perspectives. It is now an incontrovertible fact that the ability to think clearly and to analyse arguments logically is of tremendous practical importance. If, according to Blaise Pascal, “all our dignity lies in thought”, then, we must in agreement with him “strive to think well”.

Study Session Summary



Summary

In this Study Session, we noted that Logic is that branch of philosophy that deals with the study of the basic principles, techniques, or methods for distinguishing between good and bad arguments, valid and invalid arguments, deductive and inductive arguments, as well as sound and unsound arguments. In logic, there are three fundamental laws that every thinking process must follow for it to be correct. They are ‘The Law of Identity’ which states that if any statement is true, then it is true; ‘The Law of Contradiction’ which says that no statement can both be true and false at the same time and ‘The Law of Excluded Middle’ which states that any statement is either true or false..

Assessment



Assessment

SAQ1.1(tests Learning Outcome 1.1)

What are those fundamental laws that every thinking process must follow if it is to be correct?

SAQ1.2(tests Learning Outcome 1.2)

In what ways can your study of logic affect your life?

Study Session 2

The Meaning and Structure of Argument

Introduction

In this Study Session, we will discuss the meaning and structure of argument. Efforts will also be made to examine types and ways of assessing arguments.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

- 2.1 *define* and *use* correctly the term “argument”.
- 2.2 *describe* the two major types of arguments.
- 2.3 *assess* arguments.

2.1 The Meaning of Argument

Argument A statement with the structure that is defined by the ‘premises’ and ‘conclusion’ and the nature of the relationship between them.

The term ‘**argument**’ can be understood from two perspectives. In ordinary discourse, it denotes a quarrel or disagreement, whereas in logic, that is, in the technical sense, an argument is a sequence of statements, ‘declarative sentences’ or propositions in which one part known as the conclusion is claimed to follow from the others called the premises. That means that an argument is not just a mere collection of statements. An argument has a structure which is defined by the terms ‘premises’ and ‘conclusion’ and the nature of the relationship between them (Oladipo, 2008).

The conclusion of an argument is that proposition which is affirmed on the basis of some other propositions, which serve as justification for the acceptance of the conclusion. These other propositions, which go by various names such as evidence, grounds, or reasons, are more professionally called premises. In an argument, therefore, the premises are intended to provide sufficient grounds for the acceptance of the conclusion. Where there is no relationship whatsoever between the putative claim or conclusion and the reasons given for its acceptance, then there is no argument. To determine whether a group of statements is an argument or not, two questions need to be answered. First, we ask: what is the claim being made? Once we are able to identify the claim being made, we then ask the next important question: what are the reasons or evidence in support of this claim? It is only when we are able to answer these questions that we can say that there is an argument. In other words, a mere collection of statements cannot be an argument. We only have an argument when there is a claim and reasons are given in support of the claim. Let us consider the following sets of propositions:

1. The moon is made of green cheese and strawberries are red. Hence, *my dog has fleas*.
2. *Helen is a physician. So, Helen went to medical school since all physicians have gone to medical school.*

If we use the two criteria for recognising an argument mentioned earlier, we discover that whereas the first example is just a collection of unrelated propositions, the second example is not just a collection of propositions, but related propositions in which the truth of ‘Helen went to medical school’ is supported and derived from the truth of ‘Helen is a physician’ and ‘All physicians have gone to medical school’. The proposition ‘Helen went to medical school’ is the conclusion, while ‘All Physicians have gone to medical school’ and ‘Helen is a physician’ are the premises. It is the combination of both the premises and conclusion that makes up an argument. The following are examples of arguments:

1. All physicians are university graduates.
All members of the Nigerian Medical Association are physicians.
Therefore, all members of the Nigerian Medical Association are university graduates.
2. The Golden Rule (the rule of conduct, do unto others as you would wish them do unto you) is basic to every system of ethics ever devised, and everyone accepts it in some form or other. It is therefore an undeniably sound moral principle
3. Large numbers of people in this country have never had to deal with the criminal justice system. Thus, they are unaware of how it works and of the extraordinary detrimental impact it has upon many people’s lives.
4. Since the elderly have always had a higher cancer rate, and since we now have older citizens, the absolute increase in the number of cancer deaths is not an indication of any kind of environmental breakdown.
5. Since witch-doctors are not supposed to serve malevolent ends and since they often use medicines or magic to help drive off, or destroy witches, they are best classified as good magicians or wizards.
6. Human brains have the same kind of chemistry and cell receptors as rats regarding glucocorticoids. So, it seems possible that our response to being handled as infants is similar.
7. What science can’t know, mankind can’t know. Therefore, all knowledge comes from science.
8. Abortion is evil not only to the victim but also to our sense of justice. Hence it should be abolished.

In some arguments such as the foregoing, the premises of the argument are stated first and the conclusion last. But, not all arguments are so arranged. In some others, the conclusion is either stated first or is sandwiched in-between different premises offered in its support. Consider the following arguments:

1. Man is superior to other animals because he is the only animal that can ask questions about his existence.
2. The death of God is not debatable. If He is not dead, He should have punished the unjust men of the world.
3. Poetry is finer and more philosophical than history, for poetry expresses the universal and history the particular.
4. All men are mortal. Therefore the pope is mortal since the pope is a man.
5. In as much as man is created first, man should be the master of all creatures.

The above arguments are not as organised as the once given earlier. It is therefore our duty to arrange them into their respective premises and conclusions.

However, there are a few instances where the conclusions of arguments are not explicitly stated. To understand and analyse arguments of this type, we have to study the context in which they occur to supply the relevant conclusion. Such arguments are called *enthymemes*, from the Greek word *en* (in) and *thymos* (mind). The following is an example of enthymeme:

If he's smart, he isn't going to go around shooting one of them and he is smart.

The relevant conclusion in this kind of argument will be “therefore, he isn't going to go around shooting one of them”.

ITQ

Question

- How are the terms “premise” and “conclusion” related in an argument?

Feedback

- We may refer to a premise as that part of an argument which serves as the justification for the acceptance of the conclusion while we may refer to the conclusion as that part of an argument which is affirmed on the basis of the premise.

Hint

Issues pertaining to analyzing arguments into their premises and conclusions have been extensively discussed in our year one logic course called 'Arguments and Critical Thinking'. Let us now turn our attention to types of arguments and how we can assess them in the next section.

2.2 Types of Arguments

There are basically two types of argument. They are called deductive and inductive arguments. In a way, every argument involves the claim that its premises provide some grounds for the truth of its conclusion. A

deductive argument can be distinguished from an inductive one by examining what each claims (Bello, 2000). When an argument claims that the truth of its premises guarantees the truth of its conclusion, it is said to be a deductive inference. **Deductive reasoning** lays claim to a very high standard of correctness. A deductive inference succeeds only if its premises provide such absolute and complete support for its conclusion that it would be utterly inconsistent to suppose that the premises are true but the conclusion false.

On the other hand, when an argument claims merely that the truth of its premises make it likely or probable that its conclusion is also true, such an argument is said to involve an inductive inference. In other words, **inductive arguments** do not claim more than that their premises provide some grounds for the truth of their conclusions. That is why we say that the conclusion of an inductive argument follows the premise with greater or lesser degree of probability.



Tip

Deductive reasoning is the process of reasoning in which reasons are given in support of a claim. The reasons, or justifications, are called the premises of the claim, and the claim they purport to justify is called the conclusion. In a correct, or valid, deduction the premises support the conclusion in such a way that it would be impossible for the premises to be true and for the conclusion to be false. In this, deduction differs sharply from induction, a process of drawing a conclusion in which the truth of the premises does not guarantee the truth of the conclusion.

2.3 Assessing Arguments

Arguments can be assessed in two ways. First, there are some formal principles and methods that have been developed for assessing arguments. The types of arguments to which these principles can be applied are described as formal arguments. The principles and methods for assessing formal arguments will be discussed much later in this course.

On the other hand, it cannot be denied that people have the natural ability not only to proffer arguments but also to evaluate and distinguish good ones from bad ones. This does not involve any formal and stringent rules or mechanical procedures. In fact, there are many kinds of arguments in ordinary language which are not amenable to assessment in accordance with the formal principles and methods for evaluating formal arguments. All such arguments are assessed informally.

In assessing arguments informally, we try to ascertain if or to what extent the premises of the argument provide support for the acceptance of the conclusion. If the premises provide enough support, then the argument is said to be good, otherwise it is said to be a bad argument.

The criteria for assessing arguments in ordinary language have been extensively discussed in our year one course on 'Arguments and Critical Thinking'. However, the following additional guidelines may be useful in assessing arguments informally:

1. First, arrange the argument into its respective premises and conclusion. You can achieve this by asking yourself or identifying the claim being put forward in the argument and the reasons being proffered in support of the claim.
2. Next, determine whether or not the premises provide support for the acceptance of the conclusion.
3. If the premises provide support for the acceptance of the conclusion at all, determine the extent to which the premises justify the conclusion.

Let us now assess some arguments using the above guidelines.

Argument 1

All censorship exist to prevent anyone from challenging current conceptions and existing institutions. All progress is initiated by challenging current conceptions and executed by supplanting existing institutions. Consequently, the first condition of progress is the removal of censorship.

– G. Bernard Shaw

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has two premises and a conclusion.

Premise 1: All censorship exist to prevent anyone from challenging current conceptions and existing institutions.

Premise 2: All progress is initiated by challenging current conceptions and executed by supplanting existing institutions.

Conclusion: The first condition of progress is the removal of censorship.

Support for the conclusion

The premises support the conclusion

Justification of support

If all censorship exist to maintain the status quo and the only way of initiating progressive changes and executing such is by challenging conceptions and supplanting existing institutions (which the status quo seeks to protect with all manner of censorship), then the first major step towards progressive changes will be to remove those censorship that prevent any form of challenge to current conceptions and existing institutions. The conclusion that 'the first condition of progress is the removal of censorship' is therefore strongly supported by the premises.

Argument 2

The life of every civilised community is governed by rules.

Neither peace of mind for the present nor intelligent planning for the future is possible for people who either live without rules or cannot abide by the rules they make. Making rules for the community, and enforcing them, is the job of government. No community can be truly civilised, therefore, without an

effective and reasonably stable government.

- Carl Cohen

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has three premises and a conclusion.

Premise 1: The life of every civilized community is governed by rules.

Premise 2: Neither peace of mind for the present nor intelligent planning for the future is possible for people who either live without rules or cannot abide by the rules they make.

Premise 3: Making rules for the community and enforcing them is the job of government.

Conclusion: No community can be truly civilized, without an effective and reasonably stable government.

Support for the conclusion

The premises support the conclusion

Justification of support

The aim of every community is to civilize and some indices of civilization include 'peace of mind for the present' and 'intelligent planning for the future'. These, however, are only possible where people live and abide by the rules of the community. Since the responsibility for making and enforcing rules for the community is the job of government, it follows therefore that a community without an effective and reasonably stable government to make and enforce its rules, will not enjoy the peace of mind and the intelligent future planning that are the hallmarks of any civilized community. The conclusion therefore that 'no community can be truly civilized, without an effective and reasonably stable government' is supported by the premises.

Argument 3

It is far from certain that the need for government among men rests solely on 'original sin' or man's innate criminality. For no association, however constituted, can exist without a regulatory force of some kind: even a society of angels will still need some form of government if only to ensure that common tasks are assigned and coordinated.

- Mkwugo Okoye

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has one premise and a conclusion.

Premise: No association, however constituted, can exist without a regulatory force of some kind: even a society of angels will still need some form of government if only to ensure that common tasks are assigned and coordinated

Conclusion: It is far from certain that the need for government among men

rests solely on 'original sin' or man's innate criminality.

Support for the conclusion

The premise supports the conclusion

Justification of support

It is obvious that the premise provides enough support for the conclusion, for, if the state is some type of association and if no association can exist without a regulatory force of some kind, it follows that the state needs a government which serves as its regulatory force. By stressing the necessity for some form of government, even in a society of angels (who by definition cannot sin and have no criminal tendencies), the premise gives further credence to the conclusion that the need for government among men (who are guilty of original sin and have criminal propensities) does not rest solely on 'original sin' or man's innate criminality.

Argument 4

The presumption that the creation of states automatically means the creation of development is wrong. There are many areas in this country which have seen no progress even though they have been affected several times by the state creation exercise.

- A. G. A. Bello

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has one premise and a conclusion.

Premise: There are many areas in this country which have seen no progress even though they have been affected several times by the state creation exercise

Conclusion: The presumption that the creation of states automatically means the creation of development is wrong.

Support for the conclusion

The premise supports the conclusion

Justification of support

If one can identify as little as a single area that has been affected by the state creation exercise but has seen no progress in terms of development, then it becomes difficult to sustain the claim that the creation of states translates automatically into the creation of development. Therefore, if there are many areas that have seen no progress, despite their being affected by the state creation exercise, it follows then that the creation of states does not automatically mean the creation of development.

Argument 5

Since happiness consists in peace of mind, and since durable peace of mind depends on the confidence we have in the future, and since that confidence is based on the science we should have of the nature of God and the soul, it follows that science is necessary for true happiness.

– Gottfried Leibniz

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has three premises and a conclusion.

Premise 1: Happiness consists in peace of mind.

Premise 2: Durable peace of mind depends on the confidence we have in the future.

Premise 3: The confidence we have in the future is based on the science we have of the nature of God and the soul

Conclusion: Science is necessary for true happiness

Support for the conclusion

The premises support the conclusion

Justification of support

If happiness consists in peace of mind, and an enduring peace of mind depends on the confidence we have in the future, which itself is based on the science we should have of the nature of God and the soul, then it follows that science we have (of the nature of God and the soul) is necessary for true happiness.

So far, we have assessed five arguments and they all have turned out to be good arguments. Let us also look at a few examples of bad arguments.

Argument 6

The inquisition must have been justified and beneficial, if whole people evoked and defended it, if men of the loftiest souls founded and created it severally and impartially and its very adversaries applied it on their own account pry answering to pry.

– Benedetto Croce

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has three premises and a conclusion.

Premise 1: Whole people evoked and defended the inquisition

Premise 2: Men of the loftiest souls founded and created it severally and impartially.

Premise 3: Its very adversaries applied it on their own account pry answering to pry

Conclusion: The inquisition must have been justified and beneficial.

Support for the conclusion

The premises do not support the conclusion

Justification of non-support

The conclusion that the inquisition was justified and beneficial is being canvassed on the grounds that whole people evoked and defended it. But there is nothing in the conclusion to suggest that all those who created and evoked it were not mistaken and that therefore the conclusion is true. The endorsement of a claim by all or some group of people does not necessarily mean that the claim is true or correct. We have to critically look at the claim itself and not the belief or disposition of the people supporting the claim.

Argument 7

It is only when it is believed that I could have acted otherwise that I am held to be morally responsible for what I have done. For a man is not thought to be morally responsible for an action that was not in his power to avoid.

- A. J. Ayer

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has one premise and a conclusion.

Premise: A man is not thought to be morally responsible for an action that was not in his power to avoid.

Conclusion: It is only when it is believed that I could have acted otherwise that I am held to be morally responsible for what I have done.

Support for the conclusion

The premise does not support the conclusion

Justification of non-support

In this argument, the premise and conclusion are saying the same thing. For instance, the expression "thought to be morally responsible for an action" in the premise and "held to be morally responsible" in the conclusion are the same. Also, the phrase "an action that was not in his power to avoid" in the premise and "I could not have acted otherwise" in the conclusion are the same. Although, the subject-matter of the premise must be related to that of the conclusion for an argument to be good, such a relation must not be such that both the premise and the conclusion must be saying the same thing. When this is the case as we have seen above, the premise cannot be said to provide independent evidence in support of the conclusion. What we have will not be a support or justification but a repetition.

Argument 8

Those who say that Ifa is not a science are mistaken. The wisest men and women in Yoruba history have all been interested in Ifa, and Obas and Queens and Regents of all epochs in Yoruba land have believed in it and have guided the affairs of their people by it.

- A. G. A. Bello

Assessment of Argument

Arrangement of argument into premises and conclusion

The argument has two premises and a conclusion.

Premise 1: The wisest men and women in Yoruba history have all been interested in Ifa.

Premise 2: Obas and queens and regents of all epochs in Yoruba land have believed in it and have guided the affairs of their people by it.

Conclusion: Those who say that Ifa is not a science are mistaken.

Support for the conclusion

The premises do not support the conclusion

Justification of support

The conclusion that “those who say that Ifa is not a science are mistaken” is asserted on the grounds that the wisest men and women in Yoruba history have all been interested in Ifa and that Obas and Queens and Regents of all epochs in Yoruba land have believed in it and have guided the affairs of their people by it. The fact that the wisest men and women and regents of all epochs believed in Ifa does not affirm the scientificity of Ifa. There is even nothing in the argument to suggest that the wisest men and women, Obas and Queens and Regents were not all mistaken about the true worth of Ifa. It is obvious therefore that the conclusion that “those who say that Ifa is not a science are mistaken” does not follow from the premises.

ITQ

Question

- What are the two questions that you must answer in determining whether a group of statement is an argument or not

Feedback

- Whatever your answer, the two questions we must answer in determining whether a set of proposition constitutes an argument is:
 - what is the claim that is made?
 - what is/are the reason(s) for making that claim?



Note

It is expected that after a considerable practice in the recognition and analysis of arguments, you should be able to distinguish argumentative from non-argumentative discourses. And even within argumentative discourse, you should be able to identify good arguments from bad ones.

Study Session Summary



Summary

In this Study Session, we discussed in technical sense that, argument refers to a group of statements in which one part known as the 'conclusion' is claimed to follow from the others called the premises. That means that an argument is not just a mere collection of statements. Also, an argument has a structure which is defined by the terms 'premises' and 'conclusion' and the nature of the relationship between them. Arguments can be assessed both formally and informally. In assessing arguments informally, we try to ascertain if, or to what extent, the premises of the argument provide support for the acceptance of the conclusion. If the premises provide enough support, then the argument is said to be good, otherwise it is said to be a bad argument.

Assessment



Assessment

SAQ2.1 (tests Learning Outcome 2.1)

Determine if each of the following statements is an argument or not.

1. The notion of single motherhood is strange, and my neighbour has cancer.
2. Single motherhood must be tamed if our family system will not crumble.

SAQ2.2 (tests Learning Outcome 2.2)

Determine the type of argument that each of the following is.

1. Since tests have proven that it takes the copulation of a man and a woman for a woman to get pregnant, Susan obviously cannot come and tell us that no man is responsible for her pregnancy.
2. Since whatever has gone up in the past one billion years has been coming down, I will not be wrong to absolutely aver that whatever goes up tomorrow will come down.
3. That Hamilton ever held any considerable sum in securities seems highly improbable, for he was at no time a rich man, and at his death left a small estate.

SAQ 2.3(tests Learning Outcome 2.3)

Assess the following arguments, stating whether it contains one argument or more as well as dividing each of the arguments to premises and conclusion.

1. A housewife's work has no results; it simply has to done again. Bringing up children is not a real occupation, because children come up naturally, brought or not.
2. Poetry is fine and more philosophical than history; for poetry expresses the universal and history only the particulars.

Study Session 3

Formal Logic

Introduction

In the previous Study Session, we focused on arguments. In this Study Session, we will extend our study light to formal logic. Formal logic deals with the logical or formal structure of statements and arguments. It comprises of analysis of statements (that is, categorical propositions) that make up Categorical Syllogism.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

3.1 *discuss* formal arguments in natural language.

3.2 *highlight* categorical propositions.

3.1 Formal Arguments in Natural Language

What we have discussed so far in the last two Study Sessions of this course falls under the aspect of Logic called Informal Logic. Informal logic is concerned mainly with our everyday activities of making and evaluating claims, as well as detecting errors in reasoning. Formal logic, which is our main focus in this Study Session, deals with the logical or formal structures of statements and arguments (Ekanola, 2004). These statements or arguments may be either in natural language or in artificial language. Formal arguments in natural language are many and they go by various names. Amongst them are Categorical Syllogism and Relational argument. The part of formal logic that deals with formal arguments in artificial language is called symbolic logic.

This Study Session introduces us to analysis of statements that make up Categorical Syllogism. These statements are called Categorical Propositions. A syllogism is an argument that has three statements or propositions, two of which are premises and the last, the conclusion.

Categorical syllogism is about the oldest and most popular form of arguments in natural language. It was indeed one of the earliest approaches to evaluating formal arguments. It was originally developed by Aristotle, codified in greater detail by medieval logicians and then interpreted mathematically by George Boole and John Venn in the 19th century. A categorical syllogism is a form of formal argument made up of three categorical propositions. What then are categorical propositions?

3.2 Categorical Propositions

Categorical propositions are propositions of certain kind. They are about classes. We say they are about classes because they either affirm or deny that one class is included in another, either partially or wholly. There are four of such propositions represented by the following examples:

1. All men are politicians
2. No men are politicians.
3. Some men are politicians.
4. Some men are not politicians.

The above propositions either affirm that one class is included in another like 1 above or deny that one class is included in another like 2 above, or affirm that some members of a particular class are members of another class like 3 above or that some members of a particular class are not members of another class like 4 above.

These propositions have been given names for ease of reference. Proposition 1 above is an 'A' proposition and can be reduced to skeletal form as 'All S is P'. Proposition 2 is an 'E' proposition and is of the form 'No S is P'. Propositions 3 and 4 are called I and O propositions and are skeletally represented as 'Some S is P' and 'Some S is not P' respectively. Every categorical proposition has a recognisable form made up of four parts; namely, 'quantifier-word', 'subject-term', 'copula' and 'predicate-term'. Using these terms, we can analyse the four categorical propositions as follows:

Quantifier-word	Subject-term	Copula	Predicate-term
All	men	are	politicians
No	men	are	politicians
Some	men	are	politicians
Some	men	are not	politicians

3.2.1 The Nature of Categorical Propositions

The following notions should be understood, in order to be able to analyse categorical propositions. These are 'Quantity', 'Quality' and 'Distribution'.

Quantity

Every categorical proposition has a quantity. The quantity of a categorical proposition is either universal or particular, depending on whether or not the proposition refers to all or some of the members of the class designated by the subject term. For example, the A and E propositions refer to the entire members of their subject class and are therefore universal in quantity. On the other hand, the I and O propositions are particular in quantity because both refer to part of the members of the class designated by their subject terms.

ITQ**Question**

- What is the quantity of each of the propositions below:
 - 1) All men are mortals
 - 2) Some men are mortals

Feedback

- The first proposition is universal in quantity while the second is particular in quantity.

Quality

The quality of a categorical proposition is either affirmative or negative, depending on whether or not the proposition affirms that one class is included in another, either partially or wholly. For example, the A proposition affirms that ‘all’ men are included wholly in the political class. Also, the I proposition affirms that part of the class of men are included in the political class. Both propositions are therefore affirmative in quality. On the other hand, the E and O propositions both deny that all or parts of their subject class are included in their predicates. Therefore, they are negative in quality.

Distribution

This is a technical term that is used to describe the ways terms occur in categorical propositions. A categorical proposition is said to either distribute or not distribute its terms. Since every categorical proposition has a subject-term and a predicate-term, a proposition may either distribute or not distribute its subject-term or predicate-term. A categorical proposition distributes a term if it refers to all the members of the class designated by that term. Consider the following propositions:

All Nigerians are Africans

It is clear here that the intention is to talk about every Nigerian. Therefore, we say that the subject term is distributed. However, the predicate does not refer to all or every African. It refers only to these Africans that are Nigerians. Thus, we say that the predicate-term is not distributed. But the E proposition,

No Nigerians are Africans:

Asserts of each and every Nigerian that he or she is not an African. The whole of the class of Nigerians is said to be excluded from the class of Africans. The subject-term is therefore distributed since it refers to the whole of the class of Nigerians. In asserting that the whole class of Nigerians is excluded from the class of Africans, the proposition is also asserting that the whole class of Africans is excluded from the class of Nigerians. The ‘E’ proposition therefore refers to all members of the class designated by its predicate-term, and is said to distribute its predicate term. In the I proposition:

Some Nigerians are Africans:

The reference in the subject-term is to ‘some’ and not ‘all’, and as such, the subject-term is not distributed. Also the predicate-term does not refer

to all members of that class and is therefore not distributed. Lastly, in the O proposition.

Some Nigerians are not Africans

The subject-term refers to some members of the class and is therefore not distributed. What the proposition is saying is that part of the class of subject (Nigerians) is excluded from the class of predicate (Africans). When something is said to be excluded from a class, the whole of that class is referred to. Therefore the O proposition distributes its predicate term.

In summary, we may say that:

1. The A proposition is a universal affirmative proposition that distributes its subject-term but does not distribute its predicate-term.
2. The E proposition is a universal negative proposition that distributes both its subject-term and predicate-term.
3. The I proposition is a particular affirmative proposition that does not distribute its subject-term and predicate-term.
4. The O proposition is a particular negative proposition that does not distribute its subject-term but distributes its predicate term.

ITQ

Question

- Which of the following propositions has its predicate-term distributed:
 - a) Some people that are blonde are not intelligent.
 - b) All people that are blonde are intelligent.
 - c) Some people that are blonde are intelligent.

Feedback

- The first proposition has its predicate term distributed. This is because the proposition is saying that there are some people who are blonde but do not fall in the universal class of the intelligent.
- The second proposition does not have its predicate term distributed. This is because the proposition is not referring to all who are intelligent but just a section.
- The third proposition does not also have its predicate term distributed. This is because the proposition is not referring to all members of the class of 'intelligent'.

Study Session Summary



In this Study Session, we examined the nature of categorical propositions which are about classes because they either affirm or deny that one class is included in another. Also, we examined the notions of

Summary

'quantity', 'quality' and distribution as they relate to categorical propositions. The quantity of a categorical proposition is either universal or particular, depending on whether or not the proposition refers to all or some of the members of the class described by the subject term. The quality of a categorical proposition is either affirmative or negative, depending on whether or not the proposition affirms that one class is included in another, either partially or wholly. A categorical proposition distributes a term if it refers to all the members of the class designated by that term.

Assessment



Assessment

SAQ 3.1 (tests Learning Outcomes 3.1 and 3.2)

Define types of categorical proposition in terms of quantity, quality, and distribution.

Study Session 4

Inferences

Introduction

In this Study Session, we will discuss inference and the nature of relationships that exist among the four categorical propositions. It should be noted that each of the categorical propositions has a relationship that links it with other propositions. Also to be examined here is immediate inference and its forms.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

4.1 *define* the term “inference”.

4.2 *explain* at least four forms of oppositions.

4.1 Mediate and Immediate Inferences

An **inference** is the process by which one proposition is arrived at and affirmed, on the basis of one or more other propositions accepted as the starting point of the process. What distinguishes an argument from a mere collection of statements is the inference that is supposed to hold between them. An inference is either mediate or immediate. A mediate inference proceeds from two premises to a conclusion in such a way that the propositions together represent a complete argument. An inference is immediate if it proceeds from only one premise to the conclusion. Some of these inferences can be demonstrated by using what is called the traditional square of opposition.

4.2 The Notion of Opposition

The notion of opposition here describes the relationship between two categorical propositions which have the same subject and predicate terms, but differ in their quality or in their quantity. Let us now discuss some of these oppositions.

4.2.1 Contradictoriness

When two propositions are contradictories, one is a denial of the other. This means that if one is true, the other will be false and if one is false, the other will be true. In other words, they cannot both be true and they cannot both be false. For example, the A proposition:

All politicians are liars

and the O proposition:

Some politicians are not liars

are contradictories. Similarly, the E proposition:

No politicians are liars

and the I proposition:

Some politicians are liars

are contradictories.

4.2.2 Contrariety

Two propositions are contraries if they cannot both be true, that is, if the truth of either one entails that the other is false. But it is possible for both of them to be false. The A proposition:

All humans are animals

and the E proposition:

No humans are animals

are contraries.

4.2.3 Sub-Contrariety

Two propositions are sub-contraries if they cannot both be false, though they might both be true. The I proposition:

Some men are Nigerians

and the O proposition:

Some men are not Nigerians

are sub-contraries.

4.2.4 Super-alternation

If a proposition is the super-altern of another, it means from the truth of that proposition, you can derive the truth of the other. For example, the A proposition:

All men are politicians

is the super-altern of the I proposition:

Some men are politicians

Similarly, the E proposition:

No men are politician

is the super-altern of the O proposition:

Some men are not politicians.

4.2.5 Sub-alternation

If a proposition is the sub-altern of another, it means from the falsity of that proposition, you can derive the falsity of the other. For example, the I proposition:

Some men are angels

is the sub-altern of the A proposition:

All men are angels.

Similarly, the O proposition:

Some men are not angels

is the sub-altern of the E proposition:

No men are angels.



The immediate inferences from the various oppositions we have discussed so far, can be summarised as follows:

1. If the A proposition is true, then E is false, I is true and O is false.
2. If the E proposition is true, then A is false, I is false and O is true.
3. If the I proposition is true, then E is false while A and O are undetermined.
4. If the O proposition is true, then A is false, while E and I are undetermined.
5. If A is false, then O is true while E and I are undetermined.
6. If E is false, then I is true while A and O are undetermined.
7. If I is false, then A will be false, E will be true and O will be true.
8. If O is false, then A is true, E is false and I is true.

ITQ

Question

What can be inferred about the truth or falsehood of the remaining propositions in each of the following if we assume the first to be true? And, If we assume it to be false?

1. i. All successful people are intelligent.
ii. no successful people are intelligent.
2. i. Some dedicated people are successful businessmen.
ii. Some dedicated people are not successful businessmen.
iii. No dedicated people are successful businessmen.

Feedback

The first step that we have to undertake if we would arrive at the right answer is that we have to identify the kind of proposition that each of the proposition falls under. Have you done that? Good!

You will discover that in the first set, (i) is an A proposition while (ii) is an E proposition. From this, we can infer rightly that if the A proposition

is true, then the E proposition will be false and when the A proposition is false, the E proposition will be undetermined. The reason for this is that, as you have learnt in this section, A and E propositions are contraries (they cannot both be true but can both be false).

In the second set of positions, (i) is an I proposition, (ii) is an O proposition, while (iii) is an E proposition. From this, we can infer that when (i) is true, (ii) will be undetermined and (iii) will be false. This is because I and O propositions are subcontraries (they cannot both be false but might both be true) while I and E propositions are contradictories (when one is true, the other would be false and vice versa). However, if the first set of proposition were to be false, then (ii) will be true and (iii) will also be true.

Further Immediate Inferences

Apart from the foregoing inferences that are drawn from the traditional square of opposition, the following immediate inferences could also be drawn:

Conversion

The process of conversion proceeds when the subject-term replaces the predicate-term and the predicate-term replaces the subject-term. In other words, a categorical proposition undergoes conversion by interchanging the subject and predicate terms. The original proposition is called the convertend, while the new proposition is called the converse. The conversion of the four categorical propositions will therefore proceed as follows:

'A'	proposition:	
	Convertend:	All men are politicians
	Converse:	All politicians are men.
'E'	proposition:	
	Convertend:	No men are politicians
	Converse:	No politicians are men
'I'	proposition:	
	Convertend:	Some men are politicians
	Converse:	Some politicians are men
'O'	proposition:	
	Convertend:	Some men are not politicians
	Converse:	Some politicians are not men.

Now, if each set of the above propositions is taken to represent a complete argument, for which of them can we say that the inference is valid?

Conversion is not valid for the 'A' proposition (except by limitation). The process of limitation involves only universal propositions, and it consists in reducing such propositions to particular propositions. Conversion by limitation for the 'A' proposition proceeds by interchanging the subject-

term with the predicate-term and then changing the quantity of the proposition from universal to particular. Thus, from the proposition 'All dogs are animals', the conclusion 'some animals are dogs' could be validly inferred, the inference being through conversion by limitation.

Conversion is however, a valid inference when applied to 'E' and 'I' propositions, but not valid for the 'O' proposition.

ITQ

State the converse, obverse, and contrapositive of the following propositions and state whether conversion, obversion, or contrapositive is valid for them.

- i. No teacher are a graduate.
- ii. All teachers are graduates.
- iii. Some teachers are graduates.
- iv. Some teachers are not graduates.

Feedback

Always remember that the first thing you should do is to break the propositions to simplified form. In that wise, you will discover that (i) is an E proposition, (ii) is an A proposition, (iii) is an I proposition while (iv) is an O proposition. Having done this, you may now proceed with other steps.

For (i):

Converse: No graduates are politicians.

Obverse: All teachers are non-graduates.

Contrapositive: No non-graduates are non-teachers.

From the foregoing, we can infer that conversion and obversion are valid for (i) while contrapositive is not valid for it.

For (ii):

Converse: All graduates are teachers.

Obverse: No teachers are non-graduates.

Contrapositive: All non-graduates are non-teachers.

From the foregoing, we can infer that conversion is not valid for (ii) while obversion and contrapositive is valid for (ii).

For (iii),

Converse: Some graduates are teachers.

Obverse; Some teachers are not non-graduates.

Contrapositive: Some non-graduates are non-teachers.

For (iii), conversion and obversion is valid while contrapositive is not valid.

For (iv),

Converse: Some graduates are not teachers.

Obverse: Some teachers are non-graduates.

Contrapositive: Some non-graduates are not non-teachers.

From this, we can infer that obversion and contrapositive are both valid for (iv) while conversion is not valid for it.

Obversion

In obverting a proposition, the subject term remains unchanged, and so does the quantity of the proposition being obverted. In obverting a proposition, we change the quality of the proposition and then replace the predicate-term by its complement. To obtain the complement of a term, simply add the prefix 'non-' to it, or if the expression already contains 'non' then delete the 'non-' from the expression to obtain its complement. The obversion of the four categorical propositions will be as follows:

'A'	proposition	
	Obvertend:	All men are politicians
	Obverse:	No men are non-politicians
'E'	proposition	
	Obvertend:	No men are politicians
	Obverse:	All men are non-politicians
'I'	proposition	
	Obvertend:	Some men are politicians
	Obverse:	Some men are not non-politicians
'O'	proposition	
	Obvertend:	Some men are not politicians
	Obverse:	Some men are non-politicians

The original proposition is called the obvertend, while the new one is called the obverse. Obversion is a valid form of inference for all the categorical propositions.

Contrapositive

To obtain the contrapositive of a given proposition, we replace its subject-term by the complement of its predicate-term and replace its predicate-term by the complement of its subject-term. The contrapositive of the four categorical propositions will proceed as follows:

'A'	proposition	
	Premise:	All men are politicians
	Contrapositive:	All non-politicians are non-men
'E'	proposition	
	Premise:	No men are politicians
	Contrapositive:	No non-politicians are non-men
'I'	proposition	
	Premise:	Some men are politicians
	Contrapositive:	Some non-politicians are non-men
'O'	proposition	
	Premise:	Some men are not politicians
	Contrapositive:	Some non-politicians are not non-men.

Contrapositive is a valid form of inference for 'A' and 'O' propositions but not valid for E and I propositions. Contrapositive is only valid for E proposition by limitation.³¹ Thus from the expression: 'No men are

politicians', the conclusion 'some non-politician is not non-men' could be validly inferred through contraposition by limitation.

Study Session Summary



Summary

In this Study Session, we discussed broadly two types of inferences involving categorical propositions. They are mediate and immediate inferences. A mediate inference proceeds from two premises to a conclusion, whereas an immediate inference proceeds from only one premise to the conclusion. We have looked at the various relationships that exist among the four categorical propositions, following the traditional square of opposition. You learnt how to determine the value of other categorical propositions once the value of the one to which they are related is given. We concluded by looking at other immediate inferences that can be drawn using the notions of conversion, obversion and contraposition, as well as the issue of the validity of inferences resulting from these relationships.

Assessment



Assessment

SAQ 4.1 (tests Learning Outcome 4.1)

What is an inference?

SAQ 4.2 (tests Learning Outcome 4.2)

Explain at least four forms of opposition.

Study Session 5

Categorical Syllogism

Introduction

This Study Session will focus on meaning and the basic features of syllogism. Also to be explored here includes mood of categorical syllogism and the figure of categorical syllogism.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

- 5.1 *define and use* correctly the term “syllogism”.
- 5.2 *describe* the features of categorical syllogism.
- 5.3 *explain* the mood of categorical syllogism.
- 5.4 *determine* the figure of a categorical syllogism.

5.1 Syllogism

Syllogism The deductive argument in which the conclusion is drawn from two premises

As stated earlier, a **syllogism** is a deductive argument in which the conclusion is drawn from two premises. A categorical syllogism, therefore, is an argument in which the conclusion (which itself is a categorical proposition) is drawn from two categorical propositions.

5.2 Features of Categorical Syllogism

A standard form categorical syllogism has the following features:

1. It must have only three terms. These are known as the major term, the minor term and the middle term. The major term is the predicate term of the conclusion of the argument. The minor term is the subject term of the conclusion of the argument. The middle term is that term that appears in both premises of the argument but not in the conclusion.
2. We can also classify the premises of a categorical syllogism into the major and minor premises. The major premise contains the major term, while the minor premise contains the minor term.
3. In writing a categorical syllogism in a standard form, the major premise is written first, followed by the minor premise and then the conclusion comes last.

ITQ

- Identify the major, minor, and middle term in this argument below.

All proteins are organic compounds whence all enzymes are

proteins, as all enzymes are organic compounds.

Feedback

- To attempt this, we have to first identify the conclusion and this will be easily done by breaking the argument into a standard form which is represented below:
All proteins are organic compounds
All enzymes are proteins
Therefore, all enzymes are organic compounds.

5.3 Mood of a Categorical Syllogism

The mood of a categorical syllogism is determined by the types of categorical propositions which it contains. It is usually represented by three letters, each standing for the form of each of the propositions which the syllogism contains. For example, in the argument: “No heroes are cowards; some soldiers are cowards; therefore, some soldiers are not heroes”. The mood will be E, I, O.

However, the mood of a categorical syllogism does not completely characterise its form. Consider the following two syllogisms:

1. All great physicians are university graduates
Some clinic owners are university graduates
Therefore some clinic owners are great physicians
2. All swimmers are egoists
Some swimmers are paupers
Therefore some paupers are egoists.

Both arguments are of the mood. ‘A I I’, but they are different in form. We can bring out this difference most clearly by displaying their logical skeleton. Let us represent the major term with ‘P’, the minor term with ‘S’ and the middle term with ‘M’. For both arguments, we then have the representation:

Argument 1:

All P is M
Some S is M
Therefore some S is P

Argument 2:

All M is P
Some M is S
Therefore some S is P.

In the first argument, whereas the middle term (M) occupies the predicate position of both premises, in the second argument, the middle term occupies the subject position of both premises. This explains the reason

for their difference in form. The correct form of a categorical syllogism is identified by naming its mood and figure.

5.4 Figure of a Categorical Syllogism

The figure of a categorical syllogism is determined by the position of the middle term in the premises of the argument. There are four possible figures a syllogism may have. They are the following:

Figure 1: This is when the middle term occupies the subject position of the major premise and the predicate position of the minor premise.

Figure 2: This is when the middle term occupies the predicate position of both premises.

Figure 3: This is when the middle term occupies the subject position of both premises

Figure 4: This is when the middle term occupies the predicate position of the major premise and the subject position of the minor premise.

Going by our earlier alphabetical representations of the terms, we can then present the different figures in the following schema:

Figure 1:

M is P
S is M
∴ S is P

Figure 2:

P is M
S is M
∴ S is P

Figure 3:

M is P
M is S
∴ S is P

Figure 4:

P is M
M is S
∴ S is P

We can only give a complete description of the form of any standard categorical syllogism by naming its mood and figure.

Study Session Summary



Summary

In this Study Session, we examined categorical syllogism. A categorical syllogism has three terms. These are the major term, the minor term and the middle term. The major term is the predicate term of the conclusion of the argument. The minor term is the subject term of the conclusion of the argument. The middle term is that term that appears in both premises of the argument but not in the conclusion.

Also, every categorical syllogism has a mood and a figure. The mood of a categorical syllogism is determined by the types of categorical propositions which it contains; while, the figure of a categorical syllogism is determined by the position of the middle term in the premises of the argument.

Assessment



Assessment

SAQ5.1 (tests Learning Outcomes 5.1 and 5.2)

What is a categorical syllogism and what are its features?

SAQ5.2 (tests Learning Outcomes 5.3 and 5.4)

Rewrite each of the following syllogism in standard form and name its mood and figure.

1. No police van are commercial vans, so no combat vans are commercial vans, since all police vans are combat vans.
2. Some conservatives are not members of the ruling party, because all members of the ruling party are looters, and some looters are not conservatives.

Study Session 6

Determining the Validity/Invalidity of Categorical System

Introduction

In this Study Session, we will examine both the validity and invalidity of categorical syllogism; and the rules needed to be able to test for a true categorical syllogism.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

6.1 *determine* the validity or invalidity of categorical syllogism.

6.2 *use* rules test for categorical syllogism.

6.1 Validity/Invalidity

The **validity** or **invalidity** of categorical syllogisms can be determined in several ways. First, the form of a syllogism may help us to determine whether or not the argument is valid (Bello, 2000). If an argument is valid, then any argument having that form will be valid and if an argument-form is invalid, then any argument having that form will also be invalid. Again, there is the method of using diagram to determine the validity or invalidity of categorical syllogism. Two of such methods were developed by John Venn and the Swiss mathematician, Leonhard Euler (Copi, 1978; Bello, 2000). Finally, there is the Rules Test for determining the validity or invalidity of categorical syllogism. Our main concern in this lecture is with this latest method, that is, the Rules Test for determining the validity or invalidity of categorical syllogism.

6.2 Rules Test for Categorical Syllogism

There are six rules, which a standard-form categorical syllogism must not violate for it to be valid. Any argument that violates one of such rules is invalid and is said to commit a formal fallacy. Let us now examine the rules and fallacies one after the other.

Rule 1

A standard form categorical syllogism must contain exactly three terms, each of which must be used in the same sense throughout the argument. If a syllogism has more than three terms, it breaks Rule 1 and commits the

fallacy of Four Terms or what in Latin is called '*Quaternio Terminorum*'. If a term is used in different senses in the same argument, the argument also breaks Rule 1 and commits the *Fallacy of Equivocation*. Consider the following argument:

*No wealthy men are social critics, because no
wealthy men are antagonists and all labour
leaders are antagonists*

The above argument breaks Rule 1 because it contains more than three terms. Precisely, it contains exactly four terms, to wit: wealthy men, social critics, antagonists and labour leaders. The argument is therefore invalid.

Rule 2

In a valid standard form categorical syllogism, the middle term must be distributed in at least one of the premises. Any syllogism whose middle term is not distributed in at least one premise breaks Rule 2 and is said to commit the fallacy of undistributed middle term. The following argument is invalid because the middle term (militants) is not distributed in any of the premises:

All indigenes of River State are militants

All Bayelsians are militants

Therefore, all indigenes of River State are Bayelsians

Rule 3

In a standard-form categorical syllogism, if any term is distributed in the conclusion of the argument, such a term must be distributed in the relevant premise. There are two different ways in which Rule 3 may be broken.

- a. In a syllogism, if the major term is distributed in the conclusion but the same term is not distributed in the major premise, the syllogism is invalid, because it breaks Rule 3 and commits the *Fallacy of Illicit Major*. The following argument breaks Rule 3 and commits the fallacy just mentioned:

Some men are good politician

No criminals are men

No criminals are good politicians

- b. If the minor term of a syllogism is distributed in the conclusion of the argument but the same term is not distributed in the minor premise, then the syllogism violates Rule 3 and commits the '*Fallacy of Illicit Minor*'. The following argument breaks Rule 3 and commits the fallacy of illicit minor:

All good politicians are men

Some criminals are not men

No criminals are good politicians

Rule 4

No standard-form categorical syllogism with two negative premises can be valid. Any categorical syllogism with two negative premises is invalid and breaks Rule 4. Such a syllogism is said to commit the *Fallacy of Exclusive Premises*. The following argument breaks Rule 4 and commits the *Fallacy of Exclusive Premises*:

Some men are not good politicians

No criminals are men.

No criminals are good politicians

Rule 5

If any of the premises of a categorical syllogism is negative, the conclusion must be negative for the syllogism to be valid. Any argument that breaks this rule commits the *Fallacy of Drawing an Affirmative Conclusion from a Negative Premise*. The following argument breaks Rule 5 and commits the fallacy just mentioned:

Some men are not good politicians

Some criminals are men.

All criminals are good politicians.

Rule 6

No valid standard-form categorical syllogism with a particular conclusion can have two universal premises. In other words, if the conclusion of a valid categorical syllogism is particular, one of the premises must be particular. Any syllogism, which violates Rule 6, is said to commit the *Existential Fallacy*. The following argument breaks Rule 6 and commits the 'Existential Fallacy':

All men are good politicians

No criminals are men.

Some criminals are not good politicians.



To test the validity or invalidity of categorical syllogisms by using the rules, what we do is to write the argument in standard form by writing the major premise first, followed by the minor premise and then the conclusion. After this, we apply the rules to the argument one after the other. If the argument passes all the rules, then it is valid, but if an argument fails any (at least one) of the rules, then the argument is invalid.

ITQ

Question

- What does it mean for a categorical syllogism to be valid?

Feedback

- A categorical syllogism is said to be valid if it is not possible for its premises to be true and its conclusion false.

Study Session Summary



Summary

In this Study Session, we treated the six rules, which a standard-form categorical syllogism must not violate for it to be valid. Any syllogism that violates one of such rules is invalid and is said to commit a corresponding fallacy. To test whether or not a categorical syllogism is valid, what we do is to first write the argument in standard form by writing the major premise first, followed by the minor premise and then the conclusion. After this, we apply the rules to the argument one after the other. If the argument passes all the rules, then it is valid, but if an argument fails any (at least one) of the rules, then the argument is invalid.

Assessment



Assessment

SAQ 6.1 (tests Learning Outcomes 6.1 and 6.2)

Mention the rules broken and the fallacies committed by each of the syllogisms which are invalid:

1. Some snakes are not poisonous animals but all snakes are reptiles, therefore some poisonous animals are not reptiles.
2. All people who live in London are people who speak English and all people who speak English are people who like it. We may conclude then that people who live in London are people who like it.
3. All chocolate éclairs are fattening foods, because all chocolate éclairs are rich desserts, and some fattening foods are not rich desserts.
4. No coal-tar derivatives are nourishing foods because all artificial dyes are coal-tar derivatives, and no artificial dyes are nourishing foods.

Study Session 7

Rational Statements and Arguments Involving Relations

Introduction

In this Study Session, we will discuss relational propositions. We will also examine the attributes of relations and how this information can be used to determine the validity or invalidity of arguments involving relational propositions.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

7.1 *define* and *use* correctly the term “relational propositions”.

7.2 *explain* the different attributes of relation.

7.3 *determine* the validity or invalidity of arguments involving relational propositions or statements.

7.1 Relational Propositions or Statements

Relational propositions

A statement that employ terms that express a relation.

Relational propositions are propositions or statements that employ terms that express a relation. A term is said to express a relation if such a term requires more than one individual, object or entity, to make complete sense (though it is possible for an entity to express a relation to itself). The following are examples of words and phrases that express relations: father, brother, cousin, sister, married to, lover of, enemy, teacher, equal to, has the same weight as, bigger than, is the mate of, and richer than.

A relational term may express a one-place relation, or it may be two-place, three-place, or four-place, depending on the number of individuals required for the sentence expressing the relation to make meaningful sense. For example, the proposition, ‘Adebayo is snub-nosed’, expresses a one-place relation, while the proposition, ‘Bello is the teacher of Offor’, expresses a two-place relation because it requires two individuals to make complete sense. Propositions like ‘Cameroun is between Nigeria and Ghana’ and ‘The landlord traded his house rent to the tenants for second-hand clothes’ express a three-place and four-place relation respectively. A proposition that expresses a one-place relation is said to be ‘monadic’ (Bello, 2000). Where a proposition expresses a relation between two or more entities, such a proposition is said to be polyadic (Bello, 2000). If it expresses a two-place relation, it is (binary), three-place (triadic), or four-place (tetradic). A polyadic relation also has a direction. It is either uni-directional and therefore irreversible or bi-directional and reversible. An example of a uni-directional proposition is ‘Bayo is the father of Bimpe’.

On the other hand, the proposition 'Babaginda is the same age as Obasanjo' is bi-directional.

7.2 Attributes of Relations

Attributes of relations help to describe the way relational terms behave in propositions, and the way relational terms behave enables us to determine the validity or invalidity of arguments involving relational propositions. To properly understand the way relational terms behave, let us learn a little more about some attributes of relation.

- 1) A relation between two entities may be symmetrical, asymmetrical or non-symmetrical. When a relation is symmetrical, it means that if one entity 'A' has a relation to 'B', 'B' must also have the same relation to 'A'. For example, if a proposition says that 'Peter is married to Jane', it follows that Jane must be married to Peter. If a proposition also says that 'Peter is the same age as Andrew', it follows that Andrew must be the same age as Peter.

On the other hand, when it is the case that an entity 'A' has a relation to another 'B', but 'B' cannot have the same relation to 'A', then such a relation is asymmetrical. If, for example, a proposition says that 'Peter is the father of Matthew', it follows that Matthew cannot at the same time have the relation (of being father of) to Peter. All such relations as expressed by phrases like 'the husband of', 'is taller than' e.t.c, are said to be asymmetrical.

However if the situation is such that an entity 'A' has a relation to another 'B', but 'B' may or may not have the same relation to 'A', then the relation is said to be non-symmetrical. For instance, if a person 'A' is the brother of another person 'B', 'B' may or may not be the brother of 'A'.

- 2) Again, a relation may be transitive, intransitive or non-transitive. A transitive relation is such that if an entity 'A' has that relation to another 'B' and 'B' has the same relation to yet another 'C', then 'A' must have the same relation to 'C'. The expression, 'Andy is taller than John and John is taller than Patrick, then Andy is taller than Patrick expresses a transitive relation.

On the other hand, where an entity 'A' has a relation to another 'B' and 'B' has the same relation to 'C', but 'A' cannot have the same relation to 'C', then the relation is intransitive. For example, a proposition like 'Ibadan is five miles to the south of Lagos and Lagos is five miles to the south of Ijebu-Ode' is intransitive.

However, when a relation is such that if one thing 'A' has that relation to another 'B' and 'B' has the same relation to 'C', but 'A' may or may not have the same relation to 'C', then the relation is said to be non-transitive. Examples of phrases that express non-transitive relations are 'friend of' and 'enemy of'.

- 3) Finally, a relation may either be reflexive, irreflexive or non-reflexive. When a relation is reflexive, it means that a thing can have such a relation to itself. For example, it is possible for

someone to be the same age as himself or to have the same weight as himself. However, when it is not possible for an entity to have a particular relation to itself, that relation is said to be irreflexive. No one, for instance, can be said to be the father of himself or richer than himself. But, when a relation is such that an individual entity may or may not have such a relation to himself or itself, then such a relation is said to be non-reflexive. For instance, somebody may or may not love or admire himself.

ITQ

Question

- A relational term may express all of the following relations except
 - a) Two-place relation
 - b) Three-place relation
 - c) No-place relation
 - d) Four-place relation

Feedback

- A relational term may express a one-place relation and more depending on the individuals involved but it can never express a no-place relation because there must be at least one person involved.

7.3 Validity/Invalidity of Arguments Involving Relations

ITQ

Question

- In Study Session 6, we learnt how to test validity or invalidity of categorical syllogisms, can you highlight the three ways that we used to test the validity of categorical syllogism.

Feedback

- First, we can use the form of the categorical syllogism.
- Secondly, we can also use the Venn or Euler diagram.
- Thirdly, we can use the rule test.

We have seen how relational terms behave in propositions. Therefore, when we are faced with the task of analyzing (that is, determining the validity or invalidity of) arguments involving relational propositions, we have to be careful enough to remind ourselves of the following questions:

1. What is the relational term or terms involved in the argument?
2. What is the attribute(s) of such term or terms?
3. Has the relational term behaved the way it ought to behave normally in the argument under consideration?

If the answer to question 3 above is yes, then the argument in question is valid, if no, then the argument is invalid.

Consider the following argument:

Francis is the same weight as Florence

Florence is the same weight as Mercy

Therefore Francis is the same weight as Mercy.

In this example, the relational term is 'has the same weight as'. It is a transitive relation which says that if an entity 'A' has a relation to another 'B' and 'B' has the same relation to 'C', then 'A' must have the same relation to 'C'. The relational term in the argument has behaved to type and the argument is therefore valid. Consider, however, this other argument:

Adebanjo is taller than Saheed

Therefore Saheed is taller than Adebanjo.

In this second argument, the relational term 'taller than' is asymmetrical. A relation is asymmetrical if it is such that if one entity 'A' has that relation to another 'B', 'B' cannot have the same relation to 'A'. In the above argument, however, the relational term is used as if it is symmetrical. In other words, the relational term 'taller than' has not behaved in the usual manner in the argument under consideration. The argument is therefore invalid.

ITQ

Question

- What does it mean for a polyadic relation to be unidirectional or bi-directional?

Feedback

- A uni-directional relation is an irreversible one and as such the relation only applies to one and not to be shared while it is bi-directional if the relation is reversible and thus can be shared by the entities referred to.

Study Session Summary



Summary

In this Study Session, you learnt that relational propositions are statements which contain terms that express a relation. A term is said to express a relation if such a term requires more than one individual, object or entity, to make complete sense (though it is possible for an entity to express a relation to itself). When an argument contains relational propositions or statements, such an argument is called a relational argument. There are certain attributes of relations that help to describe the way relational terms should behave, and the way relational terms behave enables us to determine the validity or invalidity of arguments involving relational propositions. A relation can be:

- 1) symmetrical, asymmetrical or non-symmetrical,
- 2) transitive, intransitive or non-transitive, and

3) reflexive, irreflexive or non-reflexive.

Assessment



Assessment

SAQ7.1 (tests Learning Outcomes 7.1 and 7.2)

Identify the relational term in the following propositions:

Every girl at the party danced with every boy who was there.

Caleb is the ancestor of Ezra.

The teacher is in love with the principal.

SAQ7.2 (tests Learning Outcome 7.3)

Determine the validity or invalidity of each of the following relational arguments. Give reasons for your answer:

Agnes is shorter than Mary.

Mary is shorter than Helen.

Therefore, Agnes is shorter than Helen.

Jingo is older than Bongo

Therefore, Bongo is older than Jingo.

Jingo is an enemy of Bongo

Bongo is an enemy Bamanga

Therefore, Bamanga is an enemy to Jingo.

Study Session 8

Formal Argument in Artificial Language

Introduction

In this Study Session, we will specifically examine the nature of words and phrases that we use in forming compound statements. These words and phrases are called 'logical connectives'. Also, we will observe the logical symbols representing the various connectives.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

8.1 *discuss* the meaning of logical connectives.

8.2 *highlight* the logical symbols that represent connectives.

8.1 Logical Connectives or Logical Constants

In logic, we can identify two kinds of statements. On the one hand, we have simple or atomic statements. A statement is simple if it has no other statement as part of its component. For example, the statement 'it is raining' is a simple statement. On the other hand, we have compound or molecular statements. A compound statement is made up of at least two other statements. An example of a compound statement is 'either it is raining or the ground is wet'. Compound or molecular propositions are formed by using what we call logical constants or connectives, and there are five of such connectives.

ITQ

Question

- What is the difference between a simple or atomic statement and a compound or molecular statement?

Feedback

- A simple or atomic statement is one that has no other statement as part of its component while a molecular or compound statement is one which has another statement as part of its component.

8.2 Forms of Connectives

8.2.1 Conjunction

A conjunction consists of two propositions joined by words like ‘and’, ‘but’, ‘though’ and their equivalents. The two parts of a conjunction are called conjuncts. The logical symbol that represents all forms of conjunctions is the dot (\cdot) sign. The expression ‘Peter is in Lagos’ and ‘John is in Kaduna’ is a conjunction. Now, if we represent the statements: ‘Peter is in Lagos’ and ‘John is in Kaduna’ as ‘P’ and ‘J’, then the conjunction of both statements will be symbolised as ‘ $P \cdot J$ ’. The conditions under which expressions involving a conjunction can be true or false can be expressed using the following table:

P	\cdot	J
T	T	T
T	F	F
F	F	T
F	F	F

The above table reveals that a conjunction is true only when both conjuncts are true and false when at least one of the conjuncts is false.

8.2.2 Disjunction

A disjunction is a compound proposition in which two statements are joined by the connective ‘or’ or its equivalent. The two parts of a disjunction are called disjuncts, and the logical symbol that represents the disjunction is the wedge (\vee). The expression ‘Peter is in Lagos’ or ‘John is in Kaduna’ is a disjunction and is symbolised as ‘ $P \vee J$ ’. The truth conditions for a disjunction can be expressed as follows:

P	\vee	J
T	T	T
T	T	F
F	T	T
F	F	F

From the above table, a disjunction is true when at least one of the disjuncts is true. A disjunction is only false when both disjuncts are false.

8.2.3 Conditional

A conditional consists of two propositions joined by the connective ‘If ... then ...’ or its equivalent. The statement on the left hand side of the conditional, that is, the statement between the ‘If’ and the ‘then’ is called the ‘antecedent’. The statement on the right hand side of the conditional, that is, the statement following the ‘then’ is called the ‘consequent’. The logical symbol that represents the conditional is the ‘horse shoe’ sign (\supset).

The expression ‘If Peter is in Lagos’ then ‘John is in Kaduna’ is a conditional statement and is symbolised as ‘ $P \supset J$ ’. The truth-condition of a conditional can be represented in the following table:

P	\supset	q
T	T	T
T	F	F
F	T	T
F	T	F

The above table reveals that a conditional statement is only true when the antecedent is true and the consequent is false.

8.2.4 Bi-conditional

When two propositions are joined by the connective ‘... if and only if ...’, then the expression is called a bi-conditional. The two parts of a bi-conditional are called ‘components’ and the logical sign that represents the bi-conditional is the triple bar (\equiv). The expression ‘Peter is in Lagos’ if and only if ‘John is in Kaduna’ is a bi-conditional statement and is symbolised as ‘ $P \equiv J$ ’. The conditions under which expressions involving a bi-conditional can be true or false are shown below:

P	\equiv	J
T	T	T
T	F	F
F	F	T
F	T	F

The above table reveals that a bi-conditional statement is true either if both components are true or if both components are false. A bi-conditional is false if both components have different truth-values.

8.2.5 Negation

If someone says ‘It is raining’ and another person says ‘It is not raining’, the second person has negated what the first person said. A negation is a sentence which contains the negation sign ‘not’ or its equivalent. Ordinarily, a negation looks like a simple statement, but logically, it has a compound structure. The logical sign that represents the negation is the curl sign (\sim). When a statement is negated, the negation sign is placed immediately before the statement being negated. ‘John is not in Kaduna’ is an example of a negation and is symbolized as ‘ $\sim J$ ’. When a statement is negated, it takes on the opposite value. In other words, when a statement, say ‘ P ’, is true, ‘ $\sim P$ ’ will be false and when ‘ $\sim P$ ’ is true, then ‘ P ’ will be false. This is displayed in the table below:

P	$\sim P$
T	F
F	T

Study Session Summary



Summary

In this Study Session, we examined the nature and truth conditions of words and phrases that we use in joining two or more simple statements to form compound statements. These words and phrases have been grouped into five categories of connectives. These are the 'conjunction', 'disjunction', 'conditional', 'bi-conditional' and 'negation'.

Assessment



Assessment

SAQ8.1 (tests Learning Outcome 8.1)

What is the use of logical connectives?

SAQ8.2 (tests Learning Outcome 8.2)

What does each of these logical connectives represent:

- 1.
- 2.
- 3.
- 4.
- 5.

Study Session 9

Symbolising Statement and Arguments in Propositional Logic

Introduction

In this Study Session, we will be discussing how to symbolise statements and arguments in propositional logic. Our main focus here is to: explore how to symbolise statements, determine truth-value of propositions and symbolise arguments.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

9.1 *symbolise* statements.

9.2 *determine* the truth-value of propositions.

9.3 *symbolise* arguments.

9.1 Symbolising Statements

In propositional logic, we make use of special symbols and alphabetical letters when symbolising statements. Among the alphabetical letters, we have capital letters 'A' to 'Z', also known as propositional constants, which are used to represent actual propositions. We also have small letters 'p' to 'w', also known as propositional variables. Variables do not stand for actual proposition but may be used to represent any proposition whatsoever. Among the symbols are those representing logical connectives as well as the various punctuation marks.

In English language, punctuation is absolutely required if complicated statements are to be made clear. In writing a letter to our loved ones, for instance, we have to make use of different punctuation marks; otherwise, our sentences would remain highly ambiguous. Punctuation is equally necessary in mathematics. For instance, the question $2 \times 3 + 5$ can be interpreted as either 11 or 16; the first answer when the question is punctuated as $(2 \times 3) + 5$, and the second, when it is punctuated as $2 \times (3 + 5)$.

Punctuation is also required in the language of symbolic logic for many reasons. First, where many simple statements are compounded into more complicated ones by various connectives, the use of punctuation marks enables us to know the dominant connective in the expression. Again, punctuations help to remove ambiguity from expressions. For instance, the expression $p \bullet q \vee r$ will remain ambiguous and can be interpreted

differently unless it is properly punctuated. It could mean the conjunction of 'p' with the disjunction of 'q' with 'r' $[p \wedge (q \vee r)]$, or it might mean a disjunction whose first disjunction is the conjunction of 'p' and 'q' $[(p \wedge q) \vee r]$. That the different ways of punctuating this statement do make a difference can be seen from the following case in which 'p' is false while 'q' and 'r' are both true:

$(p \wedge q) \vee r$	and	$p \wedge (q \vee r)$
F F T T T		F F T T T

From the above, the first statement is true and the second false. Here, the difference in punctuation makes all the difference between the truth of the first statement and the falsity of the second, for it is possible for the same set of ambiguous statements to have different values, depending on how they are punctuated. In symbolic logic, we make use of three punctuation marks. These are the brackets (), parenthesis [] and braces { }. Let us now symbolise the following compound statements by using letters A, B, C, and D to abbreviate:

- i. Anambra wins its conference championship
 - ii. Benin wins its conference championship
 - iii. Calabar wins the superbowl and
 - iv. Delta wins the superbowl.
1. Either Anambra wins its conference championship and Benin wins its conference championship or Calabar wins the superbowl: $(A \wedge B) \vee C$
 2. Anambra wins its conference championship and either Benin wins its conference championship or Delta does not win the superbowl: $A \wedge (B \vee \neg D)$
 3. Anambra and Benin will not both win their conference championships but Calabar and Delta will both not win the superbowl: $\neg(A \wedge B) \wedge (\neg C \wedge \neg D)$
 4. Either Anambra or Benin will win its conference championships but neither Calabar nor Delta will win the superbowl: $(A \vee B) \wedge (\neg C \vee \neg D)$
 5. Either Calabar or Delta will win the superbowl but they will not both win the superbowl: $(C \vee D) \wedge \neg(C \wedge D)$
 6. Both Anambra and Benin win their conference championships only if Calabar does not win the superbowl: $(A \wedge B) \supset \neg C$

7. Anambra wins its conference championships if either Calabar wins the superbowl or Delta wins the superbowl: $(C \vee D) \supset A$
8. Anambra wins its conference championship and either Calabar or Delta wins the superbowl: $A \wedge (C \vee D)$
9. If Anambra does not win its conference championship, then it is not the case that either Calabar or Delta wins the superbowl: $\neg A \supset \neg(C \vee D)$
10. Anambra wins its conference championship only if either Calabar or Delta does not win the superbowl: $A \supset \neg(C \vee D)$

ITQ

Question

- What is the role of punctuation in symbolic logic?

Feedback

- They help us in removing ambiguity in statements. Also, in the case of molecular statements, punctuations help us to know the dominant connective in the statement.

9.2 Determining the Truth-Value of Propositions

Any compound statement constructed from simple statements using logical connective(s) has its truth-value completely determined by the truth or falsehood of its component simple statements, as well as the truth-condition of the connectives involved. In determining the truth value of compound statements, we always begin with their inner most components and work outwards. For example, let us assume that the statements represented by A and B in the following expressions are true, while those represented by X and Y are false. On the basis of this information, and following our knowledge of the truth-conditions of logical connectives, let us determine which of them are true and which of them are false:

$$1. \quad \neg A \vee \neg X$$

F T T T F

$$2. \quad A \vee (X \wedge Y)$$

T T F F F

$$3. \quad (A \vee B) \wedge (X \vee Y)$$

T T T F F F F

$$4. \quad A \wedge [X \vee (B \wedge Y)]$$

T F F F T F F

$$5. \quad \neg\{\neg[\neg(A \bullet \neg X) \bullet \neg A] \bullet \neg X\}$$

F T F T T T F F F T T T F

$$6. \quad [(X \bullet A) \vee \neg Y] \vee \neg[(X \bullet A) \vee \neg Y]$$

F F T T T F T F F F T T T F

$$7. \quad [X \vee (A \bullet Y)] \vee \neg[(X \vee A) \vee (X \vee Y)]$$

F F T F F F F F T T T F F F

$$8. \quad [X \bullet (\neg A \supset Y)] \equiv \neg[(X \vee A) \vee (\neg X \supset \neg Y)]$$

F F F T T F T F F T T T T F T T F

$$9. \quad A \supset [X \equiv (B \supset Y)]$$

T T F T T F F

$$10. \quad \neg[\neg(\neg A \equiv B) \bullet \neg(X \equiv \neg Y)]$$

F T F T F T T T F F T F

9.3 Symbolising Arguments

In symbolising an argument, we write each of the premises (if the argument has more than one premise) on separate lines. The conclusion is usually preceded by three dots of a triangular shape, and it is written on the last line following the last premise.

ITQ

Question

- What are the steps in symbolizing arguments?

Feedback

- a) Write out each of the premise in the argument on a single line and the last line should be the conclusion. The conclusion should be preceded by the therefore operator (\therefore).
- b) Use the notations you are given to symbolize the statement.

The following is an example of argument and its symbolisation:

If the seed catalogue is correct, then if the seeds are planted in April then the flowers bloom in July. The seeds are planted in April. Therefore, if the flowers do not bloom in July, then the seed catalogue is not correct.

Suggested notations:

S: the seed catalogue is correct
 A: the seeds are planted in April
 F: the flowers bloom in July

$$S \supset (A \supset F)$$

A

$$\neg F \supset \neg S$$

Let us now symbolise the following arguments using the suggested letters:

1. If Ed wins the first prize, then Fred wins the second prize, and if Fred wins second prize, then George is disappointed. Either Fred does not win the second prize or George is not disappointed. Therefore, Ed does not win the first prize.

Suggested notations:

E: Ed wins first prize

F: Fred wins second prize

G: George is disappointed

$$(E \supset F) \bullet (F \supset G)$$

$$\neg F \vee \neg G$$

$$\neg E$$

2. If the weather is warm and the sky is clear then either we go swimming or we go boating. It is not the case that if the sky is clear then we go swimming. Therefore, if we do not go boating then the weather is not warm.

Suggested notations:

W: the weather is warm

S: the sky is clear

G: we go swimming

B: we go boating

$$(W \bullet S) \supset (G \vee B)$$

$$\neg(S \supset G)$$

$$\neg B \supset \neg W$$

3. If either algebra is required or geometry is required then all students will study mathematics. Algebra is required and Trigonometry is required. Therefore all students will study mathematics.

Suggested notations:

A: algebra is required

G: geometry is required

S: all students will study mathematics

T: Trigonometry is required

$$(A \vee G) \supset S$$

$$A \bullet T$$

$$\therefore S$$

Study Session Summary



Summary

In this Study Session, we focused on symbolising statements and arguments in propositional logic and made use of special symbols and alphabetical letters. Among the alphabetical letters are capital letters 'A' to 'Z' (or propositional constants), representing actual propositions, and small letters 'p' to 'w' (or propositional variables), representing any proposition whatsoever. Among the symbols are those representing logical connectives as well as the various punctuation marks. In determining the value of a compound statement, we are guided by the truth-value of its component simple statements, as well as the truth-condition of the logical connectives in the statement.

Assessment



Assessment

SAQ 9.1 (tests Learning Outcome 9.1)

Symbolize the following compound statements using the suggested letters:

- i. If either Brazil wins the tournament or its ranking in world football drops then Spain would be the new number 1 footballing country in the world. (B, R, S)
- ii. Spain would be the number 1 footballing country in the world if and only if Brazil does not win the tournament. (S, B)
- iii. If Nigeria beats Brazil at the preliminary stages then Brazil wins the tournament if and only if its ranking in the world football drops or Spain is not the number 1 footballing country in the world. (N, B, R, S)

SAQ 9.2 (tests Learning Outcome 9.2)

If A and B are true statements while X and Y are false statements, which of the following statements are true and which are false?

- i. $(A \rightarrow X) \rightarrow (Y \vee B)$
- ii. $(B \vee A) \rightarrow (B \rightarrow [Y \rightarrow Y])$

SAQ9.3 (tests Learning Outcome 9.3)

Symbolize this argument using the suggested letters:

Either Bongo attends the party or Bongo was not invited to the party. If the organizers want Bongo at the party then Bongo was invited to the party. Bongo did not attend the party. Therefore, if the organizers wanted Bongo at the party and Bongo was not at the party, then something is fishy.

Suggested notations:

B: Bongo attended the party.

I: Bongo was invited to the party.

O: the Organizers want Bongo at the party.

S: something is fishy.

Study Session 10

Uses of the Truth Table

Introduction

In this Study Session we will be exploring how to use truth table to show statements that are tautologous, contradictory or contingent. We will also use truth table to show the pairs of statements that are logically equivalent.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

10.1 *discuss* the meaning of truth table.

10.3 *distinguish* a tautologous statement from a contradictory or contingent statement.

10.3 *use* the truth table to work out which pairs of statements are logically equivalent.

10.1 The Meaning of Truth Table

Truth table A table containing an array of 'Ts' and 'Fs' in columns and rows

A **truth table** is a table containing an array of 'Ts' and 'Fs' in columns and rows. The number of columns a table should have is a function of the type of compound statement under consideration, while the number of rows is determined by the number of simple statements that make up the compound statement. For instance, where you have two simple statements forming a compound statement, the number of rows will be four, whereas if the statements are three, the number of rows will be eight. Our simple explanation for this formula is this: every statement can only have two (2) possible values; it is either a statement is true or it is false. To determine the number of rows a truth table should have for a particular expression, we count the number of simple statements in that expression and then raise 2 (representing the possible values) to that number. Where you have two simple statements, it will be 2^2 , which is $2 \times 2 = 4$. It means for any compound statement having two simple statements, the truth table will have four (4) rows. Where you have three simple statements, it will be 2^3 , which is $2 \times 2 \times 2 = 8$, and where you have four or five simple statements, it will be 2^4 , which is $2 \times 2 \times 2 \times 2 = 16$, or 2^5 which is $2 \times 2 \times 2 \times 2 \times 2 = 32$ respectively. Each row in a truth table stands for a possible world of interpreting the statement or argument as the case may be. In allocating values to statements in the truth table, it is advisable to start with 'T' instead of 'F'. This is, however, a matter of convention, as one would still arrive at the same result if one starts with 'F'. The most important thing is to be consistent. The allocation must be done in such a way that the first simple statement will share the two values in equal proportion in all the rows. For instance, where we have three statements,

which will give us eight (8) rows, the first statement will have four (4) 'Ts' in the first four rows and four (4) 'Fs' in the last four rows. The second statement will have two (2) 'Ts' in the first two rows, two (2) 'Fs' in the next two rows, two 'Ts' in the fifth and sixth rows and two 'Fs' in the last two rows. The third and last statement will have 'T' in the first row, 'F' in the second row and continue in that order until it gets to the last row. In fact, in the allocation of values to statements in a truth table, the last statement must end with 'T' in the first row, 'F' in the second row and must continue in that order until it gets to the last row of the table. Let us now learn how to use the truth table to show statements that are tautologous, contradictory or contingent.

10.2 Tautology, Contradiction and Contingent Truth

A statement that is true under all interpretations or in all possible worlds is called a tautology. When it is false under all interpretations, it is contradictory. A statement is contingent when it is neither tautologous nor contradictory. Such a statement will be true in some rows and false in some others. In the final interpretation of statements as tautologous, contradictory or contingent, we check the values in all the rows under the column for the major logical connective for that expression. Let us now work out the following statements to see which of them is tautologous, contradictory or contingent:

$$1. p \vee \neg p$$

T T F T

F T T F

This statement is tautologous.

$$2. p \bullet \neg p$$

T F F T

F F T F

This statement is contradictory.

$$3. p \vee q$$

T T T

T T F

F T T

F F F

This statement is contingent.

$$4. [(p \supset q) \supset p] \supset p$$

T T T T T T T

T F F T T T T

F T T F F T F

F T F F F T F

This statement is tautologous.

$$5. (\neg p \bullet q) \bullet (q \supset p)$$

F T F T F T T T

F T F F F F T T

T F T T F T F F

T F F F F F T F

This statement is contradictory.

ITQ

Question

- When would a truth table be said to be a tautology, contradiction, or contingent truth?

Feedback

- A truth table is said to be a tautology if there is no row where the central connective is false. It is a contradiction when there is no row in the truth table where the central connective is true while it is a contingent truth if it is neither tautologous or contradictory.

10.3 Logical Equivalence

When two statements are logically equivalent, it means they have the same logical force or have the same truth value, and so can easily replace one another. Put differently, two statements are said to be logically equivalent when they are either both true or they are both false. The sign for logical equivalence is the same sign used for the bi-conditional, that is the triple bar (\equiv). To test if two statements are logically equivalent, we join the two statements together with the bi-conditional sign before working out the truth table. For two statements to be logically equivalent, we check the values in all the rows under the column for the major logical connective (this time, the triple bar) for that expression. If all the values are true, then both statements are logically equivalent, otherwise, they are not. Following this explanation, we can agree with A. G. A. Bello that “if two statements are expressed as a bi-conditional, then if the resulting expression is a tautology, then the two statements are logically equivalent (Bello, 2000). Let us now use the truth table to work out which of the following pairs of statements are logically equivalent:

$$1. (\neg p \supset \neg q) \text{ and } (p \vee \neg q)$$

To test whether or not the two statements are logically equivalent, we write out the two statements as a bi-conditional thus:

$$(\neg p \supset \neg q) \equiv (p \vee \neg q)$$

FT T TF T T T FT

FT T FT T T T TF

TF F TF T F F FT

TF T FT T F T TF

The two statements are logically equivalent.

2. $(p \supset q)$ and $(\sim p \supset \sim q)$

$(p \supset q) \equiv (\sim p \supset \sim q)$

T T T T FT T FT

T F F F FT T TF

F T T F TF F FT

F T F T TF T TF

The two statements are not logically equivalent.

ITQ

Question

What does it mean for two pairs of arguments to be logically equivalent?

Feedback

Two pairs of arguments can be said to be logically equivalent when they both have the same truth value throughout the rows in the truth table.

Study Session Summary



Summary

In this Study Session, you learnt that a truth table is a table containing an array of 'Ts' and 'Fs' in columns and rows. The number of columns a table should have is a function of the type of compound statement under consideration, while the number of rows is determined by the number of simple statements that make up the compound statement. There are many uses of the truth table. The truth table can be used to exhibit the truth-conditions of logical connectives; it can be used to characterise statements as tautologous, contradictory or contingent. It can also be used to show pairs of statements that are logically equivalent.

A statement that is true in all possible worlds is called a tautology, while a statement that is false in all possible worlds is said to be contradictory. A statement is contingent when it is neither tautologous nor contradictory.

Assessment



Assessment

SAQ10.1 (tests Learning Outcomes 10.1 and 10.2)

Use truth tables to determine if each of the following statements is tautologous, contradictory or tautologous:

- i. $(q \rightarrow r) \rightarrow (r \vee s)$
- ii. $[p \rightarrow (p \rightarrow q)] \rightarrow q$
- iii. $p \rightarrow [p \rightarrow (q \rightarrow q)]$

SAQ10.2 (tests Learning Outcome 10.1)

Use truth tables to determine which of the following pairs of expressions are logically equivalent:

- i. $(p \rightarrow q)$ and $q \rightarrow p$
- ii. $[(p \rightarrow q) \rightarrow r]$ and $[(q \rightarrow p) \rightarrow r]$

Study Session 11

Basic Valid Argument-Forms

Introduction

In this Study Session, we will examine the concept of valid argument-forms and distinguish it from the invalid ones.

Learning Outcomes



Outcomes

When you have studied this session, you should be able to:

11.1 *identify* the valid argument-form.

11.2 *distinguish* valid arguments from invalid ones.

11.1 Valid Argument-Forms

Let us start by examining nine argument-forms that are valid. *Any argument that takes on any of these forms will be valid. This is because if an argument-form is valid, any argument having that form will also be valid.*

11.2 Argument-Forms

11.2.1 Modus Ponens

Premise one: If it rains, then the ground is wet

Premise two: It rains

Conclusion: Therefore, the ground is wet

Symbolised as:

$p \supset q$

p

$\therefore q$

What this means is that given a conditional statement as a first premise, and given also another statement, which is the same as the antecedent of the first premise, we can then infer the consequent of the first premise as a conclusion.

11.2.2 Modus Tollens

Premise one: If it rains, then the ground is wet

Premise two: The ground is not wet

Conclusion: Therefore it did not rain

Symbolised as:

$$p \supset q$$

$$\sim q$$

$$\therefore \sim p$$

What Modus Tollens is saying is that given a conditional statement as the first premise and given also as the second premise, a denial of the consequent of the first premise, we can conclude by negating the antecedent of the first premise.

ITQ

Question

- What is the difference between modus ponens and modus tollens?

Feedback

- In a Modus ponens, you affirm the antecedent and then affirm the consequent while in a Modus Tollens, you deny the consequent and thus deny the antecedent.

11.2.3 Hypothetical Syllogism

Premise one: If it rains, then the ground is wet

Premise two: If the ground is wet, then there will be flood

Conclusion: Therefore, if it rains, then there will be flood

Symbolised as:

$$p \supset q$$

$$q \supset r$$

$$\therefore p \supset r$$

What is implied here is that if we have two conditional statements as the first and second premise of an argument, and it is also the case that the consequent of the first premise is the same as the antecedent of the second premise, then we can conclude that the antecedent of the first premise implies the consequent of the second premise.

11.2.4 Disjunctive Syllogism

Premise: Either it rains or the ground is wet

Premise two: It is not raining

Conclusion: Therefore, the ground is wet

Or

Premise one: Either it rains or the ground is wet

Premise two: The ground is not wet

Conclusion: Therefore it is raining

Symbolised as:

p	\vee	q		p	\vee	q
$\sim p$			or	$\sim q$		
$\therefore q$				$\therefore p$		

What this is saying is that given a disjunction as a first premise, and given a second premise, which negation of any of the disjuncts is, we can conclude by affirming the other disjunct.

ITQ

Question

- Differentiate between hypothetical syllogism and disjunctive syllogism.

Feedback

- Hypothetical syllogism involves three conditional statements where the first two are the premises and the third is the conclusion. The consequent of the first premise is the antecedent of the second premise and so we can infer that the antecedent of the first premise infers the consequent of the second premise.
- Disjunctive syllogism involves disjunctive statement as the first premise. The denial of one of the disjuncts as the second premise leads to an affirmation of the other disjunct as the conclusion.

11.2.5 Simplification

Premise: It rains and the ground is wet

Conclusion: Therefore it rains

Or

Premise: It rains and the ground is wet

Conclusion: Therefore the ground is wet

Symbolised as:

$p \cdot q$	or	$p \cdot q$
$\therefore p$		$\therefore q$

What this is saying is that from a conjunction of two statements, you can conclude by affirming any of the conjuncts.

11.2.6 Addition

Premise: It rains

Conclusion: Therefore, it rains or the ground is wet

Or

Premise one: It rains

Premise two: the ground is wet

Conclusion: Therefore, it rains or the ground is wet

Symbolised as:

p	or	p
$\therefore p \vee q$		q
		$\therefore p \vee q$

What this means is that from a statement, you can form a disjunction of which that statement is a part or you can form a disjunction of two existing statements.

11.2.7 Conjunction

Premise one: It rains

Premise two: the ground is wet

Conclusion: Therefore, it rains and the ground is wet

Symbolised as:

p
q
 $\therefore p \cdot q$

What this means is that from two separate statements, you can derive their conjunction as a conclusion.

11.2.8 Constructive Dilemma

Premise one: If it rains then the ground is wet, and if there is earthquake then there will be flood

Premise two: Either it rains or there is earthquake

Conclusion: Therefore, either the ground is wet or there will be flood

Symbolised as:

$(P \supset q) \cdot (r \supset s)$
 $p \vee r$
 $\therefore q \vee s$

What this argument form is saying is that given a conjunction of two conditional statements as a first premise, and given also as a second premise, the disjunction of their respective antecedents, we can then infer the disjunction of their consequents as our conclusion.

11.2.9 Destructive Dilemma

Premise one: If it rains, then the ground is wet and if there is earthquake, then there is flood

Premise two: Either the ground is not wet, or there is no flood

Conclusion: Therefore, either it does not rain, or there is no earthquake

Symbolized as:

$$(p \supset q) \cdot (r \supset s)$$

$$\sim q \vee \sim s$$

$$\therefore \sim p \vee \sim r$$

What is implied here is that given a conjunction of two conditional statements as a first premise, and given also as a second premise, a disjunction of their negated consequents, it is permissible to infer the disjunction of their negated antecedents as our conclusion.

Hint

The validity of the above argument-forms can be shown by using various techniques. The next two study sessions shall be devoted to the discussion of some of these techniques.

ITQ

Question

- What is the difference between a disjunctive dilemma and a constructive dilemma?

Feedback

- The two involve the conjunction of two conditional statements as premise. However, for a constructive dilemma, there is a disjunction of the antecedents of each of the conditional statements and this serves as the second premise while there is a disjunction of the consequent of each conditional statement as conclusion.
- For a disjunctive dilemma, the second premise involves the disjunction of the negated consequent of each conditional statement while the conclusion is the disjunction of the negated antecedent of each conditional statement.

Study Session Summary



Summary

In this Study Session, we examined how the form of an argument (especially formal argument) is important in determining the validity and invalidity of such an argument. This is because if an argument has a form that is valid, all arguments having that form will be valid and if an argument form is invalid, any argument having that form will also be invalid. We also looked at nine argument-forms that are valid. These are Modus Ponens, Modus Tollens, Hypothetical Syllogism, Disjunctive Syllogism, Conjunction, Simplification, Addition, Constructive Dilemma and Destructive Dilemma.

Assessment



Assessment

SAQ11.1 (tests Learning Outcomes 11.1 and 11.2)

Give the name of the valid argument-form represented by the following:

1. If Bongo attends the party then we will have a swell time and if something goes wrong then everyone will be devastated. Either we did not have a swell time or everyone will not be devastated. Therefore, either Bongo did not attend the party or something did not go wrong.
2. Bongo attends the party. We will have a swell time. Therefore, Bongo will attend the party and we will have a swell time.
3. If Bongo will attend the party then we will have a swell time and if something goes wrong then everyone will be devastated. Bongo will attend the party or something will go wrong. Therefore, we will have a swell time or everyone will be devastated.
4. If Bongo will attend the party then we will have a swell time. If we will have a swell time then something goes wrong. Therefore, if Bongo will attend the party then something goes wrong.
5. If Bongo will attend the party then we will have a swell time. Bongo will attend the party. Therefore, we will have a swell time.
6. Either Bongo will attend the party or we will have a swell time. We will not have a swell time. Therefore, Bongo will attend the party.
7. If Bongo will attend the party then we will have a swell time. We will not have a swell time. Therefore, Bongo will not attend the party.