COURSE MANUAL

Mechanics and Properties of Matter

PHY102

University of Ibadan Distance Learning Centre

Open and Distance Learning Course Series Development

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Vice-Chancellor's Message

The Distance Learning Centre is building on a solid tradition of over two decades of service in the provision of External Studies Programme and now Distance Learning Education in Nigeria and beyond. The Distance Learning mode to which we are committed is providing access to many deserving Nigerians in having access to higher education especially those who by the nature of their engagement do not have the luxury of full time education. Recently, it is contributing in no small measure to providing places for teeming Nigerian youths who for one reason or the other could not get admission into the conventional universities.

These course materials have been written by writers specially trained in ODL course delivery. The writers have made great efforts to provide up to date information, knowledge and skills in the different disciplines and ensure that the materials are user-friendly.

In addition to provision of course materials in print and e-format, a lot of Information Technology input has also gone into the deployment of course materials. Most of them can be downloaded from the DLC website and are available in audio format which you can also download into your mobile phones, IPod, MP3 among other devices to allow you listen to the audio study sessions. Some of the study session materials have been scripted and are being broadcast on the university's Diamond Radio FM 101.1, while others have been delivered and captured in audio-visual format in a classroom environment for use by our students. Detailed information on availability and access is available on the website. We will continue in our efforts to provide and review course materials for our courses.

However, for you to take advantage of these formats, you will need to improve on your I.T. skills and develop requisite distance learning Culture. It is well known that, for efficient and effective provision of Distance learning education, availability of appropriate and relevant course materials is a *sine qua non*. So also, is the availability of multiple plat form for the convenience of our students. It is in fulfilment of this, that series of course materials are being written to enable our students study at their own pace and convenience.

It is our hope that you will put these course materials to the best use.

Prof. Abel Idowu Olayinka

Vice-Chancellor

Foreword

As part of its vision of providing education for "Liberty and Development" for Nigerians and the International Community, the University of Ibadan, Distance Learning Centre has recently embarked on a vigorous repositioning agenda which aimed at embracing a holistic and all encompassing approach to the delivery of its Open Distance Learning (ODL) programmes. Thus we are committed to global best practices in distance learning provision. Apart from providing an efficient administrative and academic support for our students, we are committed to providing educational resource materials for the use of our students. We are convinced that, without an up-to-date, learner-friendly and distance learning compliant course materials, there cannot be any basis to lay claim to being a provider of distance learning education. Indeed, availability of appropriate course materials in multiple formats is the hub of any distance learning provision worldwide.

In view of the above, we are vigorously pursuing as a matter of priority, the provision of credible, learner-friendly and interactive course materials for all our courses. We commissioned the authoring of, and review of course materials to teams of experts and their outputs were subjected to rigorous peer review to ensure standard. The approach not only emphasizes cognitive knowledge, but also skills and humane values which are at the core of education, even in an ICT age.

The development of the materials which is on-going also had input from experienced editors and illustrators who have ensured that they are accurate, current and learner-friendly. They are specially written with distance learners in mind. This is very important because, distance learning involves non-residential students who can often feel isolated from the community of learners.

It is important to note that, for a distance learner to excel there is the need to source and read relevant materials apart from this course material. Therefore, adequate supplementary reading materials as well as other information sources are suggested in the course materials.

Apart from the responsibility for you to read this course material with others, you are also advised to seek assistance from your course facilitators especially academic advisors during your study even before the interactive session which is by design for revision. Your academic advisors will assist you using convenient technology including Google Hang Out, You Tube, Talk Fusion, etc. but you have to take advantage of these. It is also going to be of immense advantage if you complete assignments as at when due so as to have necessary feedbacks as a guide.

The implication of the above is that, a distance learner has a responsibility to develop requisite distance learning culture which includes diligent and disciplined self-study, seeking available administrative and academic support and acquisition of basic information technology skills. This is why you are encouraged to develop your computer skills by availing yourself the opportunity of training that the Centre's provide and put these into use.

In conclusion, it is envisaged that the course materials would also be useful for the regular students of tertiary institutions in Nigeria who are faced with a dearth of high quality textbooks. We are therefore, delighted to present these titles to both our distance learning students and the university's regular students. We are confident that the materials will be an invaluable resource to all.

We would like to thank all our authors, reviewers and production staff for the high quality of work.

Best wishes.

Professor Bayo Okunade

Director

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Contents

About this course manual

Mechanics and Properties of MatterPHY102 has been produced by University of Ibadan Distance Learning Centre. All course manuals produced by University of Ibadan Distance Learning Centreare structured in the same way, as outlined below.

How this course manual is structured

The course overview

The course overview gives you a general introduction to the course. Information contained in the course overview will help you determine:

- If the course is suitable for you.
- What you will already need to know.
- What you can expect from the course.
- How much time you will need to invest to complete the course.

The overview also provides guidance on:

- Study skills.
- Where to get help.
- Course assignments and assessments.
- Margin icons.

We strongly recommend that you read the overview *carefully* before starting your study.

The course content

The course is broken down into Study Sessions. Each Study Session comprises:

- An introduction to the Study Session content.
- Study Sessionoutcomes.
- Core content of the Study Sessionwith a variety of learning activities.
- A Study Session summary.
- Assignments and/or assessments, as applicable.
- Bibliography

Your comments

After completing Mechanics and Properties of Matter we would appreciate it if you would take a few moments to give us your feedback on any aspect of this course. Your feedback might include comments on:

- Course content and structure.
- Course reading materials and resources.
- Course assignments.
- Course assessments.
- Course duration.
- Course support (assigned tutors, technical help, etc.)

Your constructive feedback will help us to improve and enhance this course.

Course Overview

Welcome to Mechanics and Properties of MatterPHY102: Is this Course for you

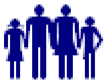
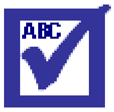
Mathematics is the language of Physics. It is highly important that learners should revise basic mathematics before going through the course of this study. Such includes basic calculus, trigonometric functions, logarithmic functions and derivatives/identities. Many things will be assumed, but if the learner has updated his/her mathematical skills, most of the concepts that is to be discussed will be assimilated more properly.

Getting around this course manual

Margin icons

While working through this course manual you will notice the frequent use of margin icons. These icons serve to “signpost” a particular piece of text, a new task or change in activity; they have been included to help you to find your way around this course manual.

A complete icon set is shown below. We suggest that you familiarize yourself with the icons and their meaning before starting your study.

 **Activity**	 **Assessment**	 **Assignment**	 **Case study**
 **Discussion**	 **Group Activity**	 **Help**	 **Outcomes**
 **Note**	 **Reflection**	 **Reading**	 **Study skills**
 **Summary**	 **Terminology**	 **Time**	 **Tip**

Study Session 1

Essential Mathematics I

Introduction

In this study session, we will examine trigonometric, logarithm and derivatives functions. We will also attempt examples to explain each of them.

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

1.1 Explain the following functions with an example each:

- Trigonometric functions
- Derivatives functions
- Logarithm function

Terminology

Trigonometry	a branch of mathematics that studies relationships involving lengths and angles of triangles
Logarithm	a quantity representing the power to which a fixed number (the base) must be raised to produce a given number

1.1 Essential Mathematics Functions

The concept of function plays an important role in the study of calculus. A function classically is also called a mapping from **A** to **B,** the function takes each element x in **A** into its corresponding element y in **B**. while, the modern way defines it as the set of ordered pairs. Functions are used to represent relationship between measurable, observable, quantities etc. Also, there are basically two approaches to mathematics functions namely the classical and modern definitions. We will discuss the following essential mathematical functions

i. Trigonometric functions

ii. Logarithmic functions

iii. Derivatives functions

1.1.1 TRIGONOMETRIC FUNCTIONS

The sine, cosine and tangent of an angle are all defined in terms of trigonometry, but they can also be expressed as functions. In this section, we shall use information about the trigonometric ratios sine, cosine and tangent to define functions f(x) = sinx, f(x) = cosx and f(x) = tanx.

The sine function f(x) = sinx

We shall start with the sine function, f(x) = sinx. This function can be defined for any number x. The function f(x) = sinx has all real numbers in its domain, but its range is $-1 \leq \sin x \leq 1$. The values of the sine function are different, depending on whether the angle is in degrees or radians. The function is periodic with periodicity 360 degrees or 2π radians.

The cosine function f(x) = cosx

The function f(x) = cosx has all real numbers in its domain, but its range is $-1 \leq \cos x \leq 1$. The values of the cosine function are different, depending on whether the angle is in degrees or radians. The function is periodic with periodicity 360 degrees or 2π radians.

The tangent function f(x) = tanx

The function f(x) = tanx has all real numbers except odd multiples of 90 degree in its domain (in the case where x is expressed in degrees), or all real numbers except odd multiples of π/2 (in the case where x is expressed in radians. The range of the tangent function contains all real numbers. The function is periodic with periodicity 180 degrees or π radians.

TRIGONOMETRIC TABLE

Figure 1.1

X	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
Sinx	**0**	**0.71**	**1**	**0.71**	**0**	**-0.71**	**-1**	**-0.71**	**0**
Cosx	**1**	**0.71**	**0**	**-0.71**	**-1**	**-0.71**	**0**	**0.71**	**1**
Tanx	**0**	**1**	*	**-1**	**0**	**1**	*	**-1**	**0**

1.1.2 LOGARITHMIC FUNCTIONS

Every exponential function has an inverse function. The inverse of the exponential function with base b, f (x) = b x , is called the logarithmic function base b and is denoted log b x. The logarithmic function base e is called the natural logarithm function and is denoted ln(x). The logarithmic function base 10 is called the common logarithm function and is denoted log(x). These logarithmic functions can be evaluated directly by using a scientific calculator.

Example

Let f (x) = ln(x). Find f (2.349)

f (2.349) = In(2.349) = 0.85399 (rounded)

The logarithmic functions satisfy the identities

log_b AB = log_b A + log_bB (the Product Identity),

$log_b \frac{A}{B} = log_b$ A $-$ log_b B(the Quotient Identity) and

$log_b A^P$ = Plog_b (the Power Identity)

Where A and B may be positive numbers or variables, algebraic expressions or functions that take on positive values. These identities are used to write expressions involving logarithmic functions in different forms to suit different purposes.

Example 1:

Solve the equation log x^{81} = 4.

Solution:

Rewrite the logarithmic equation as the exponential equation x^4 = 81. Apply the fourth root function to both sides of the equation to find x = $81^{\frac{1}{4}}$ = 3

Example 2:

Solve $\log_2 (x - 2) + \log_2 (x + 1) = 2$.

Solution:

Use the Product Identity for Logarithms to write the equation as

$\log_2 (x - 2)(x + 1) = 2$. Rewrite this equation as the exponential equation

$(x - 2)(x + 1) = 2^2$,

or $X^2 - x - 6 = 0$.

The solutions to this quadratic equation are x = 3 and x = -2. On checking by substitution in the original equation, we find that x = 3 is a solution and x = -2 is not a solution.

ITQ

Question

What are logarithmic functions bases 'e' and base '10' called and how can they be represented?

Feedback

Logarithmic function base 'e' is called the natural logarithm function and is denoted by ln(x).

Logarithmic function base 10 is called the common logarithm function and is denoted as well by log(x)

1.1.3 DERIVATIVES FUNCTIONS

The derivative of a function of a real variable measures the sensitivity to change a quantity (a function value or dependent variable), which is determined by another quantity (the independent variable). Derivatives are a functional tool of calculus. For example, the derivative of a position of a moving object with respect to time is the object's velocity: this measures how quickly the position of the object changes when time is advanced.

The process of finding a derivative is called differentiation. The derivative of a function f at a number x, denoted by f′(x), is

If y = f(x) the derivative is defined to be f' (x) = $\lim_{k \to 0} \left(\frac{f(x+h)-f(x)}{h}\right)$

Other notations are:

f' (x) = y' = $\frac{dy}{dx}$ = ẏ

Some common derivatives

$\frac{d}{dx}$ **(x) = 1**

$\frac{d}{dx}$ **(Sin x) = Cos x**

$\frac{d}{dx}$ **(Cos x) = - Sin x**

$\frac{d}{dx}$ **(Tan x) =** sec^2 **x**

$\frac{d}{dx}$ **(Sec x) = Sec x Tan x**

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}$ **In(x) =** $\frac{1}{x}$

ITQ

Question

What were the two essential mathematical functions discussed in this section?

Feedback

The two essential mathematics functions discussed are:

i. Trigonometric functions

ii. Logarithmic functions

Study Session Summary

Summary

In this Study Session, we explained trigonometric functions, logarithmic functions and derivatives. We gave some solved examples and analysed the solutions.

Assessment

Assessment

SAQ 1.1 (tests Learning Outcome 1.1)
Fill the table below with their correct values

X	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
Sinx	**0**	**0.71**		**0.71**	**0**	**-0.71**	**-1**		**0**
Cosx	**1**		**0**	**-0.71**		**-0.71**	**0**	**0.71**	
tanx	**0**		*	**-1**	**0**	**1**		**-1**	**0**

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 2

Essential Mathematics II

Introduction

There are many examples in practice where two forces, acting together, exert a moment or turning-effect on some object. In this study session, we will explain cases like two strings be tied to a wheel and two equal opposite forces applying tangentially to the wheel, we will observe what will happen if the wheel is pivoted at its centre.

Learning Outcomes

Outcomes

When you have studied this session, you should be able to

2.1 explain the following terms:

- torque and angular momentum
- conservation of angular momentum
- kinetic energy of a rolling object
- angular momentum of rotating rigid bodies

Terminology

Angular momentum	the quantity of rotation of a body, which is the product of its moment of inertia and its angular velocity
Kinetic energy	energy that a body possesses by virtue of being in motion

2.1 Effects of Forces and their Reactions

2.1.1 Torque and Angular Momentum

Torque is the effect of a force about an axis. The torque is equal to the moments of the force F about the axis of rotation.

$$Torque, \Gamma = Fr$$

Note

Torque has a unit of ***Nm*** *and t*his unit is not equivalent to Joule, the unit of Energy since, since Torque is not an energy

Torque in rotational motion is analogous to force in linear motion. Since moments of inertia I is analogous to mass m. The relationship between torque Γ and moments of inertia is given by

$$Torque, \Gamma = I\alpha = I\frac{dw}{dt}$$

where $\alpha = angular\ acceleration.$

The work done by a constant torque Γ when the body is turned through an angular displacement θ is given by;

$$Work\ done, W = \ \Gamma\theta.$$

Angular momentum in rotational motion is analogous to linear momentum in rotational motion.

$$Angular\ momentum, L = (mv)r$$

$$= m(rw)r\ , since\ v = rw$$

$$\therefore \boldsymbol{L = mr^2w}$$

where m is the mass of the particle, r is the radial distance and w is the angular velocity.

For a rigid body containing many particles;

$$\boldsymbol{L = Iw}$$

where I is the moment of inertia of the body about the axis of rotation.

The unit for angular momentum is $\boldsymbol{kgm^2s^{-1}}$.

Recall that:

$$Torque, \Gamma = I\alpha = I\frac{dw}{dt}$$

So, $Torque, \Gamma = \frac{d}{dt}(Iw)$

But, $\boldsymbol{L = Iw}$

Therefore, $Torque, \Gamma = I\frac{dL}{dt} = rate of change of angular momentum.$

ITQ

Question

If Torque in rotational motion is analogous to force in linear motion and moments of inertia I, is analogous to mass m. How can you represent the relationship between torque Γ and moments of inertia?

Feedback

This can be represented thus: $Torque, \Gamma = I\alpha = I\frac{dw}{dt}$

2.1.2 Conservation of Angular Momentum

The principle of Conservation of Angular Momentum states that the angular momentum about an axis of rotation for a system is constant if no unbalanced external Torque acts on it.

$$i.e.\ \boldsymbol{L = Iw = constant}.$$

For detailed understanding of torque and angular momentum, let us consider rolling motion.

Rolling Motion

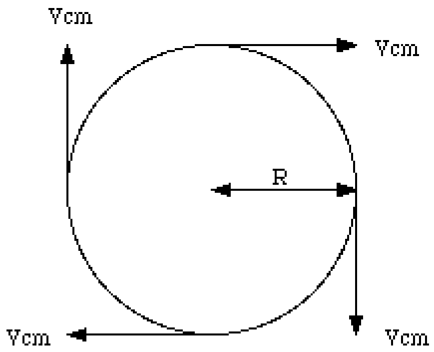


Figure 2.: Rotational Motion of Wheel

A wheel rolling over a surface has both a linear and a rotational velocity. Suppose the angular velocity of the wheel is [omega]. The corresponding linear velocity of any point on the rim of the wheel is given by

$$v_{cm} = \omega R$$

Where R is the radius of the wheel (see Figure 2.1). When the wheel is in contact with the ground, its bottom part is at rest with respect to the ground. This implies that besides a rotational motion the wheel experiences a linear motion with a velocity equal to + v_{cm} (see Figure 2.2). We conclude that the top of the wheel moves twice as fast as the centre and the bottom of the wheel does not move at all.

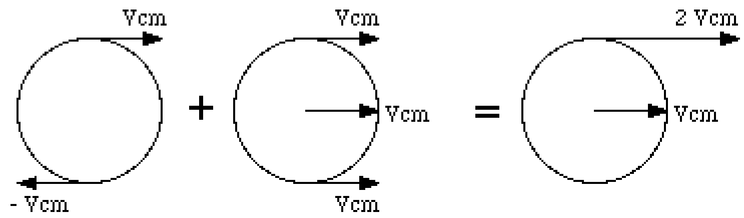


Figure 2.2: Motion of Wheel is Sum of Rotational and Translational Motion

An alternative way of looking at the motion of a wheel is by regarding it as a pure rotation (with the same angular velocity [omega]) about an instantaneous stationary axis through the bottom of the wheel (point P, Figure 2.3).

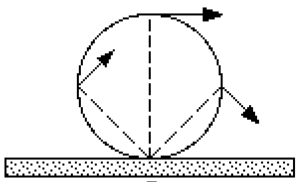


Figure 2.3: Motion of Wheel around Axis through P

ITQ

Question

State the principle of Conservation of Angular Momentum

Feedback

The principle of Conservation of Angular Momentum states that the angular momentum about an axis of rotation for a system is constant if no unbalanced external Torque acts on it

2.1.3 Kinetic Energy of Rolling Object

The kinetic energy of the wheel shown in Figure 2.3 can be calculated easily using the following formulas;

$$K = \frac{1}{2} I_P \omega^2$$

where I_P is the rotational inertia around the axis through P, and [omega] is the rotational velocity of the wheel. The rotational inertia around an axis through P, I_P, is related to the rotational inertia around an axis through the centre of mass, I_{cm}

$$I_P = I_{cm} + M R^2$$

The kinetic energy of the wheel can now be rewritten as

$$K = \frac{1}{2}\left(I_{cm} + M R^2\right)\omega^2 = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} M v_{cm}^2$$

where the first term is the kinetic energy associated with the rotation of the wheel about an axis through its center of mass and the second term is associated with the translational motion of the wheel.

Problem

Figure 2.4 shows a disk with mass M and rotational inertia I on an inclined plane. The mass is released from a height h. What is its final velocity at the bottom of the plane? The disk is released from rest. Its total mechanical energy at that point is equal to its potential energy

$$E_i = M g h$$

When the disk reaches the bottom of the plane, all of its potential energy is converted into kinetic energy. The kinetic energy of the disk will consist out of rotational and translational kinetic energy:

$$E_f = K = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2$$

The moment of inertia of the disk is given by

$$I = \frac{1}{2} M R^2$$

where R is the radius of the disk. The kinetic energy of the disk can now be rewritten as

$$E_f = \frac{1}{2}\left(\frac{1}{2} M R^2\right)\omega^2 + \frac{1}{2} M v^2 = \frac{3}{4} M v^2$$

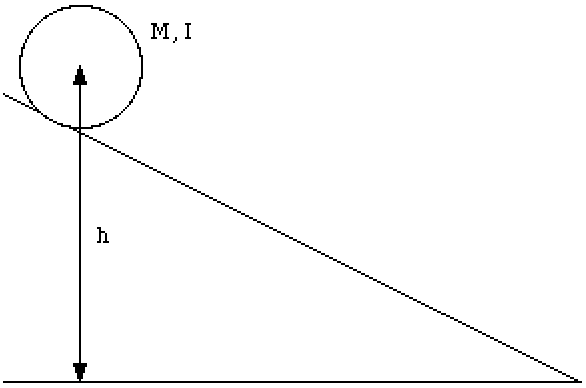


Figure 2.4: Mass on an Inclined Plane

Conservation of mechanical energy implies that $E_i = E_f$, or

$$M g h = \frac{3}{4} M v^2$$

This shows that the velocity of the disk is given by

$$v = \sqrt{\frac{4}{3} g h}$$

Consider now two different disks with identical mass M but different moments of inertia. In this case, the final kinetic energy can be written as

$$E_f = \frac{1}{2} \frac{I}{R^2} v^2 + \frac{1}{2} M v^2 = \frac{1}{2}\left(M + \frac{I}{R^2}\right) v^2$$

Conservation of energy now requires that

$$\frac{1}{2}\left(M + \frac{I}{R^2}\right) v^2 = M g h$$

or

$$v = \sqrt{\frac{2gh}{MR^2 + I} MR^2}$$

We conclude that in this case, the disk with the smallest moment of inertia has the largest final velocity.

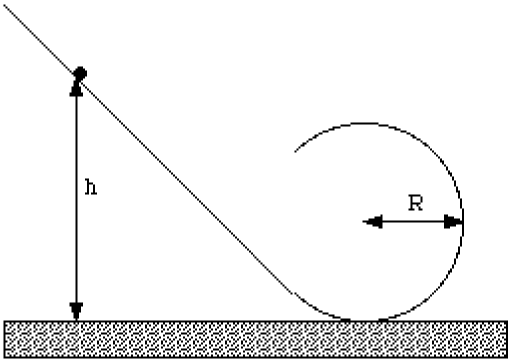


Figure 2.5: Problem

Problem

A small solid marble of mass m and radius r rolls without slipping along a loop-the-loop track shown in Figure 2.5, having been released from rest somewhere along the straight section of the track. From what minimum height above the bottom of the track must the marble be released in order not to leave the track at the top of the loop?

The marble will not leave the track at the top of the loop if the centripetal force exceeds the gravitational force at that point:

$$m \frac{v^2}{R} \geq mg$$

or

$$v^2 \geq gR$$

The kinetic energy of the marble at the top consists out of rotational and translational energy

$$K_r = \frac{1}{2} I \omega^2 + \frac{1}{2} M v^2 = \frac{1}{2}\left(\frac{I}{r^2} + M\right) v^2$$

where we assumed that the marble is rolling over the track (no slipping). The moment of inertia of the marble is given by

$$I = \frac{2}{5} M r^2$$

Using this expression, we obtain for the kinetic energy

$$K_r = \frac{1}{2}\left(M + \frac{2}{5} M\right) v^2 = \frac{7}{10} M v^2$$

The marble will reach the top if

$$K_r = \frac{7}{10} M v^2 \geq \frac{7}{10} M g R$$

The total mechanical energy of the marble at the top of the loop-the-loop is equal to

$$E_r = K_r + U_r = \frac{7}{10} m v^2 + 2 m g R \geq \frac{7}{10} m g R + 2 m g R = \frac{27}{10} m g R$$

The initial energy of the marble is just its potential energy at a height h

$$E_i = m g h$$

Conservation of energy now implies that

$$m g h \geq \frac{27}{10} m g R$$

or

$$h \geq \frac{27}{10} R$$

Problem: the yo-yo

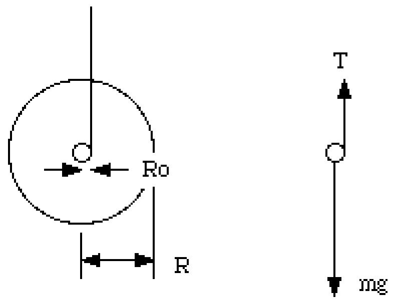


Figure 2.6: The yo-yo.

Figure 2.6 shows a schematic drawing of a yo-yo. What is its linear acceleration? There are two forces acting on the yo-yo: an upward force equal to the tension in the cord, and the gravitational force. The acceleration of the system depends on these two forces:

$$\sum F = mg - T = ma$$

The rotational motion of the yo-yo is determined by the torque exerted by the tension T (the torque due to the gravitational force is zero)

$$\sum \tau = I\alpha = TR_0$$

The rotational acceleration **a** is related to the linear acceleration **a**:

$$a = R_0 \alpha$$

We can now write down the following equations for the tension T

$$T = \frac{I\alpha}{R_0} = \frac{Ia}{{R_0}^2}$$

$$T = mg - ma$$

The linear acceleration a can now be calculated

$$a = g \frac{1}{1 + \frac{I}{m R_0^2}}$$

Thus, the yo-yo rolls down the string with a constant acceleration. The acceleration can be made smaller by increasing the rotational inertia and by decreasing the radius of the axle.

2.1.4 Torque

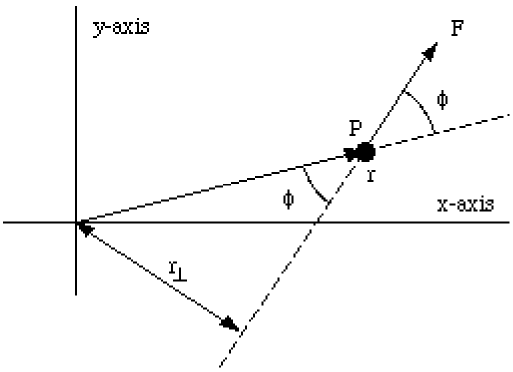


Figure 2.7: Motion of a particle P in the x-y plane.

A particle with mass m moves in the x-y plane (see Figure 2.7). A single force F acts on the particle and the angle between the force and the position vector is [phi]. Per definition, the torque exerted by this force on the mass, with respect to the origin of our coordinate system, is given by

$$\bar{\tau} = \bar{r} \times \bar{F}$$

and

$$|\bar{\tau}| = |\bar{r} \times \bar{F}| = |r F \sin(\phi)| = r_\perp F$$

where $r_{[invtee]}$ is called the arm of the force F with respect to the origin. According to the definition of the vector product, the vector [tau] lies parallel to the z-axis, and its direction (either up or down) can be determined using the right-hand rule. Torque defined in this way has meaning only with respect to a specified origin. The direction of the torque is always at right angles to the plane formed by the vectors r and F. The torque is zero if r = 0 m, F = 0 N or r is parallel or anti-parallel to F.

2.1.5Angular Momentum

The angular momentum L of particle P in Figure 6.7, with respect to the origin, is defined as

$$\bar{L} = \bar{r} \times \bar{p} = m(\bar{r} \times \bar{v}) = m r v \sin(\phi)\hat{z}$$

This definition implies that if the particle is moving directly away from the origin, or directly towards it, the angular momentum associated with this motion is zero. A particle will have a different angular momentum if the origin is chosen at a different location. A particle moving in a circle will have an angular momentum (with respect to the centre of the circle) equal to

$$\bar{L} = \bar{r} \times \bar{p} = m r v \hat{z} = m r^2 \omega \hat{z} = I \omega \hat{z}$$

Again we notice the similarity between the definition of linear momentum and the definition of angular momentum.

A particle can have angular momentum even if it does not move in a circle. For example, Figure 2.8 shows the location and the direction of the momentum of particle P. The angular momentum of particle P, with respect to the origin, is given by

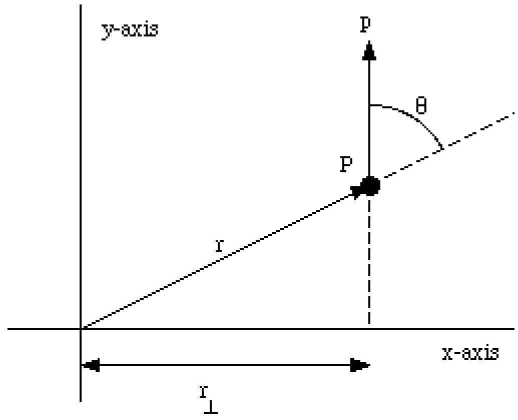


Figure 2.8: Angular Momentum of Particle P.

$$\bar{L} = \bar{r} \times \bar{p} = m\, r\, v \sin(\theta)\,\hat{z} =$$
$$= m\, v\, r_{\perp}\,\hat{z} = p\, r_{\perp}\,\hat{z}$$

The change in the angular momentum of the particle can be obtained by differentiating the equation for l with respect to time

$$\frac{d\bar{L}}{dt} = \frac{d}{dt}(\bar{r} \times \bar{p}) = m\left(\bar{r} \times \frac{d\bar{v}}{dt} + \frac{d\bar{r}}{dt} \times \bar{v}\right) = m(\bar{r} \times \bar{a} + \bar{v} \times \bar{v}) = \bar{r} \times \sum \bar{F} = \sum \bar{\tau}$$

We conclude that

$$\sum \bar{\tau} = \frac{d\bar{L}}{dt}$$

Note

This equation shows that if the net torque acting on the particle is zero and its angular momentum will be constant

Problem

Figure 2.9 shows object P in free fall. The object starts from rest at the position indicated in Figure 2.9. What is its angular momentum, with respect to the origin, as function of time?

The velocity of object P, as function of time, is given by

$$v(t) = v_0 - g\,t = -g\,t$$

The angular momentum of object P is given by

$$|\bar{L}| = |\bar{r} \times \bar{p}| = m|\bar{r} \times \bar{v}| = m\,g\,t\,d$$

Therefore

$$\frac{dL}{dt} = m\,g\,d$$

which is equal to the torque of the gravitational force with respect to the origin.

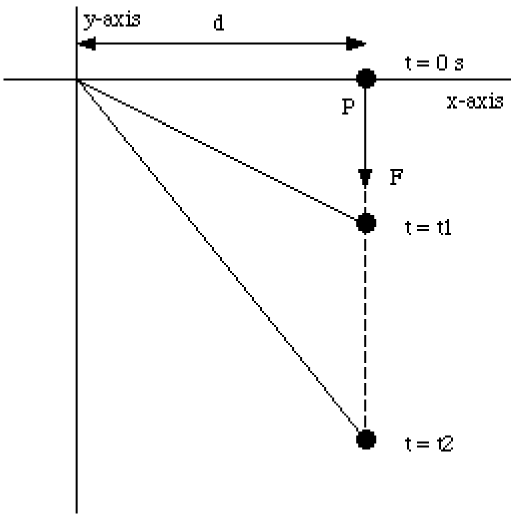


Figure 2.9: Free Fall and Angular Momentum

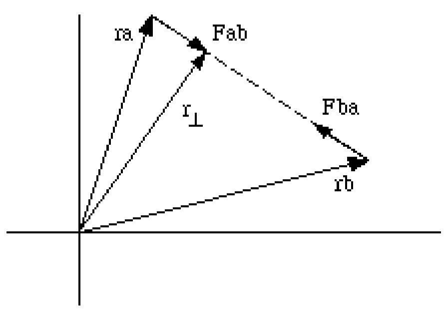


Figure 2.10: Action - Reaction Pair.

If we look at a system of particles, the total angular momentum L of the system is the vector sum of the angular momenta of each of the individual particles:

$$\bar{L} = \bar{L}_1 + \bar{L}_2 + \bar{L}_3 + \ldots = \sum_{i=1}^{n} \bar{L}_i$$

The change in the total angular momentum L is related to the change in the angular momentum of the individual particles

$$\frac{d\bar{L}}{dt} = \sum_{i=1}^{n} \frac{d\bar{L}_i}{dt} = \sum_{i=1}^{n} \bar{\tau}_i$$

Some of the torques are internal, some are external. The internal torques come in pairs, and the vector sum of these is zero. This is illustrated in Figure 2.10. Figure 2.10 shows the particles A and B which interact via a central force. Newton's third law states that forces come in pairs: if B exerts a force F_{AB} on A, than A will exert a force F_{BA} on B. F_{AB} and F_{BA} are related as follows

$$\bar{F}_{AB} = -\bar{F}_{BA}$$

The torque exerted by each of these forces, with respect to the origin, can be easily calculated

$$\tau_{AB} = \left| \bar{r}_A \times \bar{F}_{AB} \right| = r_\perp F_{AB}$$

and

$$\tau_{BA} = \left| \bar{r}_B \times \bar{F}_{BA} \right| = r_\perp F_{BA}$$

Clearly, these two torques add up to zero

$$\sum \bar{\tau} = \bar{r}_A \times \bar{F}_{AB} + \bar{r}_B \times \bar{F}_{BA} = r_\perp \left(F_{AB} + F_{BA} \right) \hat{z} = 0$$

The net torque for each action-reaction pair, with respect to the origin, is equal to zero.

We conclude that

$$\frac{d\bar{L}}{dt} = \sum \bar{\tau}_{ext}$$

This equation is another way of expressing Newton's second law in angular quantities.

2.1.6 Angular Momentum of Rotating Rigid Bodies

Suppose we are dealing with a rigid body rotating around the z-axis. The linear momentum of each mass element is parallel to the x-y plane, and perpendicular to the position vector. The magnitude of the angular momentum of this mass element is

$$L = r(\Delta m\, v)$$

The z-component of this angular momentum is given by

$$L_z = L\sin(\theta) = (r\sin(\theta))(\Delta m\, v) = r_\perp(\Delta m\, v)$$

The z-component of the total angular momentum L of the rigid body can be obtained by summing over all mass elements in the body

$$L_z = \sum L_z = \sum r_\perp(\Delta m\, v) = \sum \Delta m\,(\omega\, r_\perp)\, r_\perp = \omega \sum \Delta m\, r_\perp^{\,2}$$

From the definition of the rotational inertia of the rigid body we can conclude that

$$L_z = I\,\omega$$

This is the projection of the total angular momentum onto the rotation axis. The rotational inertia I in this equation must also be calculated with respect to the same rotation axis.

Note

Only if the rotation axis is a symmetry axis of the rigid body will the total angular momentum vector coincide with the rotation axis

2.1.7 Conservation of Angular Momentum

If no external forces act on a system of particles or if the external torque is equal to zero, the total angular momentum of the system is conserved. The angular momentum remains constant, no matter what changes take place within the system.

Problem

The rotational inertia of a collapsing spinning star changes to one-third of its initial value. What is the ratio of the new rotational kinetic energy to the initial rotational kinetic energy?

The final rotational inertia I_f is related to the initial rotational inertia I_i as follows

$$I_f = \frac{1}{3} I_i$$

No external forces act on the system, and the total angular momentum is conserved

$$L_f = L_i$$

The initial rotational kinetic energy is given by

$$K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} \frac{L_i^2}{I_i}$$

The final rotational kinetic energy is given by

$$K_f = \frac{1}{2}\frac{L_f^2}{I_f} = \frac{1}{2}\frac{L_i^2}{\frac{1}{3}\cdot I_i} = 3K_i$$

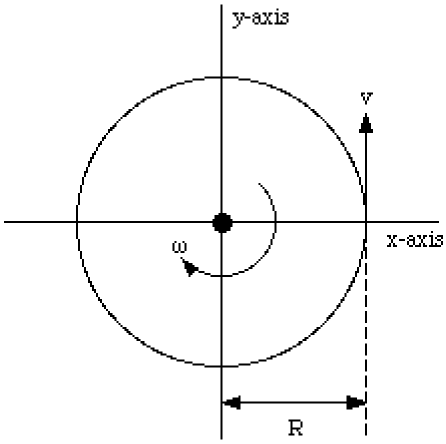


Figure 2.11: Problem

A cockroach with mass m runs counter clockwise around the rim of a lazy Susan (a circular dish mounted on a vertical axle) of radius R and rotational inertia I with frictionless bearings. The cockroach's speed (with respect to the earth) is v, whereas the lazy Susan turns clockwise with angular speed [omega]$_0$. The cockroach finds a bread crumb on the rim and, of course, stops. (a) What is the angular speed of the lazy Susan after the cockroach stops? (b) Is mechanical energy conserved?

Assume that the lazy Susan is located in the x-y plane (see Figure 2.11). The linear momentum of the cockroach is m · v. The angular momentum of the cockroach, with respect to the origin, is given by

$$\bar{L}_c = |\bar{r} \times \bar{p}| = R m v \hat{z}$$

The direction of the angular momentum can be found using the right-hand rule. The direction of the z-axis is chosen such that the angular momentum of the cockroach coincides with the positive z-axis. The lazy

Susan is moving clockwise (see Figure 2.11) and its angular momentum is pointing along the negative z-axis. Its angular momentum is given by

$$\bar{L}_d = |\bar{r} \times \bar{p}| = I\omega_0 \hat{z}$$

where I is the rotational inertia of the dish. Note that since the rotation is clockwise, ω_0 is less than zero. The total angular momentum of the system is given by

$$\bar{L} = \bar{L}_c + \bar{L}_d = (Rmv + I\omega_0)\hat{z}$$

The rotational inertia of the dish plus cockroach is given by

$$I_f = I + mR^2$$

Since the external torque acting on the system is zero, the total angular momentum is conserved. The rotational velocity of the system after the cockroach stops is given by

$$\omega_f = \frac{L_f}{I_f} = \frac{Rmv + I\omega_0}{I + mR^2}$$

The initial kinetic energy of the system is equal to

$$K_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega_0^2$$

The final kinetic energy of the system is equal to

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(I + mR^2\right)\left(\frac{Rmv + I\omega_0}{I + mR^2}\right)^2$$

The change in kinetic energy of the system is

$$\Delta K = K_f - K_i = -\frac{1}{2}\frac{Im}{I + mR^2}\left(v - R\omega_0\right)^2$$

The change in the kinetic energy of the system is negative, and we conclude that mechanical energy is not conserved. The loss of mechanical energy is due to the work done by the friction force between the surface of the lazy Susan and the legs of the cockroach.

Study Session Summary

Summary

In this Study Session, we treated torque and angular momentum. Numerous problems were also solved on each sub-topics to further drive home the concepts of torque and angular momentum forces.

Assessment

Assessment

SAQ 2.1 (tests Learning Outcome 2.1)

Torque is the effect of a force about an axis. The torque is equal to the moments of the force F about the axis of rotation. How can this expression be represented mathematically?

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.
http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 3

Scalars and Vectors in Dimension

Introduction

In this study session, we will discuss physical quantity with respect to scalar and vector quantities. We will also examine vector addition and multiplication with valid examples to be solved

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

3.1 *define* and give three (3) examples each of scalar and vector quantities

Terminology

Scalar Quantity	a quantity that can be completely described by a magnitude, that is, by a number and a unit
Vector Quantity	a measurement that refers to both the magnitude of the medium as well as the direction of the movement the medium has taken

3.1 Mathematical Quantities used in Physics

Physics is a mathematical science. The underlying concepts and principles have a mathematical basis. Throughout the course of our study of physics, we will encounter a variety of concepts that have a mathematical basis associated with them. While our emphasis will often be upon the conceptual nature of physics, we will give considerable and persistent attention to its mathematical aspect.

The motion of objects can be described by words. Even a person without a background in physics has a collection of words that can be used to describe moving objects. Words and phrases such as going fast, stopped, slowing down, speeding up, and turning provide a sufficient vocabulary for describing the motion of objects. In physics, we use these words and many more. We will be expanding upon this vocabulary list with words such as distance, displacement, speed, velocity, and acceleration and we will soon see that these words are associated with mathematical quantities that have strict definitions. The mathematical quantities that are used to describe the motion of objects can be divided into two categories. The quantity is either a vector or a scalar. These two categories can be distinguished from one another by their distinct definitions

3.1.1 Scalar Quantity

A **scalar quantity** is a quantity that can be completely described by a magnitude, that is, by a number and a unit. Some examples of scalar quantities are mass, length, time, density, and temperature. The characteristic of scalar quantities is that they add up like ordinary numbers. That is, if we have a mass m1 = 3 kg and another mass m2 = 4 kg then the sum of the two masses is

m = m1 + m2 = 3 kg + 4 kg = 7 kg

3.1.2Vector Quantity

A **vector quantity**, on the other hand, is a quantity that is described by both magnitude and direction. Some examples of vector quantities are force, displacement, velocity, and acceleration. The velocity of a car moving at 50 km per hour (km/hr) due east can be represented by a vector. Velocity is a vector because it has a magnitude, 50 km/hr, and a direction, due east. A vector quantity can be represented by an arrow drawn to scale.

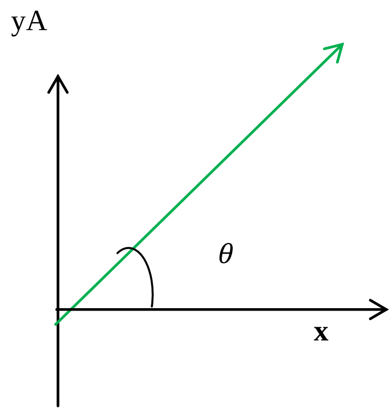


Fig. 3.1: Representation of a Vector

The length of the arrow is proportional to the magnitude vector quantity. The direction of the arrow represents the direction of the vector as shown in the figure 3.1 above. That is, the length of the arrow represents the magnitude of the vector, while the direction of the arrow represents the direction of the vector. The direction is specified by the angle θ that the vector makes with an axis, usually the x-axis, and is shown in figure 1.1. The magnitude of vector A is written as the absolute value of A called |A|, or simply by the letter A.In mechanics, we often find the component of a vector in a certain direction. The component of a vector is the effective part of the vector in that direction. We can illustrate it by considering figure 3.2 as follow:

A vector $\underline{A}$ on the x – y plane can be specified by its two components, which are mutually perpendicular to each other and directed along the X – and Y – axes form the origin of i.e.

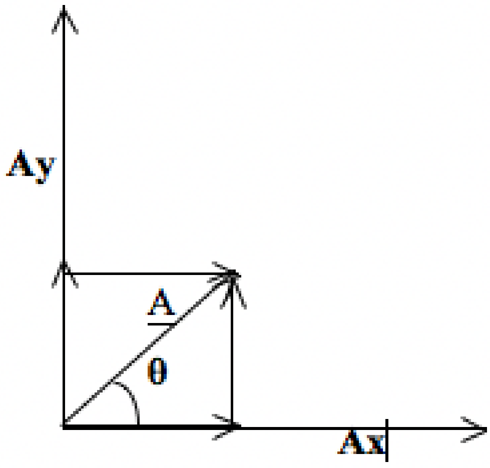


Fig. 3.2: Resolution of vectors

The magnitudes of the resolved components along X-and Y- axes are written as Ax and Ay. These can be determined in terms of θ the angle A makes with the positive X- axis as shown in fig 3.2.

$$Ax \quad = \ A \cos \theta$$

$$Ay \quad = A \text{ Sin } \theta$$

Where A is the magnitude of vector A in symbol, A = /A/

Therefore, we can write A in terms of the components as;

A = Axî + Ayĵ

Where î and ĵ are unit vectors in the x- and y- directions respectively;

By applying Pythagoras theorem, we obtain the magnitude of A from fig 1.2.

$$A = \sqrt{Ax^2 + Ay^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad 3.1$$

And its direction as

$$\theta \ = tan^{-1}(Ay/Ax) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 3.2$$

Vector A can also be resolved in a 3-dimensional space i.e.

$$A \ = Ax\ \hat{i} + Ay\ \hat{j} + Az\ \hat{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 3.3$$

Where k is a unit vector in the z – direction. It should be noted that only vectors lying in the quadrants and not those lying on any of the axis can be resolved.

3.1.3 Vector Addition

Let us assume we add any two arbitrary vectors **A** and **B**. The result of adding the two vectors **A** and **B** forms a new resultant vector **R**, which is the sum of **A** and **B**. This can be shown graphically by laying off the first vector **A**in the horizontal direction and then placing the tail of the second vector **B** at the tip of vector **A**, as shown in figure 3.3. The resultant vector **R** is drawn from the origin of the first vector to the tip of the last vector. The resultant vector is written mathematically as

$\underline{\mathbf{R}} = \underline{\mathbf{A}} + \underline{\mathbf{B}}$ ……………………………………3.4

Let $\underline{\mathbf{A}} = \mathbf{A_x\hat{i} + A_y\hat{j}}$ and

$\underline{\mathbf{B}} = \mathbf{B_x\hat{i} + B_y\hat{j}}$

$\underline{\mathbf{R}} = \underline{\mathbf{A}} + \underline{\mathbf{B}} = (\mathbf{A_x + B_x})\,\mathbf{\hat{i}} + (\mathbf{A_y + B_y})\,\mathbf{\hat{j}}$ ………………….3.5

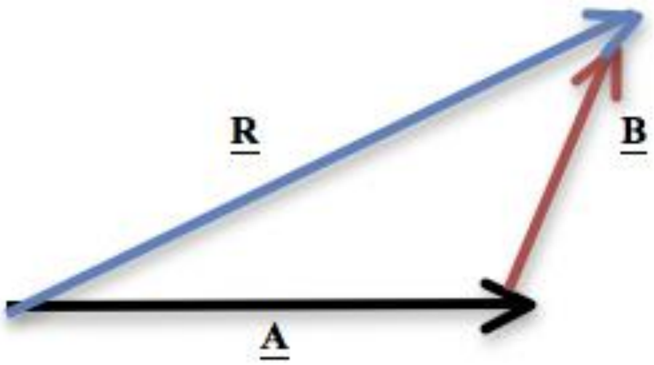


Fig.3.3: Addition of Vectors

Note that in this sum we do not mean scalar addition. The resultant vector is the vector sum of the individual vectors **A** and **B**.

1) Vector addition is commutative i.e., it does not matter in which order we add vectors.

   $\underline{\mathbf{A}} + \underline{\mathbf{B}} = \underline{\mathbf{B}} + \underline{\mathbf{A}}$

2) Vector addition is associative

   $\underline{\mathbf{A}} + (\underline{\mathbf{B}} + \underline{\mathbf{C}}) = (\underline{\mathbf{A}} + \underline{\mathbf{B}}) + \underline{\mathbf{C}}$

3) Subtraction is simply addition with one of vectors changing sign.

   $\underline{\mathbf{A}} - \underline{\mathbf{B}} = \underline{\mathbf{A}} + (-\underline{\mathbf{B}})$

3.1.4Multiplication of Vectors

Dot Product

The multiplication of two vectors yielding a scalar quantity is called a scalar or dot product. The dot product of **A** and **B** is given by

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} = /\underline{\mathbf{A}}/ /\underline{\mathbf{B}}/ \mathbf{Cos}\ \theta$$

Where θ is the angle between **A** and **B**, in terms of the resolved component of A and B, the dot product is

$$\underline{\mathbf{A}} \cdot \underline{\mathbf{B}} = \mathbf{AxBx + AyBy + AzBz}$$

Vector Product

The vector product of **A** and **B** equals another vector **C** which is perpendicular to the plane of **A** and **B** in symbol;

$$\underline{\mathbf{C}} = \underline{\mathbf{A}} \wedge \underline{\mathbf{B}} = /\underline{\mathbf{A}}/ /\underline{\mathbf{B}}/ \sin\theta\ \hat{r}$$

Where θ is the angle between **A** and **B**and r is unit vector in the direction of**C.**

This operation is also called cross-product; the direction of **C**is usually determined by the right hand screw rule.

Therefore, we can conclude that some vector products do not commute

i.e.,

$$\underline{\mathbf{A}} \wedge \underline{\mathbf{B}} - \underline{\mathbf{B}} \wedge \underline{\mathbf{A}} \neq 0$$

Analytically, **C** can be expressed in terms of the resolved component of the vector **A** and **B** as

$$\underline{\mathbf{C}} = \begin{vmatrix} \hat{\mathbf{I}} & \hat{\mathbf{j}} & K \\ \mathbf{Ax} & \mathbf{Ay} & \mathbf{Az} \\ \mathbf{Bx} & \mathbf{By} & \mathbf{Bz} \end{vmatrix}$$

Or,

$$\underline{\mathbf{C}} = \hat{\imath} \begin{vmatrix} \mathbf{Ay} & \mathbf{Az} \\ \mathbf{By} & \mathbf{Bz} \end{vmatrix} \mathbf{-\hat{j}} \begin{vmatrix} \mathbf{Ax} & \mathbf{Az} \\ \mathbf{Bx} & \mathbf{Bz} \end{vmatrix} \mathbf{+k} \begin{vmatrix} \mathbf{Ax} & \mathbf{Ay} \\ \mathbf{Bx} & \mathbf{By} \end{vmatrix}$$

$$\mathbf{= (AyBz - AzBy)}\, \hat{\imath} + \mathbf{(AzBx - AxBz)}\, \hat{\mathbf{j}} \quad + \quad \mathbf{(AxBy - AyBx)k}$$

3.1.5 Unit Vector

This is a vector of a unit magnitude, it is usually associated with very vector i.e., for vector **A** on the 3-dimensional plane.

$\underline{\mathbf{A}} = /\underline{\mathbf{A}}/\hat{r}$

Where $\hat{r}$ is a unit vector in the direction of **A**.

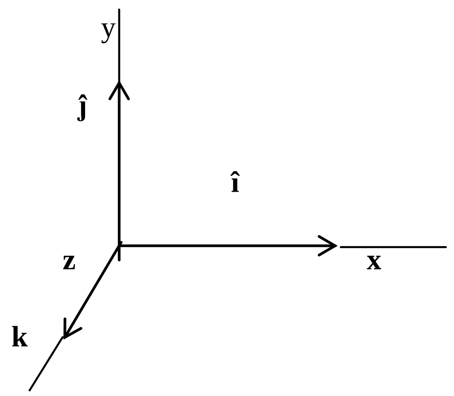


Fig. 3.4: Unit Vector

The defining three unit vectors î, ĵ, κ are parallel to the x-, y- and z respectively.

Hence, we write r as

$$r = \frac{\mathbf{Ax\hat{\imath} \;+\; Ay\hat{\jmath} \;+Akz}}{\mathbf{/A/}}$$

$$= \frac{\mathbf{Ax\hat{i} \ + \ Ay\hat{j} \ + Akz}}{\sqrt{Ax^2 + Ay^2 + Az^2}}$$

Note that the product of same unit vector is unity i.e.,

î.î = ĵ.ĵ = k.k = 1

and î.ĵ = ĵ.κ = κ.î = 0

Similarly, the cross products among the unit vectors are;

$\hat{i} \wedge \hat{j} \ = \ -\hat{j} \wedge \hat{i} \ = \ k$

$\hat{j} \wedge \hat{i} \ = \ -k \wedge \hat{j} \ = \ \hat{i}$

$k \wedge \hat{i} \ = \ -\hat{i} \wedge k \ = \ \hat{j}$

$\hat{i} \wedge \hat{i} = \hat{j} \wedge \hat{j} = k \wedge k = 0$

SOLVED PROBLEMS

While the direction θ = tan^{-1} (-12/6) = -63.43 degrees

2. Determine the angle between the vectors

<u>A</u> = -î + ĵ + 2K and

<u>B</u> = 2î + 3ĵ –K

<u>Solution</u>

Recall that;

<u>A</u> . <u>B</u> = /<u>A</u> / / <u>B</u> / Cos θ and

<u>A</u> . <u>B</u> = AxBx + AyBy + AzBz

<u>A</u> . <u>B</u> = (-1) (2) + (1) (3) + (2) (-1)

= -2 +3-2

= -1

/<u>A</u> / = $\sqrt{1 + 1 + 4}$ = 2.45 units

/B / = $\sqrt{4+9+1}$ = 3.74 units

Cos θ = $\frac{A\ .\ B}{/A//B/}$

θ = cos^{-1}(-1/9.17) = cos^{-1}(-1.09)

θ = 96.3 degrees.

1. What is the vector product of the vectors

A = 2î + 3ĵ - K

B = -î + ĵ +2K

Solution

A∧ B = | **î** **ĵ** ***k*** | ***î*** ***3*** ***-1*** **-ĵ** | **2** **-1** ***K*** | **2** **3** |

2 | **3** **-1** = **1** | **2** **-1** **1** | **-1** | **1**

-1 | **1** **2**

= 7î - 3ĵ + 5K

ITQ

Question

The following are examples of scalar quantity except--------

(a) mass

(b) length

(c) power

(d) temperature

Feedback

The correct answer is power because a scalar quantity is a quantity that can be completely described by a magnitude, that is, by a number and a unit

Study Session Summary

Summary

In this Study Session, we discussed scalar and vector quantities. We also explained addition and subtraction of vectors. Special attention was paid to dot product and cross product (multiplication of vectors).

Assessment

Assessment

SAQ 3.1 (tests Learning Outcome 3.1)
Find the resultant of the following vectors:

$\underline{A}1 \quad = -3\hat{\imath} + 2\,\hat{\jmath}$

$\underline{A}2 \quad = 2\hat{\imath} - 6\,\hat{\jmath}$

$\underline{A}3 \quad = 7\hat{\imath} - 8\,\hat{\jmath}$

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 4

Kinematics

Introduction

It is a fact of everyday experience that objects in the real world appear to be in a continual state of relative motion because motion is defined as a change of position of a body, depending on time. In this study session, we will describe motion in a straight line with four parameters. We will also discuss projectile and give examples. Finally, we will examine the term uniform circular motion.

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

4.1 *describe* motion in a straight line

4.2 *discuss* projectile and give examples

4.3 *examine* the term uniform circular motion

Terminology

Kinematics	the branch of mechanics concerned with the motion of objects without reference to the forces that cause the motion
Projectile	an object upon which the only force is gravity

4.1 Motion in a Straight Line

This is also known as a linear motion or rectilinear motion that is, a motion of object in a straight line. There are four parameters required to describe it. These are distance or displacement (s), speed or velocity (v), acceleration (a) and time (t). There are also four types of motion, which are; Random, Translational, Rotational, and Oscillatory motion.

4.1.1 Displacement

This is defined as the distance travelled in a specified direction (s). It is an example of vector quantity because it has both size and direction. E.g.

supposed a car moves from point A to point B in North-East direction and covered a distance of 5 *m*. It is illustrated in figure 4.1 below.

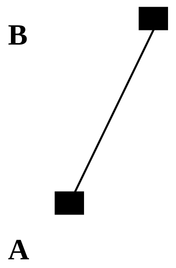


Figure 4.1:Illustrating the idea of displacement

4.1.2 Speed

This is the rate of change of distance with time (***v***).

V = d/t where ***s*** refers to change in distance and ***t*** change in time. It is a scalar quantity. It is measured in ***m***/***s***.

Uniform Speed

Supposed a body travels a distance ***s*** in a time interval ***t***, and then the average speed ***v*** of the body is

$$V = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \text{a constant}$$

4.1.3 Velocity

This is defined as the rate of change of distance with time in a given direction or is a speed measured in a given direction. It is therefore a vector quantity. The SI unit is m/s.

$$V = \frac{\text{change in distance}}{\text{Time taken}}$$

Instantaneous Velocity

This is the rate of change of displacement with time during a very short interval of time around that instant. If the short time interval is Δt a change of displacement Δs take place. Then, the instantaneous velocity is the limit of the ratio $\frac{\Delta s}{\Delta t}$ as the interval approaches zero.

$$V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

ITQ

Question

The following parameters are used to describe motion in a straight line except------------

(a) distance or displacement (s)

(b) Newton (n)

(c) acceleration (a)

(d) time (t)

Feedback

The correct answer is (b), Newton, n.

4.1.4 Acceleration

This is defined as the rate of change of velocity with time. It is also a vector quantity, the SI unit of acceleration is ms^{-2}.

$$a = \frac{\text{velocity change}}{\text{time taken to make change}} = \frac{v-u}{t}$$

Uniform Acceleration

If the rate of change of velocity with time is constant, the acceleration is said to be uniform i.e. if the motion of an object whose velocity increases by equal amounts in equal time intervals.

$$a = \frac{v}{t} = \text{constant.}$$

The opposite of acceleration is **deceleration** that is if the velocity of a body is decreasing with time. In this case, acceleration is negative.

4.1.5 The Equations of Motion

By definition, acceleration a = rate of change of velocity and since a is constant we have

$$a = \frac{v-u}{t}$$

Where u = initial velocity of the body,

v = velocity at time t,

a = the uniform acceleration

Then, at = v-u

V = u + at .... (i)

The distance s, travelled by the body during the time interval is average velocity × time i.e.

S = $\frac{v+u}{2} \times$ t .... (ii)

Putting equation (i) into (ii) in order to eliminate v

S = $\frac{u+at+u}{2} \times$ t

S = $\frac{2ut+at^2}{2}$

S = ut + ½at^2 .... (iii)

From equation (ii) substitute for t

S = $\frac{v+u}{2} \times \frac{v-u}{a}$

2as = v^2 - u^2

v^2 = u^2 + 2as .... (iv)

WORKED EXAMPLES

Example 1

A car travels from rest with an acceleration of $2ms^{-2}$. Calculate its velocity after travelling 9m.

Solution

From rest, u = 0, a =$2ms^{-2}$, s = 9m

$Recall, v^2 = u^2 + 2as$

$= 0 + 2 \times 2 \times 9$

$= 36$

$v = \sqrt{36}$

$v = 6 ms^{-1}$

Example 2

A train has a uniform velocity of 108 km/h. how far does it travel in ½ minute?

Solution

108 km/h = $\frac{108 \times 1000}{60 \times 60}$ = 30 m/s

S =vt

V = 30 m/s

t = 1/2 minute = 30s

$\therefore$ s = 30 × 30 = 900m = 0.9km

Example 3

An aeroplane lands on the runway with a velocity of 50 ms^{-1} and decelerate at 10 ms^{-2} to a velocity of 20ms^{-1}. Calculate the distance travelled on the runway.

Solution

Recall,

$v^2 = u^2 + 2as$,

$20^2 = 50^2 + (2\times -10 \times s)$

400 = 2500 – 20s

$S = \frac{2500-400}{20}$

S = 105 m

4.2 Projectile

This is a two-dimensional motion of an object launched into space without any motive power of its own; therefore, it travels freely under the action of gravity and air resistance alone. A projectile is an object whose curved motion in space is affected or influenced by gravity alone.

EXAMPLES

i. Arrow or bullet shot into space;
ii. A stone shot from a catapult;
iii. A basket or football kicked into space;
iv. A tennis ball thrown against a vertical wall.

The motion of the projectile is always in two forms;

a) A constant horizontal motion along X-part,
b) A vertically downward acceleration of free fall due to gravity along Y-part.

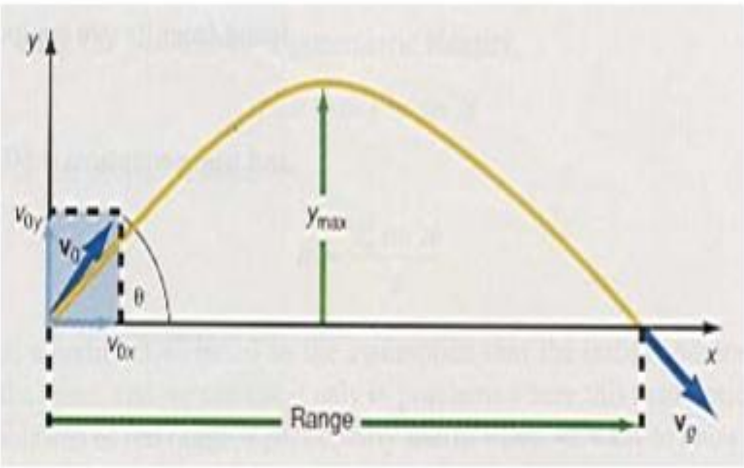


Figure 4.2: Projectile Motion

Table 3.1: The descriptive equations are tabulated as follows.

Description	X- Part	Y-Part
a. initial velocities (resolved U_0) b. Velocities at time (t) c. Particle Co-ordinate at time (t)	U_0Cos θ $V_x = U_0$ Cos θ X = (U_0 Cos θ)t	U_0Sin θ $V_y = U_0$ Sin θ – gt Y = U_0Sin θt - ½gt^2

Other useful equations are derived from the equations above. The time (t) taken to reach the maximum height i.e. the peak is gotten from the y- part (b) by setting V_y = 0

0 = U_0 Sin θ – gt

gt = U_0 Sin θ

$$t = \frac{U_0 \sin\theta}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

The time of flight, (T) of a projectile is the time required for the projectile to return to the level from which it was projected.

$$T = 2t = \frac{2U_0 \sin\theta}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

The maximum height, (H), of a projectile is defined as the highest vertical distance reached by the projectile as measured from the horizontal projection plane

We set y = H and t = $\frac{U_0 \sin\theta}{g}$ in y-part of equation (c)

Y = U_0Sin θt - ½gt^2

$$H = U_0 \sin\theta \times \frac{U_0 \sin\theta}{g} - ½g\frac{U_0 \sin\theta}{g}$$

$$H = \frac{U_0{}^2 \sin^2\theta}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

The range, (R), of a projectile is the horizontal distance from the point of projection to the point where the projectile hits the projectile plane.

Substitute equation 1 into the x-part of equation (c)

X = (U_0 Cos θ) T

$$X = (U_0 \cos\theta) \times \frac{2U_0 \sin\theta}{g}$$

X = 2 U_0^2 Cos θ Sin θ/ g

2 Cos θ Sin θ = sin2θ

$$R = \frac{U_0^2 \sin 2\theta}{g} \text{ .... (4)}$$

To get the maximum range, we set θ = 45^0 and the maximum peak θ = 90^0

$$R_{max} = \frac{U_0^2}{g}$$

ITQ

Question

-------- is an object whose curved motion in space is affected or influenced by gravity alone

(a) force

(b) projectile

(c) acceleration

(d) distance

Feedback

The Correct answer is (b) that is to mean that a projectile is an object whose curved motion in space is affected or influenced by gravity alone

The descriptive equations formulated are valid provided the following assumption holds:

a. The range should be small so that the earth curvature may be neglected
b. The peak (maximum height) should be small so that variation of gravity with altitude is neglected
c. The initial velocity should also be small for air resistance to be neglected.

WORKED EXAMPLES

Example 1

A ball is projected upwards with a speed of 20m/s, and attains a maximum height of 5 m. Calculate (a) the angle of projection (b) the time of flight, and (c) the horizontal range of the ball.

Solution

(a) The maximum height, H, is given by

$$H = \frac{U_0^2 \, Sin^2 \theta}{2g}$$

By substitution

$$5 = \frac{20^2 \, Sin^2 \theta}{2 \times 10}$$

Sin θ = $\sqrt{0.25}$ = 0.5

$\theta = 30^0$

(b) The time of flight, T, is given by

$$T = \frac{2U_0 \, \text{Sin}\,\theta}{g}$$

$$= \frac{2 \times 20 sin 30^0}{10}$$

= 2s

(c) The horizontal range, T, is given by

$$R = \frac{U_0^2 \, \sin 2\theta}{g}$$

$$= \frac{20^2 \times \sin 60^0}{10}$$

= 34.6 m.

4.3 Uniform Circular Motion

There are many examples of bodies involving in circular path.

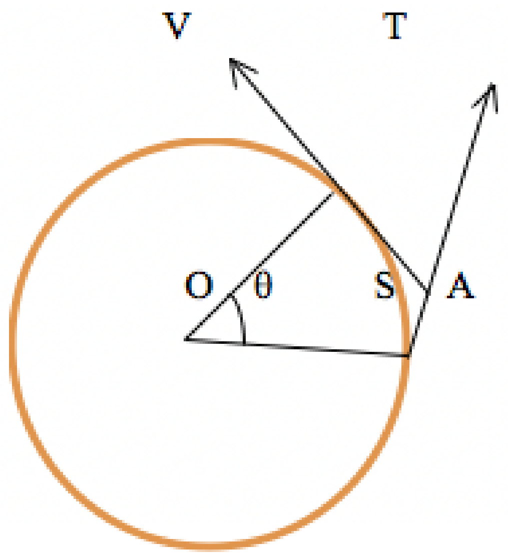


Figure 4.3: Circular motion

Examples are:

i. The earth rotation round the sun
ii. Clothes circling in a spin drier
iii. Planetary revolution;
iv. Thee moon circling round the earth
v. A car moving in a circle at the constant speed of 20 km/hr is an example of a body in uniform circular motion. At every point on that circle the car would be moving at 20 km/hr.
vi. Motion in a circle with changing speeds

4.3.1 Definition of Terms

Uniform circular motion: This is defined as motion in a circle at constant speed.

Angular Speed: This can be define as the change of the angle per second

$\boldsymbol{\omega} = \frac{\boldsymbol{\theta}}{\boldsymbol{t}}$

$\boldsymbol{\theta} = \boldsymbol{\omega t}$, $\boldsymbol{\theta}$ is the angle measured in radians**.** **(**$\boldsymbol{2\pi\ radians = 360^0}$ **)**

Period: T = $\frac{\boldsymbol{2\pi}}{\boldsymbol{\omega}}$

2π Radian is the angle in 1 revolution(360^0)

Length of an arc s = r θ = distance

$\theta = \frac{s}{r}$

Speed = s/t = $\frac{r\theta}{t}$

Hence, v = r $\boldsymbol{\omega}$

Acceleration in a circle = $\frac{v^2}{r}$

Since v = r $\boldsymbol{\omega}$**,**

$a = \frac{r^2\omega^2}{r} = r\omega^2$

Centripetal force: This is the force F, required to keep an object of mass m moving in a circle of radius r

$F = ma = \frac{mv^2}{r}$

Worked Examples

A model car moves round a circular track of radius 0.3 m at 2 revolutions per second. What is?

(a) The angular speed $\boldsymbol{\omega}$
(b) The period **T**
(c) The speed v of the car?

Solution

(a) For 1 revolution, angel turned $\theta = 2\pi\ radians 360^0$

$\omega = 2 \times 2 = 4\pi$ **rad/s**

(b) Period T = time for I rev = $\frac{2\pi}{\omega} = \frac{2\pi}{4\pi}$ = 0.5 s

(c) Speed v = **r** $\boldsymbol{\omega}$= 0.3 × 4π 3.8 m/**s**

ITQ

Question

……..can be defined as the change of the angle per second

(a) Uniform circular motion
(b) Angular speed
(c) Acceleration/deceleration
(d) Centripetal force

Feedback

Angular speed can be defined as the change of the angle per second. The correct answer is (b)

Study Session Summary

Summary

In this Study Session, we were made able to differentiate between distance and displacement; also, speed, velocity and acceleration were explained. We also discussed projectile and gave examples. Finally, we examined the term uniform circular motion and defined some terms.

Assessment

Assessment

SAQ 4.1 (tests Learning Outcome 4.1)
What are the four types of motion?

SAQ 4.2 (tests Learning Outcome 4.2)
What are the two forms of motion of a projectile? Give two examples of a motion

SAQ 4.3 (tests Learning Outcome 4.3)

A car travels from rest with an acceleration of $2ms^{-2}$. Calculate its velocity after travelling $4m$.

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Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 5

Newton's Law of Motion

Introduction

In this study session, we will discuss Newton's law of motion and define all the three (3) types of Newton's law.

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

4.1 *define* the three Newton's law of motion

4.2 *list* and *explain* various types of force

Terminology

Inertial force	a force that resists a change in velocity of an object. It is equal to—and in the opposite direction of—an applied force, as well as a resistive force
Centrifugal force	a force that acts on a body moving in a circular path and is directed towards the center around which the body is moving
Centripetal force	an apparent force that acts outward on a body moving around a center, arising from the body's inertia

5.1 Newton's Laws of Motion

In 1687, Sir ISAAC NEWTON published a work called *Principia Mathematical, in* which he set out clearly the laws of Mechanics. We shall consider these laws as he gave them:

First law: Everybody continues to be in a state of rest or to move with uniform velocity unless a resultant force acts on it.

Second law: The change of momentum per second is proportional to the applied force and the momentum change takes place in the direction of the force.

Third law: To every action there is equal and opposite reaction.

5.1.1 Force and Motion

INERTIAL: The inertial of a body is its reluctance to start moving, and its reluctance to stop after it has begun moving. This is expressed by the first law of Newton which states that an object will remain in its state of rest or continue in motion unless it is acted upon by an external force. Inertial of a body depends on its mass i.e. the quantity of matter contained, the greater the mass the higher the inertial. The mass of a body is constant at every point on the earth surface whereas the weight varies from place to place.

5.1.2 Atwood's Machine

Atwood's machine is a system that consists of a pulley, with a mass mA on one side, connected by a string of negligible mass to another mass mB on the other side, as shown in figure 4.1. We assume that mA is larger than mB. When the system is released, the mass mA will fall downward, pulling the lighter mass mB, on the other side, upward. We would like to determine the acceleration of the system of two masses. When we know the acceleration, we can determine the position and velocity of each of the masses at any time from the kinematic equations. Let us start by drawing all the forces acting on the masses in figure 4.1 and then apply Newton's second law to each mass.

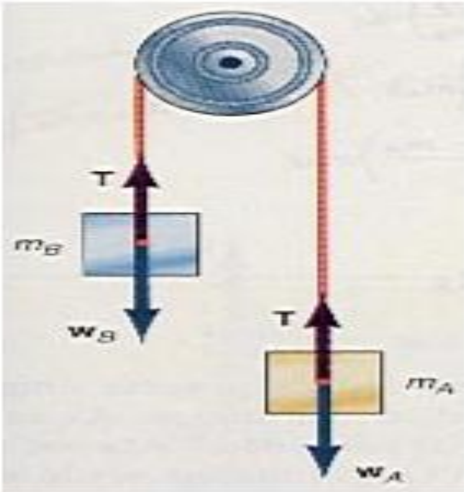

Figure 5.1: **Atwood's machine**

(The assumption that the tension T, in the rope is the same for each mass is again utilized. On rotational motion where the rotating pulley is massive and hence the tensions on both sides of the pulley are not the same). For mass A, Newton's second law is:

F_A=mA_a …………………………………1

or

T + wA = mAa ……………………......2

We can simplify this equation by taking the upward direction as positive and the downward direction as negative, that is,

T − wA = −mAa …………………………3

We cannot yet solve for the acceleration of the system, because the tension T in the string is unknown. Another equation is needed to eliminate T. We obtain this equation by applying Newton's second law to mass B:

FB = mB a T + wB = mB a ………………4

Simplifying again by taking the upward direction as positive and the downward direction as negative, we get

T − wB = + mBa…………………………..5

We thus have two equations, 3 and 5, in the two unknowns of acceleration a and tension T. The tension T is eliminated by subtracting equation 5 from equation 3. That is,

T − wA = −mAa

Subtract T − wB = mBa

$$T - wA - T + wB = -mAa - mBa$$

$$wB - wA = -(mA + mB)a$$

Solving for a, we obtain

$$a = \frac{wA - wB}{mA + mB}$$

$$= \frac{mA\,g - mBg}{mA + mB}$$

Hence, the acceleration of each mass of the system is

$$\propto \;=\; \left(\frac{mA - mB}{mA + mB}\right)g$$

We find the tension T in the string from equation 3 as

$$T = wA - mAa$$

$$T = mAg - mAa$$

Hence,

$T = mA(g - a)$ is the tension in the string of the Atwood's machine.

A Passenger in a Lift

Example

A man stands on a scale in an elevator. If the scale reads 600N when the elevator is stationary, what will it read when the elevator is

i. Ascending with an acceleration of 2 ms^{-2}
ii. Descending with an acceleration of 2 ms^{-2}
iii. Moving with a constant speed
iv. Descending with an acceleration due to gravity

Comment on your results.

Solution

N-W = 0 (i.e. no acceleration)

Or N = W, the man feels his own weight which is equal to the reaction force.

The mass of the man, **m =** $\frac{w}{g}$ **= 600/9.8 = 61.2kg**

i. The elevator and the man ascend with a = 2 ms^{-2}

N – W = ma

N = ma + W

N = (61.2kg) (2ms^{-2}) + 600N

And N = 722.4 newton

i.e. the man feels heavier than his normal weight.

ii. The elevator and the descend with a = 2 ms^{-2}

W – N = ma

N = W – ma = 600 – (61.2) (2 ms^{-2})

N = 477.6 newton

i.e. the man feels higher than his normal weight

iii. At constant speed, the acceleration, a = 0ms^{-2}

Therefore, $N - W = 0$

$N = W$ the man feels his true weight (W)

iv Elevator and the man descending with $a = g$

$W = N = ma$

But $a = g$,

$W - N = mg$

and $N = W - mg = mg - mg = 0$

The man feels weightless.

ITQ

Question

Isaac Newton's ----------law of motion states that the change of momentum per second is proportional to the applied force and that the momentum change takes place in the direction of the force.

(a) First

(b) Second

Feedback

The correct answer is (b). Second law of motion states that the change of momentum per second is proportional to the applied force and the momentum change takes place in the direction of the force.

5.2 Force and its Types

Force in general, is the agency of change. In mechanics, it is a pull or a push that changes the velocity of an object. Force is a vector quantity, having both magnitude and direction.

Note

A force is a push or pull upon an object resulting from the object's interaction with another object. Whenever there is an interaction between two objects, there is a force upon each of the objects

There are external and net force, an external force is one whose source lies outside of the system being considered. Net or resultant external force acting on an object causes the object to accelerate in the direction of that force. The acceleration is proportional to the force and inversely proportional to the mass of the object. The SI unit of force is Newton (N).

F = $\propto ma$,

F = kma,

if $k = 1$,

then F = ma

5.2.1 Types of Forces

There are different types of force such as:

Tensile Force: The tensile force ($\boldsymbol{F_T}$ **)** acting on a string, chain or tendon is an applied force tending to stretch it.

Normal Force: The normal force($\boldsymbol{F_N}$) is the force acting perpendicular to the surface.

Frictional Force: This is the tangential force **(** $\boldsymbol{F_f}$**)** acting on an object that opposes the sliding of that object on an adjacent surface with which it is in contact**.**

The Coefficient of Kinetic Friction: This is defined for the case of one surface sliding across another at constant speed.

$$\boldsymbol{\mu_k} = \frac{\textbf{Frictional force}}{\boldsymbol{Normal\ force}} = \frac{\boldsymbol{F_f}}{\boldsymbol{F_N}}$$

Centripetal Force: This is the force $F_{C,}$, which must act on a mass m, moving in a circular path of radius r to give the centripetal acceleration$\boldsymbol{\frac{v^2}{r}}$**.**

From **F = ma**

$$\mathbf{F} = \boldsymbol{\frac{mv^2}{r}} = \boldsymbol{m\omega^2}$$

5.2.2 Conical Pendulum

If a small object, A of mass *m,* is tied to a string O-A of length *l,* and then whirled round in a *horizonta*l circle of radius *r,* with O fixed directly above the centre B of the circle. Suppose the circular speed of A is constant, the string turns at a constant angle θ to the vertical. This is called conical pendulum.

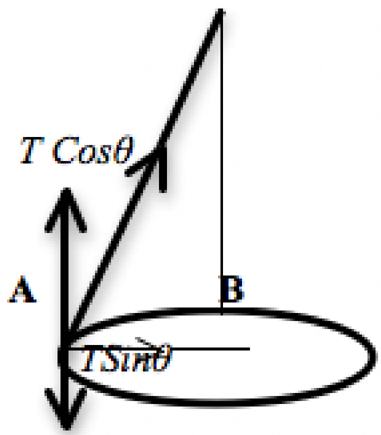

Figure 5.2: Conical pendulum

Since A moves with a constant speed *v* in a circle of radius *r*, there must be a centripetal force $\frac{mv^2}{r}$acting towards the centre B. The horizontal component, *T Sin θ,* of the tension *T* in the string provides this force along AB. So

$$T \text{ Sin } \theta = \frac{mv^2}{r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

Since the mass does not move vertically, its weight *mg* must be counterbalance by the vertical component *T Cos θ* of the tension.

$$\therefore T \text{ Cos } \theta = mg \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

Dividing equation 1 by 2,

$$\text{Tan } \theta = \frac{v^2}{rg} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

Centrifugal Force: This is the reaction force to the centripetal force. The reaction force does not act on the same body as the centripetal force. That is, if a string was tied to a rock and the rock was swung in a horizontal circle at constant speed, the centripetal force would act on the rock while the centrifugal force would act on the string.

ITQ

Question

The following are types of force except --------

(a) Tensile Force

(b) Normal Force

(c) motion force

(d) Frictional Force

Feedback

The correct answer is ‘C’, Motion force

Study Session Summary

Summary

In this Study Session, we highlighted and defined Newton’s first, second and third law of motion. We also discussed force and noted that frictional forces, normal force, tensile force among others are types of force.

Assessment

Assessment

SAQ 5.1 (tests Learning Outcome 5.1)

Differentiate between the first and third Newton’s law of motion

SAQ 5.2 (tests Learning Outcome 5.2)

List and explain three types of force

Bibliography

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http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 6

Gravitational Force of Attraction

Introduction

The space round the earth where the mass of an object experiences a gravitational pull, or force due to gravity is called the gravitational field of the earth. In this study session, we will discuss gravitational force thereby explaining acceleration due to gravity

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

5.1 *examine* gravitational force and *explain* acceleration due to gravity

Terminology

Gravitational force	a force that attracts any object with mass
Satellite	an artificial object which has been intentionally placed into orbit

6.1 Gravitational Force of Attraction

6.1.1 Acceleration Due to Gravity

When an object falls to the ground under gravitational pull that is freely falling under gravity, it is shown that the object has a constant or uniform acceleration of about 9.8 ms^{-2} or approximately to 10 ms^{-2} when it is falling. The acceleration due to gravity ‘‘g ’’ is directed downward towards the earth as shown in the figure below and varies slightly from place to place on the earth’s surface i.e. its value is least at the equator while greatest at the poles.

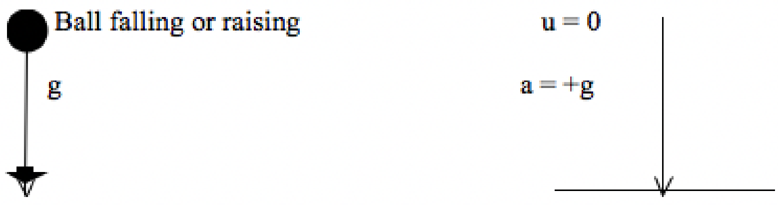

Figure 6.1: Motion under gravity-free fall

g is given as $g = \frac{F}{M}$

The equations of motion now become;

$V = u \pm gt$

$S = ut \pm ½gt^2$

$v^2 = u^2 \pm 2gs$

Note for raising objects, u = 0, v = 0 and g = -ve

While, for falling objects, u = 0, v ≠ 0 and g = +ve in value.

Example 1

A ball is thrown vertically upwards with an initial velocity of 30 m/s. Find (i) the time taken to get to its highest point (ii) the distance travelled (assume g = 10 ms^{-2})

SOLUTION

U = 30 m/s, V = 0, a = g = -10 ms^{-2}

(i) Recall, $V = u + at$
0 = 30 – 10 × t

10t = 30

t = 3 s

(ii) Distance $S = ut + ½at^2$
S = (30 × 3) + ½ × (-10) × 3^2
= 90 – 45 = 45 m

6.1.2 Orbit Round the Earth

Satellites

These are bodies, which move in orbits around the moon or planet. It can be lunched from the earth's surface to circle the earth. They are kept in space by the gravitational attraction of the earth.

Let us consider a satellite of mass m which circle the earth of mass M in an orbit, close to its surface as seen in figure 6.2

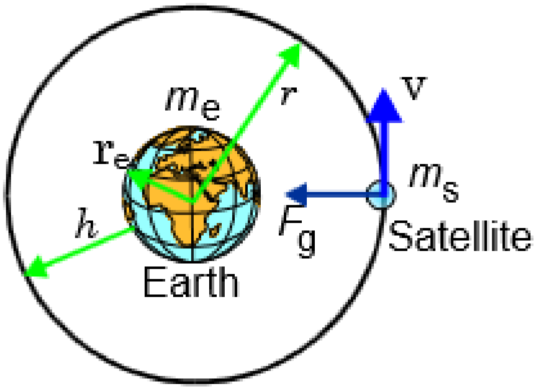

Figure 6.2: Orbit round the Earth

Let us assume that the earth is spherical and its radius r, the centripetal force = the gravitational force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} = \mathbf{mg}$$

Where g = acceleration due to gravity, and v = the velocity of m in its orbit

$$\therefore v^2 = \mathbf{rg}$$

And $\mathbf{v} = \sqrt{rg} = \sqrt{6.4 \times 10^6 \times 9.8} = 8\ kms^{-1}$

Parking of Orbits

If a satellite of mass m is circling the Earth in a plane of the equator in an orbit concentric with the earth, and if it moves with velocity v, in the same direction of rotation as the earth at a distance R from the centre of the earth, therefore;

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

But **GM =** gr^2 where r = radius of the earth

$$\therefore \frac{mv^2}{R} = \frac{mgr^2}{R^2}$$

$$\therefore v^2 = \frac{gr^2}{R}$$

And $v = \sqrt{\frac{gr^2}{R}}$

If the period in the second orbit is T_1, then

$$V = \frac{2\pi R}{T_1}$$

$$\therefore \frac{4\pi^2 R^2}{{T_1}^2} = v^2 = \frac{gr}{R}$$

$${T_1}^2 = \frac{4\pi^2 R^3}{gr^2} \text{ and } T_1 = \sqrt{\frac{4\pi^2 R^3}{gr^2}}$$

If the period of the satellite in its orbit equals the period of the earth as it turns about its axis i.e. 24 hours, the satellite will stay at the same place above as the earth rotates. The orbit is called ‘‘parking orbit’’

6.1.3 Earth Gravitational Potential

The gravitational potential energy = Mgh. The potential *V*, at a point due to the gravitational field of the earth is defined as numerically equal to the work done in taking a unit mass from infinity to that point.

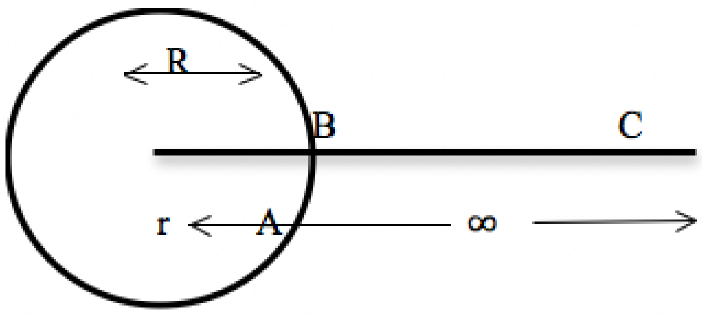

Figure 6.3: Gravitational potential

If the earth is spherical, it can be imagined that the whole mass M of the earth, is concentrated at its centre. The potential at a distance r from the earth centre is given as $V = -\frac{GM}{r}$

Note that the negative sign is an indication that the potential at infinity (zero) is higher than the potential close to that of earth.

On the earth surface of radius R, the gravitational potential is

$V = -\frac{GM}{R}$ *where* $G = 6.7 \times 10^{-11} N\, m^2 kg^{-2}$, $M = 6.0 \times 10^{24}\ kg$,

$R = 6.4 \times 10^{6}\ m$

∴ The potential V, at the earth's surface is – $6.3 \times 10^{7} J\, kg^{-1}$.

ITQ

Question

Satellites are bodies which move in orbits around the moon or planet. They can be lunched from surface of the earth to circle the earth. TRUE/FALSE

Feedback

The correct answer is TRUE

Study Session Summary

Summary

In this Study Session, we discussed gravitational force of attraction and examined aacceleration due to gravity, satellites, parking orbits and gravitational potentials with examples

Assessment

Assessment

SAQ 6.1 (tests Learning Outcome 6.1)

A ball is thrown vertically upwards with an initial velocity of 90 m/s. Find (i) the time taken to get to its highest point (ii) the distance travelled (assume g = 10 ms^{-2})

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http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 7

Energy and Work

Introduction

In everyday language, we speak of mental work, physical work, medical work, scientific work, and so on. In this study session, we will differentiate kinetic energy from potential energy. We will also examine conservative force and explain its properties

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

7.1 *define* kinetic energy and *relate* it with work

7.2 *distinguish* potential energy from energy conservation

Terminology

Conservative force	a force with the property that the work done in moving a particle between two points is independent of the taken path
Kinetic energy	the energy that a body possesses by virtue of being in motion
Potential energy	the energy possessed by a body by virtue of its position relative to others, stresses within itself, electric charge, and other factors

7.1 Kinetic Energy and Work

In physics, energy is a property of objects, which can be transferred to other objects or converted into different forms. Energy is that which makes it possible for a force to do work? Energy may be thought of, as the property of something, which enables it to do work. When we say that something has energy, we suggest that it is capable of exerting a force on something else and performing work on it. When work is done on something, energy has been added to it. Energy is measured in Joules, the same units as work.

ITQ

Question

Energy is measured in……?

a. Newton
b. Joules
c. Metre per second
d. Volts

Feedback

The correct answer is ‘b’ because energy is measured in Joules

Energy takes on many forms. One type is the energy a moving body possesses by virtue of its motion. Every moving object has the capacity to do work. By striking another object that is free to move, the moving object can exert a force and cause the second object to shift its position. While the object is moving, it has the capacity for doing work. Energy means the ability to do work, so all moving things have energy by virtue of their motion. This type of energy is called **kinetic energy**.

The energy that an object possesses because it is moving, its kinetic energy, is defined as:

$$K.E. = \frac{1}{2}mv^2$$

Therefore, **work is done when a force moves**. An object acted upon by a force moves when work is done by the force. The force may accelerate the object, raise it, or change it shapes. The object that had been accelerated does work when it comes to rest. A raised object does work when it returns to its initial position. The ability to do work is known as **energy**. The energy of an object due to its motion, position or physical condition is known as **mechanical energy.**

Mechanical energy can either be in the form of **kinetic energy** or **potential energy**.

Recall that, kinetic Energy (K.E) is the energy of a body due to its motion. Suppose an object of mass m is acted upon by a constant force F and is displaced a distance s in the direction of the force. Work done by the force F is force applied in direction of displacement. This means that if the force F acts at an angle θ with respect to the direction of motion, then:

$$W = \vec{F} \cdot \vec{s} = Fs\,cos\theta$$

Work is a scalar quantity - it has magnitude but no direction.

Work has dimensions: $M \times (LT^{-2}) \times L = ML^2T^{-2}$

Work has units: 1 Newton x 1 Metre = 1 Joule (J)

If $\theta = 0$, then;

$$W = Fs$$

Since $F = ma$ where $a = acceleration\ produced$

$$W = (ma)s$$

If $u = initial\ velocity = 0\ and,$

$$v = final\ velocity\ of\ the\ body, then;$$

$$v^2 = u^2 + 2as = 0 + 2as$$

$$\therefore s = \frac{v^2}{2a}$$

$$W = (ma)s$$

$$= ma(\frac{v^2}{2a})$$

$$= \boldsymbol{\frac{1}{2}\,mv^2}$$

Work done by the force F is changed into the kinetic energy of the body.

Therefore, kinetic energy of the body, $K.E = \frac{1}{2}\,mv^2$.

If the initial velocity of the body $u \neq 0$,

Then, work done $W = Fs$

$$= (ma)s$$

From, $v^2 = u^2 + 2as$,

$$s = \left(\frac{v^2 - u^2}{2a}\right)$$

$$W = ma\left(\frac{v^2 - u^2}{2a}\right)$$

$$= \frac{1}{2}\,mv^2 - \frac{1}{2}\,mu^2$$

$$= increase\ in\ kinetic\ energy$$

i.e. $W = \Delta K.E.$

7.2 Potential Energy (P.E.) and Conservation of Energy

Potential energy is, as the word suggests, the energy "locked up" somewhere and which can do work. The energy stored in the object is called potential energy (P.E.). A stretched spring or rubber band has potential energy due to its stretched condition. Work is done in lifting an object against the earth's gravity. The raised object possesses more gravitational potential energy. Therefore, objects may contain the potential to do work, even if they aren't moving. This is call **potential energy**. It comes in several varieties:

1. Gravitational potential energy (GPE):
$$GPE = mgH$$
2. Spring potential energy (SPE):
$$SPE = \frac{1}{2}kx^2$$

The gain in gravitational potential energy when an object of mass m is raised through a height h is equal to the work done against gravity. The force required to raise a mass m without any acceleration is equal to the gravitational force, mg, acting on it.

$$\therefore gainingravitationalpotentialenergy$$
$$= workdoneagainstgravitationalforce$$
$$= Fh$$
$$= mgh$$

The energy stored in a stretched spring that obeys Hooke's law is given by $\frac{1}{2}kx^2$, where k is the force constant of the spring and x is the extension. As stated earlier, the energy of an object due to its motion, position or physical condition is known as **mechanical energy.** Mechanical energy can either be in the form of **kinetic energy** or

potential energy. Therefore, the **mechanical energy** of a system is the sum over all the energy of the bodies. That is:

$$Total\ Energy, E = K.E. + P.E.$$

Under some circumstances, the mechanical energy of a system is conserved, so that:

$$E_{initial} = E_{final}$$

However, if ***friction*** is present, then the mechanical energy is not conserved: friction does negative work on moving objects, which decreases their kinetic energy without adding any potential energy to make up for the loss. In this case, conservation of energy can be written as:

$$E_{initial} = E_{final} + (work\ done\ by\ friction)$$

Thus, the Principle of Conservation of Energy states that: In a closed system, energy can neither be created nor destroyed but, can only be transformed from one form to another. This principle can be summarized as: The total energy of a closed system is conserved.

In this context, system means situation that consists of one or more bodies. A closed system is a system where no external forces act on the body (or bodies) in the system.

Assuming a system where the total energy, (T.E.) consists of kinetic energy (K.E.) and potential energy (P.E.), then;

$$K.E. + P.E. = T.E,\ a\ constant$$

Therefore,

$$\Delta K.E. + \Delta P.E. = 0$$

$$\Delta K.E. = -\Delta P.E.$$

This means that any increase in kinetic energy ($\Delta K.E. positive$) is equal to the decrease in potential energy ($\Delta P.E. negative$).

For instance, when a mango fruit of mass m falls out from a mango tree, after falling through a distance h, its velocity is v.

Then kinetic energy of the $mango\ fruit = \frac{1}{2}mv^2$ and it's initial $kinetic\ energy = 0$.

$$\therefore\ \Delta K.E. = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2.$$

Using $v^2 = u^2 + 2as = 0 + 2gh\ (since\ u = 0, a = g\ and\ s = h)$

$$\Delta K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m(2gh)$$

$$= mgh$$

$$= lossinggravitationalpotentialenergy$$

Gravitational force on the mango, $F = mg$

Work done by the force F is $W = Fh$

$= mgh = \Delta K.E.$

This implies that the work done by the gravitational force is converted into the gain in kinetic energy. Furthermore, when an object has potential to have work done on it, it is said to have potential energy, e.g. a ball in your hand has more potential energy than a ball on the ground. If you release the ball, gravity will perform work on the ball and its kinetic energy will increase. If a spring with a block attached is compressed, the block has potential energy because if the block is released, the spring will perform work on the block and give it kinetic energy.

In general,

- Potential energy is energy associated with the configuration of two or more objects, e.g. the ball and the earth or the block and the spring.
- A force is acting between objects. When the configuration of the system changes and work is performed on one of the objects, the potential energy of the system is transferred to the kinetic energy of that object.
- If the system returns to the original configuration, the force between the objects removes kinetic energy from the objects and stores it in the potential energy of the system.

- The amount of work done in the original change and the reversal are equal in magnitude but differ by a sign: $W_i = -W_f$
- The force in the system is known as a conservative force.

7.2.1 Conservative Force

We will consider the following, which can be regarded as conservative force

a. Can store energy in the system as potential energy

b. Can retrieve that energy and give it to an object in the system as kinetic energy.

c. Gravitational and spring forces are conservative forces.

d. Friction is a non-conservative force. If we let a block scrape along a rough floor, the friction force will take kinetic energy away from the box. However, if we reverse the block and attempt to put it back in its original position, we do not retrieve the energy out of the system. The friction force has converted the kinetic energy into thermal energy by heating up the block and the floor. The thermal energy cannot be turned back into the kinetic energy of the block.

Properties of Conservative Force:

The net work done by a conservation force as the object moves from point 1 to point 2 and back to point 1 is ***zero***.

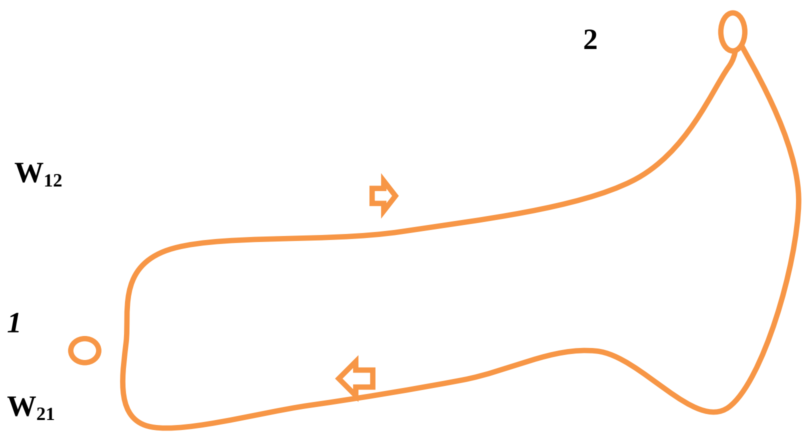


Figure 7.1: Illustration of Conservative Force

Thus;

$$W_{12} = -W_{21}$$

The work done by a conservative force in moving the object from point 1 to point 2 does not depend on the path between the two points.

ITQ

Question

The energy of an object due to its motion, position or physical condition is known as-----------

(a) mechanical energy

(b) electrical energy

(c) chemical energy

Feedback

The correct answer is (a).Mechanical energy can also either be in the form of **kinetic energy** or **potential energy**.

Study Session Summary

Summary

In this Study Session, we differentiated kinetic energy from potential energy. We also examined conservative force and explained its properties

Assessment

Assessment

SAQ 7.1 (tests Learning Outcome 7.1)

What is the mathematical representation for kinetic energy?

SAQ 7.2 (tests Learning Outcome 7.2)

Differentiate between conservative force and the principle of conservation of energy

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 8

Linear Momentum and Collision

Introduction

Newton defined the force acting on an object as the rate of change of its momentum, the momentum being the product mass and velocity. In this study session, we will consider linear momentum and collisions. We will state the principle of conservation of linear momentum. Lastly, we will define collisions, list its features and highlight its two types as well.

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

8.1 *state* the principle of linear momentum

8.2 *define* collision and *list* its features

Terminology

Collision	an instance of one moving object or person striking violently against another
Momentum	the quantity of motion of a moving body, measured as a product of its mass and velocity

8.1 Linear Momentum and Collisions

The linear momentum of a body can be defined as the product of its mass and its velocity.

That is,
$$linearmomentum = mass \times velocity$$
$$\boldsymbol{linearmomentum(p) = mv}$$

The momentum of a body is a vector quantity, and it occurs in the direction of the velocity.

8.1.1 The Principle of Conservation of Linear Momentum

The Principle of Conservation of linear momentum states that the linear momentum of a closed system before collision is equal to the linear momentum after collision.

This means that; **the total linear momentum of a system is constant, if no external forces act on the system.**

8.1.2 Collisions

A **collision** is an event in which two or more bodies exert forces on each other for a relatively short time. Although the most common colloquial use of the word "collision" refers to incidents in which two or more objects collide, the scientific use of the word "collision" implies nothing about the magnitude of the forces.

Some examples of physical interactions that scientists would consider collisions:

- An insect touches its antenna to the leaf of a plant. The antenna is said to collide with leaf.
- A cat walks delicately through the grass. Each contact that its paws make with the ground is a collision. Each brush of its fur against a blade of grass is a collision.

Some colloquial uses of the word collision are:

- automobile collision, two cars colliding with each other
- mid-air collision, two planes colliding with each other
- ship collision, two ships colliding with each other

Features of Collision

A collision is a phenomenon, which has these features:

1. It occurs in a short time interval.
2. What happen after the collision differs from what happens before the collision.
3. The colliding bodies may be assumed to constitute a closed system.
4. Momentum and energy are conserved during the collision.

Therefore, collision is short-duration interaction between two bodies or more than two bodies simultaneously causing change in motion of bodies involved due to internal forces acted between them during this time. Collisions involve forces (there is a change in velocity). The magnitude of the velocity difference at impact is called the closing speed. All collisions **conserve momentum.** What distinguishes different types of collisions is whether they also conserve kinetic energy.

Line of impact – It is the line which is common normal for surfaces are closest or in contact during impact. This is the line along which internal force of collision acts during impact and **Newton's coefficient of restitution** is defined only along this line.

Specifically, collisions can either be ***elastic***, meaning they conserve both momentum and kinetic energy, or ***inelastic***, meaning they conserve momentum but not kinetic energy. An inelastic collision is sometimes also called a ***plastic collision***.

ITQ

Question

The colloquial uses of collision include all of the following except one..

a. automobile collision, two cars colliding with each other
b. An insect touches its antenna to the leaf of a plant. The antenna is said to collide with leaf
c. mid-air collision, two planes colliding with each other
d. ship collision, two ships colliding with each other

Feedback

The correct answer is (b) that is ‘‘an insect touches its antenna to the leaf of a plant. The antenna is said to collide with leaf’’.

A “perfectly inelastic” collision (also called a "perfectly plastic" collision) is a limiting case of inelastic collision in which the two bodies stick together after impact. The degree to which a collision is elastic or inelastic is quantified by the coefficient of restitution, a value that generally ranges between zero and one. A perfectly elastic collision has a coefficient of restitution of ***one***; a perfectly inelastic collision has a coefficient of restitution of ***zero***.

Types of Collisions

There are two types of collisions between two bodies. They include:

a. Head on collisions or one-dimensional collisions - where the velocity of each body just before impact is along the line of impact.
b. Non-head on collisions, oblique collisions or two-dimensional collisions - where the velocity of each body just before impact is not along the line of impact

According to the coefficient of restitution, there are two special cases of any collision as written below:

1. A ***perfectly elastic collision*** is defined as one in which there is no loss of kinetic energy in the collision. In reality, any macroscopic collision between objects will convert some kinetic energy to internal energy and other forms of energy, so no large-scale impacts are perfectly elastic. However, some problems are sufficiently close to perfectly elastic that they can be approximated as such. In this case, the coefficient of restitution equals one.
2. An ***inelastic collision*** is one in which part of the kinetic energy is changed to some other form of energy in the collision. Momentum is conserved in inelastic collisions (as it is for elastic collisions), but one cannot track the kinetic energy through the collision since some of it is converted to other forms of energy. In this case, coefficient of restitution does not equal one.

In any type of collision there is a phase when for a moment colliding bodies have the same velocity along the line of impact. Then the kinetic energy of bodies reduces to its minimum during this phase and may be called a maximum deformation phase for which momentarily the coefficient of restitution becomes one.

Collisions in ideal gases approach perfectly elastic collisions, as do scattering interactions of sub-atomic particles which are deflected by the electromagnetic force. "Collisions" in which the objects do not touch each other, such as Rutherford scattering or the slingshot orbit of a satellite off a planet, are elastic collisions. In atomic or nuclear scattering, the collisions are typically elastic because the repulsive Coulomb force keeps the particles out of contact with each other.

Collisions in ideal gases are very nearly elastic, and this fact is used in the development of the expressions for gas pressure in a container.

Collisions between hard spheres may be nearly elastic, so it is useful to calculate the limiting case of an elastic collision. The assumption of conservation of momentum as well as the conservation of kinetic energy makes possible the calculation of the final velocities in two-body collisions.

Deflection

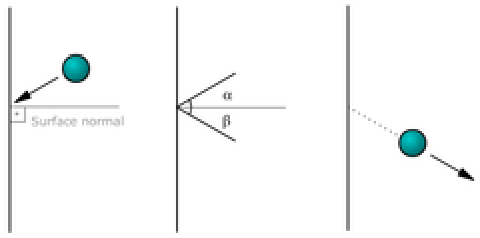


Figure 8.1: Deflection

Considering the diagram above, deflection happens when an object hits a plane surface. If the kinetic energy after impact is the same as before impact, it is an elastic collision. If kinetic energy is lost, it is an inelastic collision. It is not possible to determine from the diagram whether the illustrated collision was elastic or inelastic, because no velocities are provided. The most one can say is that the collision was not perfectly inelastic, because in that case the ball would have stuck to the wall.

Suppose that two bodies of masses m_1 and m_2 are moving with velocities u_1 and u_2 respectively before collision. During collision, m_1 exerts a force F_1 on m_2 and m_2 also exerts a force F_2 on m_1.

According to Newton's Third Law of motion,

$$F_1 = -F_2$$

Using Newton's Second Law of Motion,

$$force\ F = rate\ of\ change\ of\ momentum.$$

So, if $t = time\ of\ collision, and\ v_1, v_2 are\ the\ velocities\ of\ m_1\ and\ m_2$ respectively after collision, then;

$$\frac{m_2(v_2 - u_2)}{t} = -\frac{m_1(v_1 - u_1)}{t} \quad (since\ F_1 = -F_2)$$

$$\therefore\ m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

$$\therefore\ sum\ of\ linear\ momentum\ after\ collision = sum\ of\ linear\ momentum\ before\ collision$$

Thus, linear momentum is conserved.

ITQ

Question

The principle of conservation of linear momentum states that the linear momentum of a closed system before collision is equal to the linear momentum after collision.**YES/NO**

Feedback

YES

Study Session Summary

Summary

In this Study Session, we considered linear momentum and collisions. We were able to state the principle of conservation of linear momentum. Lastly, we defined collisions, listed its features and highlighted its two types as well.

Assessment

Assessment

SAQ 8.1 (tests Learning Outcome 8.1)
Define collisions and highlight its features

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 9

Equilibrium and Elasticity

Introduction

In this study session, we will explain the requirements for equilibrium. We will also consider the relationship between linear momentum, centre of mass and angular momentum as well as equilibrium and the force of gravity. Subsequently, we will discuss the relationship between applied force and the extension produced.

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

9.1 *explain*

- Equilibrium and the force of gravity
- Stacking blocks
- Rigid body
- Elasticity
- Hooke's law
- Young modulus
- Bulk modulus

Terminology

Equilibrium	a state in which opposing forces or influences are balanced
Elasticity	the ability of an object or material to resume its normal shape after being stretched or compressed; stretchiness

9.1 Equilibrium and Elasticity

An object is in **equilibrium** if the linear momentum (P) of its centre of mass is constant and if its angular momentum (L) about its centre of mass is constant:

That is; P = constant

L = constant

An object is in **static equilibrium** if its linear momentum and angular momentum is equal to zero:

P = 0 kg m/s

L = 0 kg m^2/s

9.1Requirements for Equilibrium

If a body is in **translational equilibrium** then dP/dt = 0, or

$$\frac{d\bar{P}}{dt} = \sum \bar{F}_{ext} = 0$$

If a body is in **rotational equilibrium** then dL/dt = 0, or

$$\frac{d\bar{L}}{dt} = \sum \bar{\tau}_{ext} = 0$$

In summary, the following equations must be satisfied for an object in static equilibrium

$$\begin{array}{lcl} \sum F_x = 0 & & \sum \tau_x = 0 \\ \sum F_y = 0 & \text{and} & \sum \tau_y = 0 \\ \sum F_z = 0 & & \sum \tau_z = 0 \end{array}$$

If we restrict ourselves to two dimensions (the x-y plane) the following equations must be satisfied:

$$\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum \tau_z = 0 \end{array}$$

9.1.2 Equilibrium and the Force of Gravity

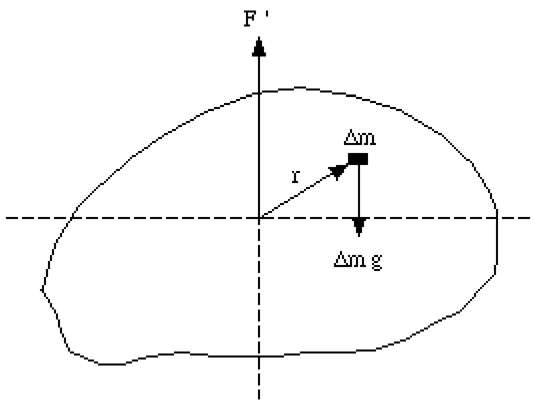


Figure 9.1: Weight of an object balanced by a single force.

Figure 9.1 shows a body of arbitrary shape balanced by a single force. The origin of the coordinate system is defined such that it coincides with the **centre of gravity** of the object, which is the point upon which the balancing force acts. An object that is supported at its centre of gravity will be in static equilibrium, independent of the orientation of the object. If the body is in equilibrium, the net force acting on it must be zero. Figure 9.1 shows that

$$\sum \bar{F} = \left(F' - \sum \Delta m\, g\right)\hat{y} = \left(F' - g\sum \Delta m\right)\hat{y} = (F' - M g)\hat{y}$$

Since the body is in equilibrium

$$\sum \bar{F} = 0$$

and therefore

$$F' = M g$$

In obtaining this result, **we have assumed that the gravitational acceleration is the same for every point of the body**. The net torque acting on the body is given by

$$\sum \bar{\tau} = \sum \bar{r} \times (m \bar{g}) = \left(\sum m \bar{r}\right) \times \bar{g} = M \bar{r}_{cm} \times \bar{g}$$

Since the body is in static equilibrium

$$\sum \bar{\tau} = 0$$

and therefore

$$M\,\bar{r}_{cm} \times \bar{g} = 0$$

This shows that $r_{cm} = 0$ or r_{cm} is parallel to g. We conclude that **for a body to be in equilibrium, its centre of mass must coincide with its centre of gravity**.

Sample Problem

A uniform beam of length L whose mass is m, rest with its ends on two digital scales (see Figure 4.2). A block whose mass is M rests on the beam, its centre one-fourth away from the beam's left end. What do the scales read?

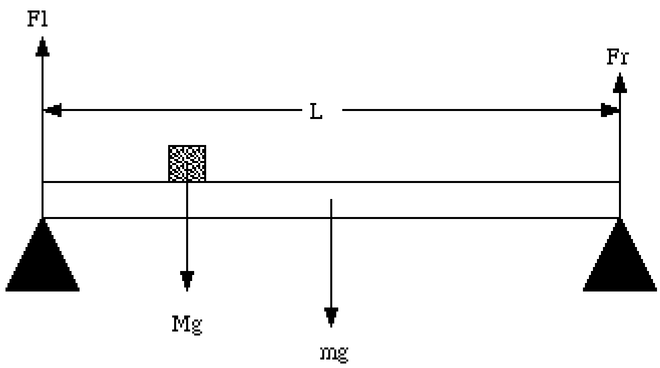


Figure 9.2: Sample problem

For the system to be in equilibrium, the net force and net torque must be zero. Figure 9.2 shows that

$$\sum F_y = F_l + F_r - Mg - mg = 0$$

Here we have replaced the force acting on the beam with single force acting on its centre of gravity. The net torque of the system, with respect to the left scale, is

$$\sum \tau = F_1 0 + F_r L - M g \frac{L}{4} - m g \frac{L}{2} = 0$$

This shows immediately that

$$F_r = \frac{M g \frac{L}{4} + m g \frac{L}{2}}{L} = \frac{g}{4}(M + 2m)$$

From the equation of the net force we obtain

$$F_1 = M g + m g - F_r = \frac{g}{4}(3M + 2m)$$

Sample Problem

A ladder with length L and mass m rests against a wall. Its upper end is a distance h above the ground (see Figure 4.3). The centre of gravity of the ladder is one-third of the way up the ladder. A fire-fighter with mass M climbs halfway up the ladder. Assume that the wall, but not the ground, is frictionless. What is the force exerted on the ladder by the wall and by the ground?

The wall exerts a horizontal force F_W on the ladder (the normal force); it exerts no vertical force. The ground exerts a force F_g on the ladder with a horizontal component F_{gx} and a vertical component F_{gy}. If these two components were not present, the system would not be in equilibrium. The net force in the x and y directions is given by

$$\sum F_x = F_W - F_{gx} = 0$$

and

$$\sum F_y = F_{gy} - M g - m g = 0$$

The net torque, with respect to O (which is the contact point between the ladder and the ground), is given by

$$\sum \tau = h F_W - M g \frac{a}{2} - m g \frac{a}{3} = 0$$

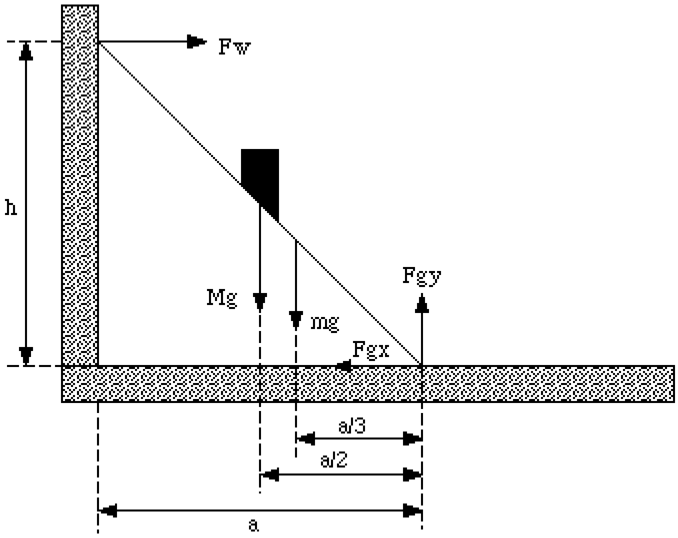


Figure 9.3: Sample Problem

This immediately shows that

$$F_W = \frac{M g \frac{a}{2} + m g \frac{a}{3}}{h} = \frac{g a}{h}\left(\frac{1}{2} M + \frac{1}{3} m\right)$$

We can now calculate the force Fg:

$$F_{gx} = F_W = \frac{g a}{h}\left(\frac{1}{2} M + \frac{1}{3} m\right)$$

and

$$F_{gy} = M g + m g$$

We observe that F_{gx} depends on the position of the fire-fighter. Suppose that the fire-fighter is a distance f L up the ladder. In this case F_{gx} is given by

$$F_{gx} = \frac{g\,a}{h}\left(f\,M + \frac{1}{3}\,m\right)$$

If the coefficient of static friction between the ladder and the ground is u_s, than the maximum distance the fire-fighter can climb is reached when

$$F_{gx} = \mu_s F_{gy}$$

or

$$\frac{g\,a}{h}\left(f\,M + \frac{1}{3}\,m\right) = \mu_s\,(M\,g + m\,g)$$

This shows that

$$f = \frac{1}{M}\left[\frac{h\,\mu_s}{g\,a}(M\,g + m\,g) - \frac{1}{3}\,m\right] = \frac{h\,\mu_s}{a}\left[1 + \frac{m}{M}\right] - \frac{1}{3}\,\frac{m}{M}$$

9.1.3 Stacking Blocks

Two bricks of length L and mass m are stacked. Using conditions of static equilibrium we can determine the maximum overhang of the top brick (see Figure 9.4).

The two forces acting on the top brick are the gravitational force F_g and the normal force N, exerted by the bottom brick on the top brick. Both forces are directed along the y-axis. Since the system is in equilibrium, the net force acting along the y-axis must be zero. We conclude that

$$N = F_g = m\,g$$

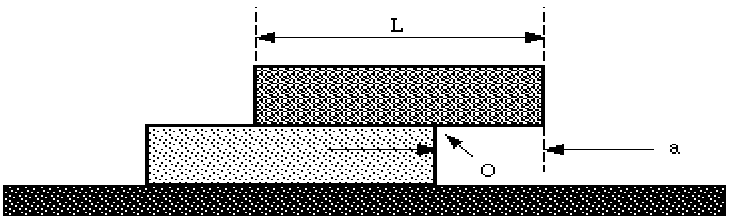


Figure 9.4: Two stacked bricks.

If the top block is on the verge of falling down, it will rotate around O. The torque exerted by the two external forces with respect to O can be easily calculated (see Figure 9.5). The gravitational force F_g acting on the whole block is replaced by a single force with magnitude mg acting on the centre of mass of the top block. The normal force N acting on the whole contact area between the top and the bottom block is replaced by a single force N acting on a point a distance d away from the rotation axis O. The torque of the normal force and the gravitational force with respect to O is given by

$$\tau_N = -N d$$

$$\tau_g = -m g \left(a - \frac{L}{2}\right)$$

The net torque acting on the top brick is given by

$$\tau = -N d - m g \left(a - \frac{L}{2}\right) = -m g \left(d + a - \frac{L}{2}\right)$$

If the system is in equilibrium, then the net torque acting on the top brick with respect to O must be zero. This implies that

$$d + a - \frac{L}{2} = 0$$

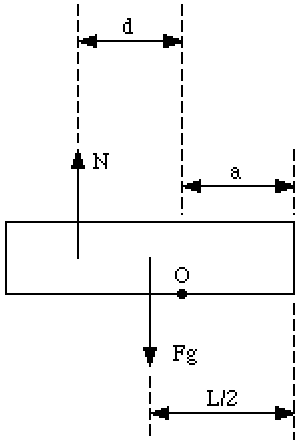


Figure 8.5: Forces acting on top brick.

or

$$d = \frac{L}{2} - a$$

This equation shows that the system can never be in equilibrium if a > L/2 (since d < 0 in that case). The system will be on the verge of losing equilibrium if a = L/2. In this case, d = 0. We conclude that the system cannot be in equilibrium if the centre of mass of the top brick is located to the right of the edge of the bottom brick. The system will be on the verge of losing equilibrium if the centre of mass of the top brick is located right over the edge of the bottom brick. Finally, if the centre of mass of the top brick is located to the left of the edge of the bottom brick, the system will be in equilibrium.

9.1.4 Rigid Body

A rigid body under the action of a number of coplanar forces is in equilibrium if:

1. The resultant forces is zero, and
2. The algebraic sum of the moments of the forces about any axis is zero.

Therefore, a rigid body under the action of coplanar forces is not in equilibrium if;

1. There is a resultant force acting on it, or
2. The algebraic sum of the moments of the forces about any axis is not zero.

The general conditions for equilibrium of a body are:

$$sum\ of\ upward\ forces = sum\ of\ downward\ forces, i.e. \sum x = \sum y = 0$$

$$sum\ of\ anticlowise\ moment = sum\ of\ clockwise\ moment, i.e. \sum \tau = 0.$$

Where $\sum x , \sum y$are the sums of the resolved components about two perpendicular axis and $\sum \tau$ is the sum of moments about any axis.

ITQ

Question

What will happen if a rigid body under the action of a number of coplanar forces is in equilibrium?

Feedback

A rigid body under the action of a number of coplanar forces is in equilibrium if the resultant forces is zero and the algebraic sum of the moments of the forces about any axis is zero

9.1.5 Defining Elasticity

Elasticity is that property of a body, which enables the body to regain its original dimensions (length, breath and height) when the deforming force acting on the body is removed. Elasticity is the ability of an object or material to resume its normal shape after being stretched or compressed.

Thus, **elastic** materials (e.g. rubber) are materials that will return to their original shapes after the deforming force is removed within an elastic limit while, **inelastic** materials are materials that do not return to their original shapes after the deforming force is removed. Examples of inelastic materials are plasticine, clay and dough.

Most materials are elastic up to a certain limit known as the **elastic limit.** Beyond this limit a material will not return to its original dimensions when the deforming force is removed.

ITQ

Question

1. A material is said to be ____ if it changes shape when a deforming force acts on it and returns to its original shape when the deforming force is removed.

A. elastic

B. inelastic

C. plastic

D. stretchy

E. rigid

Feedback

A material is said to be elastic if it changes shape when a deforming force acts on it and returns to its original shape when the deforming force is removed so the correct answer is A i.e. elastic

9.1.5 Hooke's Law of Elasticity

Hooke's law states that within elastic limits the extension (or compression) e of an elastic material is directly proportional to the force applied.

i.e. $e \propto F$

thus, $Force\ F = ke$

where k is the force constant.

Again, Again, Hooke's Law is when an elastic object - such as a spring - is stretched, the increased length is called its extension. The extension of an elastic object is directly proportional to the force applied to it:

F = k × e. F is the force in newton's, N

Young Modulus, E: This is the ratio of stress to strain of a wire within the elastic limit.

$$\therefore Young\ Modulus, E = \frac{stress}{strain}$$

where $$stress, \sigma = \frac{force\ (F)}{cross\ sectional\ area\ (A)}$$

and $$strain, \varepsilon = \frac{extension\ (e)}{original\ lenth\ (l)}$$

so, $$Young\ Modulus, E = \frac{\frac{F}{A}}{\frac{e}{l}} = \frac{Fl}{Ae}$$

Bulk Modulus of Elasticity: This is the ratio of change in pressure to change in volume

thus, $$Bulk\ Modulus, B = \frac{-dP}{\frac{dV}{V_0}}$$

where dP is the change in pressure and dV is change in volume, V_0 is initial volume. The negative sign is included in the definition of B because an increase in pressure always causes a decrease in volume. The reciprocal of bulk modulus is called compressibility (k).

$$k = \frac{1}{B} = \frac{\frac{-dV}{V_0}}{dP} = -\frac{1}{V_0}\frac{dV}{dP}$$

Thus, compressibility is a fractional increase in volume per unit increase in pressure. The unit of Bulk modulus is N/m^2 and the unit of compressibility is m^2/N.

ITQ

Question

Hooke's Law relates the

A. distance a spring stretches to the force applied to the spring.

B. distance a spring stretches to the mass of the spring.

C. distance a spring stretches to the density of the spring.

D. density of a spring to the force applied to the spring.

E. density of a spring to the mass of the spring

Feedback

The correct answer is A i.e. Hooke's Law relates the distance a spring stretches to the force applied to the spring

Study Session Summary

Summary

In this Study Session, we explained the requirements for equilibrium. We also considered the relationship between linear momentum, centre of mass and angular momentum as well as equilibrium and the force of gravity. Eventually, we discussed the relationship between applied force and the extension produced.

Assessment

Assessment

SAQ 9.1 (tests Learning Outcome 9.1)

Answer the following questions

1. Define elasticity
2. Differentiate between elastic and inelastic

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Study Session 10

Fluids

Introduction

Unlike solid objects, fluids can flow and does not have its shape or form. In this study session, we will define fluid and highlight both the properties and characteristics of fluids. Finally, we will describe viscosity and list the various factors affecting viscosity

Learning Outcomes

Outcomes

When you have studied this session, you should be able to:

9.1 *list* both the characteristics and properties of fluids

9.2 *enumerate* the factors affecting viscosity

Terminology

Fluid	a state of matter, such as liquid or gas, in which the component particles (generally molecules) can move past one another.
Viscosity	a quantity expressing the magnitude of internal friction, as measured by the force per unit area resisting a flow in which parallel layers unit distance apart have unit speed relative to one another

10.1 What are Fluids?

A **fluid** is a substance that continually deforms (flows) under an applied shear stress. Fluids are a subset of the phases of matter and include liquids, gases, plasmas and, to some extent, plastic solids. Fluids can be defined as substances that have zero shear modulus or in simpler terms, a fluid is a substance, which cannot resist any shear force applied to it.

Although the term "fluid" includes both the liquid and gas phases, in common usage, "fluid" is often used as a synonym for "liquid", with no implication that gas could also be present. For example, "brake fluid" is hydraulic oil and an hydraulic oil cannot perform its required incompressible function if there is gas in it. This colloquial usage of the term is also common in medicine and in nutrition ("take plenty of fluids"). Liquids form a free surface (that is, a surface not created by the container) while gases do not. The distinction between solids and fluid is not entirely obvious.

10.1.1 Properties of Fluids

Having defined fluids, we shall now list properties displayed by fluids as follow:

- not resisting deformation, or resisting it only slightly (viscosity)
- the ability to flow (also described as the ability to take on the shape of the container).This also means that all liquids have the property of fluidity

These properties are typically a function of their inability to support a shear stress in static equilibrium. Solids can be subjected to shear stresses, and to normal stresses — both compressive and tensile. In contrast, ideal fluids can only be subjected to normal, compressive stress, which is called pressure. Real fluids display viscosity and so are capable of being subjected to low levels of shear stress. In a solid, shear stress is a function of strain, but in a fluid, shear stress is a function of strain rate. A consequence of this behaviour is Pascal's law, which describes the role of pressure in characterizing a fluid's state.

10.1.2 Characteristics of Fluids

Depending on the relationship between shear stress, and the rate of strain and its derivatives, fluids can be characterized as one of the following:

- **Newtonian Fluids** : where stress is directly proportional to rate of strain
- **Non-Newtonian Fluids**: Where stress is not proportional to rate of strain, its higher powers and derivatives.

The behaviour of fluids can be described by the Navier–Stokes equations — a set of partial differential equations, which are based on:

a. continuity (conservation of mass)
b. conservation of linear momentum
c. conservation of angular momentum
d. conservation of energy

The study of fluids is fluid mechanics, which is sub-divided into fluid dynamics and fluid statics depending on whether the fluid is in motion.

The study of fluids in motion is known as ***hydrodynamics***. The flow of the fluid is possible because matter contains molecules. Brownian motion, diffusion, osmosis, convection, expansion, viscosity etc, are some of the evidences of the molecular nature of matter.

10.1.3 Laminar (Uniform) and Turbulence (Disorder) Flow of Fluids

Laminar and turbulent flow are the two types of fluid flow. In a laminar flow, the particles of the liquid at the same distance from the axis always have equal velocities directed and parallel to the axis. In a turbulent flow the particles at the same distance from the axis have different velocities varying in magnitude and direction with time.

ITQ

Question

Fluids can be characterized as one of the following except ----------

(a) Newtonian Fluids
(c) Mini-Newtonian fluids
(b) Non-Newtonian Fluids

Feedback

The correct answer is B which is Mini-Newtonian Fluid

10.2Defining Viscosity

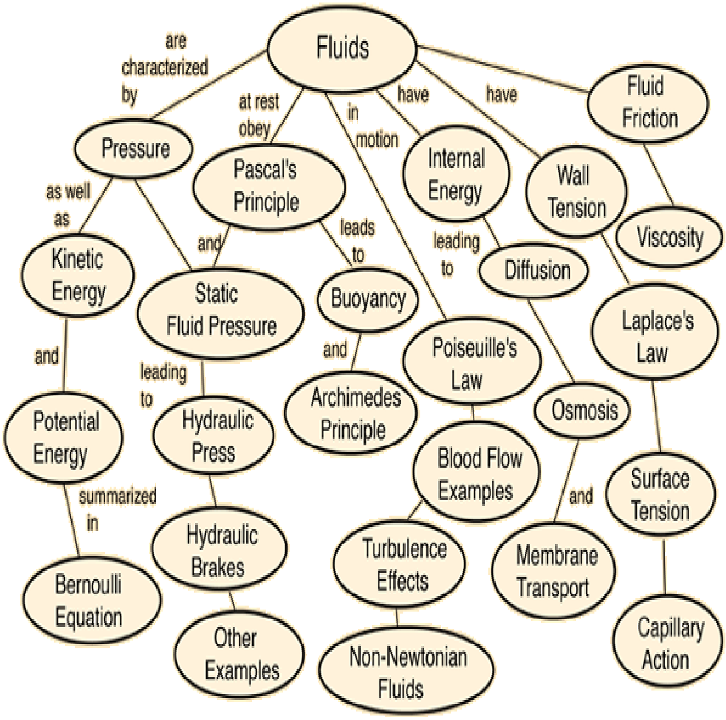
Viscosity is the frictional force in fluids. The use of a liquid as lubricant depends on its viscosity. The coefficient of viscosity μ is defined as the force acting on a fluid per unit area in a region of unit gradient.

Thus, Viscosity is: $\mu = \frac{Force}{Area \times velocity\ gradient}$

10.2.1 Factors Affecting the Viscosity of a Fluid

1. Viscosity, unlike solid friction increases as the surface area of the fluid in contact increases.
2. Viscosity, unlike solid friction depends on the velocity with which one fluid moves across the other.
3. Viscosity depends on the concentration of the fluid.
4. Pressure and Temperature affect viscosity. The higher the pressure the higher the viscosity but increases in temperature decreases viscosity of a fluid.
5. Presences of impurities affect the viscosity of a fluid.

Generally, fluids can be sub-divided into the following groups.



ITQ

Question

One of the factors affecting the viscosity of a fluid is that Viscosity depends on the concentration of the fluid. YES/NO

Feedback

YES

Study Session Summary

Summary

In this Study Session, we considered Newtonian and non-Newtonian fluids. We discussed as well the properties and characteristics of fluids. We also considered conditions under which we may have either laminar or non-laminar flow and factors that determine the viscosity of a fluid.

Assessment

Assessment

SAQ 10.1 (tests Learning Outcome 10.1)

Answer the following questions. List:

- Characteristics of fluids
- Properties of fluids

SAQ 10.2 (tests Learning Outcome 10.2)

Highlight the factors affecting the viscosity of a fluid

Bibliography

Reading

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter13/chapter13.

http://teacher.pas.rochester.edu/phy121/lecturenotes/chapter12/chapter12.

Notes on Self-Assessment Questions

SAQ 1.1

x	0^0	45^0	90^0	135^0	180^0	225^0	270^0	315^0	360^0
Sinx	**0**	**0.71**	**1**	**0.71**	**0**	**-0.71**	**-1**	**-0.71**	**0**
Cosx	**1**	**0.71**	**0**	**-0.71**	**-1**	**-0.71**	**0**	**0.71**	**1**
tanx	**0**	**1**	*	**-1**	**0**	**1**	*	**-1**	**0**

SAQ 2.1

Torque is the effect of a force about an axis. The torque is equal to the moments of the force F about the axis of rotation. Can be expressed thus: $Torque, \Gamma = Fr$

SAQ 3.1

Solution

$\underline{\mathbf{R}}$ = $\underline{A}$1+ $\underline{A}$2+ $\underline{A}$3

$\underline{\mathbf{R}}$ = (-3+2+7) î + (2-6-8) ĵ

$\underline{\mathbf{R}}$ = 6 î -12 ĵ

The magnitude /$\underline{\mathbf{R}}$/ = $\sqrt{(\mathbf{6})^2 + (-\mathbf{12})^2}$

= 13.42 units

SAQ4.1

There are four types of motion and they are Random, Translational, Rotational, and Oscillatory motion

SAQ4.2

The motion of the projectile is always in two forms, they are:

c) A constant horizontal motion along X-part

d) A vertically downward acceleration of free fall due to gravity along Y-part

Examples of motion

v. Arrow or bullet shot into space;
vi. A stone shot from a catapult;
vii. A basket or football kicked into space;
viii. A tennis ball thrown against a vertical wall.

SAQ 4.3

A car travels from rest with an acceleration of $2ms^{-2}$. Calculate its velocity after travelling $4m$.

Solution

From rest, u = 0, a =$2ms^{-2}$, s = $4m$

Recall,$v^2 = u^2 + 2as$

$= 0 + 2 \times 2 \times 4$

$= 16$

$v = \sqrt{16}$

$v = 4ms^{-1}$

SAQ 5.1

Newton's first law of motion states that everybody continues to be in a state of rest or to move with uniform velocity unless a resultant force acts on it WHILE Newton's third law of motion states that to every action there is an equal and opposite reaction.

SAQ 5.2

If you have answered using any of these, you are very correct

Normal Force: The normal force($\boldsymbol{F_N}$) is the force acting perpendicular to the surface.

Tensile Force: The tensile force ($\boldsymbol{F_T}$) acting on a string, chain or tendon is an applied force tending to stretch it.

Frictional Force: This is the tangential force ($\boldsymbol{F_f}$) acting on an object that opposes the sliding of that object on an adjacent surface with which it is in contact**.**

Centripetal Force: This is the force $F_{C,}$, which must act on a mass m, moving in a circular path of radius **r** to give the centripetal acceleration$\boldsymbol{\frac{v^2}{r}}$**.**

From **F = ma**

Centrifugal Force: This is the reaction force to the centripetal force. The reaction force does not act on the same body as the centripetal force. That is, if a string was tied to a rock and the rock was swung in a horizontal circle at constant speed, the centripetal force would act on the rock while the centrifugal force would act on the string

SAQ 6.1

SOLUTION

U = 90 m/s, V = 0, a = g = -10 ms^{-2}

Recall, V = u $+$ at

0 = 90 – 10 × t

10t = 90

t = 9 s

(iii)Distance S = ut + ½at^2

S = (90 × 9) + ½ × (-10) × 3^2

= 810– 405 = 405 m

SAQ 7.1

Kinetic energy can be mathematically represented by = $K.E. = \frac{1}{2}\ mv^2$

SAQ 7.2

Conservative force can store energy in the system as potential energy and can retrieve that energy and give it to an object in the system as kinetic energy. It can also be gravitational and spring forces BUT the Principle of Conservation of Energy states that in a closed system, energy can neither be created nor destroyed but, can only be transformed from one form to another

SAQ 8.1

1. A **collision** is an event in which two or more bodies exert forces on each other for a relatively short time

Features of collisions include:

a) It occurs in a short time interval.
b) What happen after the collision differs from what happens before the collision.
c) The colliding bodies may be assumed to constitute a closed system
d) Momentum and energy are conserved during the collision

SAQ 9.1

Elasticity is that property of a body, which enables the body to regain its original dimensions (length, breath and height) when the deforming force acting on the body is removed

Elastic materials (e.g. rubber) are materials that will return to their original shapes after the deforming force is removed within an elastic limit while, **Inelastic** materials are materials that do not return to their original shapes after the deforming force is removed

SAQ 10.1

Fluids can be characterised into:

- **Newtonian Fluids** : where stress is directly proportional to rate of strain
- **Non-Newtonian Fluids**: Where stress is not proportional to rate of strain, its higher powers and derivatives.

Properties of fluids are as follow:

- not resisting deformation, or resisting it only slightly (viscosity)
- the ability to flow (also described as the ability to take on the shape of the container).This also means that all liquids have the property of fluidity

SAQ 10.2

Factors affecting the viscosity of a fluid

- Viscosity, unlike solid friction increases as the surface area of the fluid in contact increases.
- Viscosity, unlike solid friction depends on the velocity with which one fluid moves across the other.
- Viscosity depends on the concentration of the fluid.
- Pressure and Temperature affect viscosity. The higher the pressure the higher the viscosity but increases in temperature decreases viscosity of a fluid.
- Presences of impurities affect the viscosity of a fluid

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