Mathematics for Management

EME 208

University of Ibadan Distance Learning Centre
Open and Distance Learning Course Series Development

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Vice-Chancellor's Message

The Distance Learning Centre is building on a solid tradition of over two decades of service in the provision of External Studies Programme and now Distance Learning Education in Nigeria and beyond. The Distance Learning mode to which we are committed is providing access to many deserving Nigerians in having access to higher education especially those who by the nature of their engagement do not have the luxury of full time education. Recently, it is contributing in no small measure to providing places for teeming Nigerian youths who for one reason or the other could not get admission into the conventional universities.

These course materials have been written by writers specially trained in ODL course delivery. The writers have made great efforts to provide up to date information, knowledge and skills in the different disciplines and ensure that the materials are user-friendly.

In addition to provision of course materials in print and e-format, a lot of Information Technology input has also gone into the deployment of course materials. Most of them can be downloaded from the DLC website and are available in audio format which you can also download into your mobile phones, IPod, MP3 among other devices to allow you listen to the audio study sessions. Some of the study session materials have been scripted and are being broadcast on the university's Diamond Radio FM 101.1, while others have been delivered and captured in audio-visual format in a classroom environment for use by our students. Detailed information on availability and access is available on the website. We will continue in our efforts to provide and review course materials for our courses.

However, for you to take advantage of these formats, you will need to improve on your I.T. skills and develop requisite distance learning Culture. It is well known that, for efficient and effective provision of Distance learning education, availability of appropriate and relevant course materials is a *sine qua non*. So also, is the availability of multiple plat form for the convenience of our students. It is in fulfilment of this, that series of course materials are being written to enable our students study at their own pace and convenience.

It is our hope that you will put these course materials to the best use.

Prof. Abel Idowu Olayinka

Vice-Chancellor

Foreword

As part of its vision of providing education for "Liberty and Development" for Nigerians and the International Community, the University of Ibadan, Distance Learning Centre has recently embarked on a vigorous repositioning agenda which aimed at embracing a holistic and all encompassing approach to the delivery of its Open Distance Learning (ODL) programmes. Thus we are committed to global best practices in distance learning provision. Apart from providing an efficient administrative and academic support for our students, we are committed to providing educational resource materials for the use of our students. We are convinced that, without an up-to-date, learner-friendly and distance learning compliant course materials, there cannot be any basis to lay claim to being a provider of distance learning education. Indeed, availability of appropriate course materials in multiple formats is the hub of any distance learning provision worldwide.

In view of the above, we are vigorously pursuing as a matter of priority, the provision of credible, learner-friendly and interactive course materials for all our courses. We commissioned the authoring of, and review of course materials to teams of experts and their outputs were subjected to rigorous peer review to ensure standard. The approach not only emphasizes cognitive knowledge, but also skills and humane values which are at the core of education, even in an ICT age.

The development of the materials which is on-going also had input from experienced editors and illustrators who have ensured that they are accurate, current and learner-friendly. They are specially written with distance learners in mind. This is very important because, distance learning involves non-residential students who can often feel isolated from the community of learners.

It is important to note that, for a distance learner to excel there is the need to source and read relevant materials apart from this course material. Therefore, adequate supplementary reading materials as well as other information sources are suggested in the course materials.

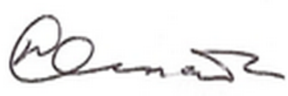
Apart from the responsibility for you to read this course material with others, you are also advised to seek assistance from your course facilitators especially academic advisors during your study even before the interactive session which is by design for revision. Your academic advisors will assist you using convenient technology including Google Hang Out, You Tube, Talk Fusion, etc. but you have to take advantage of these. It is also going to be of immense advantage if you complete assignments as at when due so as to have necessary feedbacks as a guide.

The implication of the above is that, a distance learner has a responsibility to develop requisite distance learning culture which includes diligent and disciplined self-study, seeking available administrative and academic support and acquisition of basic information technology skills. This is why you are encouraged to develop your computer skills by availing yourself the opportunity of training that the Centre's provide and put these into use.

In conclusion, it is envisaged that the course materials would also be useful for the regular students of tertiary institutions in Nigeria who are faced with a dearth of high quality textbooks. We are therefore, delighted to present these titles to both our distance learning students and the university's regular students. We are confident that the materials will be an invaluable resource to all.

We would like to thank all our authors, reviewers and production staff for the high quality of work.

Best wishes.



Professor Bayo Okunade

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Introduction to the Course

Mathematics for management is designed to expose you to basic mathematics concept in its reflection to institutional management. It is assumed that after the course, you will have acquired the calculative and manipulative skills necessary in solving problems met in everyday life, in institutional management, in economics of education, demographic study in education, education modelling, and in other social sciences.

To do this, you will initially be exposed to a general discussion on historical background of mathematics, relevance of mathematics to management and other human endeavour.

In the next lecture, you will be acquainted with set theory, under which you will be familiar with the meaning of set, presentation of set, types of set, notation, operations and applications of set theory in relation to institutional management. Subsequently in lectures three, four and five, you will be exposed to the algebraic process such as: ratios: percentages, proportion and rate, indices and logarithms. AP and GP, mathematical investment such as: simple and compound interest, annuities and discounting problems.

The second section of the course which include lecture six, seven and nine are devoted to functional graphs and equations, also matrices and determinant and its applications to organizational management were treated with probability theory.

The last section of the course which is lectures ten to twelve focused on differential, integral calculus and modelling in educational management. In doing these, we have endeavoured to make the lectures clear and our treatment brief. The approach in this lecture (course work) is new and innovative. It is a six-tier approach:- introduction, objectives, Pre-Test, main Text, Summary and Pre-Test. Our treatment of each lecture is in line with this format.

More specifically, after working through the lectures, in this course, you should be able to:-

- Demonstrate your understanding of the above listed topics by solving related problems in institutional management and everyday life.
- Apply appropriate concept(s) to solve real life problem on
  - i. Demand and supply equilibrium in education
  - ii. Consumer producer surplus in education
  - iii. Marking projections for future
  - iv. Analysis of demographic data in educational management

Table of content

Study Session 1: An Overview of Mathematics for Management

Introduction

Mathematics for management is designed to expose you to mathematics concepts with its reflection to educational system management. After the study, you would have acquired the calculative and manipulative skills necessary for solving problems met in everyday life, in institutional management, in economics of education, demographic study in education, education modelling, and in other social sciences.

This study session is an introduction to the whole unit, an overview of the mathematics for management and its everyday applications.

Learning Outcomes for Study Session 1

At the end of this study, you should be able to:

1.1 Discuss mathematics in management

1.2 Identify the uses of mathematics in management

1.1 An overview of Mathematics for management

Mathematics is the act of man, the expression of people throughout their daily activity. Mathematics is both language and tool of management science which enables the researchers and scientists to carry out their research, interpret their work, solve their problems, interpret their findings and inventions, investigate and predict the future and generally improve the world system.

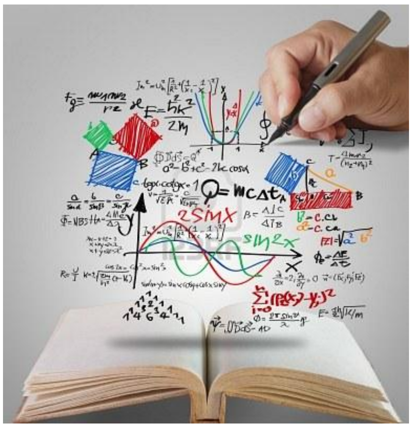


Figure1.1: *Mathematical illustrations*

Source: *http://jw.princetonk12.org/teachers/msullivan/030912E2-000F50D3.0/Book%20and%20Notes%20of%20Math.jpg*

The power of Mathematics lies in the fact that, since it expresses basic human thought processes, it is a universal language. Indeed, it is the only universal language, international in its use and enabling those who use it, of whatever race or tongue they may be able to communicate with each other.
It overcomes also, the boundaries of mental discipline which separate managers, administrator and the chemist, the economist and the astronomer, the statistician and the accountants.
It achieves this with the utmost precision and conciseness. One of the powerful attributes of the language of mathematics is that, it makes possible the cross – fertilization of ideas from one field of human knowledge to another.
Consider, for example, an education planner working on population growth and forecasting enrollment, an education economist working on cost benefit analysis and project management, the medical research worker working on cancer research, the school counselor working on the functions of psychological Test.
On the face of it, they are all doing different things and yet every one of the four is dealing with the same mathematical concepts in various dimensions. In educational planning, it is obvious that the quantitative mathematical approach is of the utmost importance. The dimensions of the future educational system have direct and significant financial implications.
All projects for improvements and expansion of educational system must, be properly cost before the policy maker can take their final decisions. The reliability of such figures on the future cost of education naturally depends on the adequacy and accuracy of the collected data, the mathematical skills and competence displayed in analyzing the material and performing the required calculations and projections.

Figure 1.2: *Logical thinking (Impacts of mathematics)*
Source: *http://newtonswindow.com/_borders/man_thinking_numbers.jpg*

It is, therefore, extremely important that all possible efforts are made to broaden and improve the basic mathematical information and tools to be used by potential educational planners and to develop and refine the mathematical methodology in the field of educational management.

More specifically, after working through the study, you should be able:

- To generate interest in mathematics and have a solid foundation for everyday life,
- To develop computational skills and foster the desire and ability to be accurate to a degree relevant educational management,
- To develop precise, logical and abstract thinking.
- To develop ability to recognize problems and to solve them with related mathematical knowledge,
- To provide necessary mathematical background for educational planning,
- To stimulate and encourage creativity.

In this study, you will initially be exposed to a general discussion of historical background of mathematics, relevance of mathematics to management and other human endeavor.

In the next study, you will be acquainted with set theory, under which you will be familiar with the meaning of the set, presentation of set, types of settings, notation, operations and applications of set theory in relation to institutional management.

Subsequently, in study three, four and five, you will be exposed to the algebraic process such as: ratios: percentages, proportion and rate, indices and logarithms. AP and GP, mathematical investment such as: simple and compound interest, annuities and discounting problems.

The second section of the unit includes studying six, seven and nine are devoted to functional graphs and equations, also matrices and determinant and its applications to organizational management were treated with probability theory.

The last section of the course which is studys, ten to twelve focused on differential, integral calculus and modelling in educational management. In doing these, the studies are clear and the treatment brief.

More specifically, after working through this study, you should be able to:-

a) Demonstrate your understanding of the above listed topics by solving related problems in institutional management and everyday life.

b) Apply appropriate concept(s) to solve real life problem on

c) Demand and supply equilibrium in education
d) Consumer producer surplus in education
e) Marking projections for future
f) Analysis of demographic data in educational management

In-Text Question

The power of Mathematics lies in the fact that, since it expresses basic human thought processes, it is a universal language. True or False

In-Text Answer

True

1.2 Uses of Mathematics in Management

Mathematics is an important subject and it helps individuals in developing reasoning and thinking skills. Mathematics helps in understanding of almost every subject; science and technology, medicine, the economy, or business and finance. It can also be used to forecast market trends. Statistics and probability which are branches of mathematics are used in everyday business and economics.

Figure: *Accounts, checking –Use of business management*
Source:*https://targetjobs.co.uk/sites/targetjobs.co.uk/files/public/An-introduction-to-professional-accountancy-bodies.jpg*

Mathematics is a significant part of accounting, business and economics. The practical applications of the use mathematics in business management include: checking accounts, price discounts, markups and markdowns, payroll calculations, simple and compound interest, consumer and business credit, and mortgages.

Activity 1.1 Overview of mathematics for management

Time Allowed: 2hours

Do personal further reading on mathematics for management

Summary for Study Session 1

In this study session 1, you have learnt that:

1. Mathematics is the act of man, the expression of people throughout their daily activity.
2. Mathematics is an important subject and it helps individuals in developing reasoning and thinking skills.
3. Mathematics is a significant part of accounting, business and economics.

Self-Assessment Questions (SAQs) for Study Session 1

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study

SAQ 1.1 (Testing Learning Outcomes 1.1)

Highlight the focus of mathematics in management

SAQ 1.2 (Testing Learning Outcomes 1.2)

Identify the uses of mathematics in management

Notes for Study Session 1

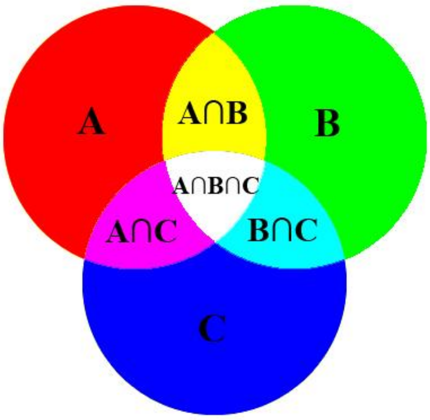
SAQ 1.1

- To generate interest in mathematics and have a solid foundation for everyday life,
- To develop computational skills and foster the desire and ability to be accurate to a degree relevant educational management,
- To develop precise, logical and abstract thinking.
- To develop ability to recognize problems and to solve them with related mathematical knowledge,

SAQ 1.2

Mathematics is a significant part of accounting, business and economics. The practical applications of the use mathematics in business management include: checking accounts, price discounts, markups and markdowns, payroll calculations, simple and compound interest, consumer and business credit, and mortgages.

Set Theory



Study Session 2: Set theory

Introduction

Set theory is the branch of mathematical logic that studies sets, which informally are collections of objects. Set theory is commonly employed as a mathematician. In this study, you will learn about set theory, its notations, Venn diagrams and some of its basic operations with real life.

Learning Outcomes for Study Session 2

In this study, you should be able to:

2.1 Define set
2.2 Identify some notations in set theory with their meaning
2.3 List the types of settings
2.4 Use Venn diagrams to solve problems in the setting

2.1 Set

A set is a collection of elements having some common properties, e.g. the set of all months with 30 days. An object belonging to a particular set is said to be a member or an element of that set.

Illustration: Consider the set of all months with less than 31 days, members of the set are: February, April, June, September, and November. Let A represent this set; A = {February, April, June, September, November} .In set A, February is a number. This is written as February$E\ A$. July is not a member of A is written as July Ɇ A.

In-Text Question

_______is a collection of elements having some common properties

A. Set
B. Venn diagram
C. Notation
D. Venn Notation

In-Text Question

A. Set

2.2 Set Notation

Symbol	Meaning	Examples
{…….}	A set of	#5, #10, #20, #50, #100, #500, #1000
E	Belongs to	a E (vowels)
Ɇ	Does not belong to	Ɇ (even integers)
:	Such that	A=[a:2a=10] implies A = {5}
n(x) s	Number of elements in set x	X ={a,b,c}, n(x)=3
{} or ∅	Empty set or null set	Motor bike with one tire
Aᴵ	Compliment of sex A	Aᴵ Ɇ A; A={a,b,c,d} Aᴵ={1,2}
∪	Union of set	(A∪B), set of element in set A and B
∩	Intersection of set	(A∩B) element that is common to both sets

2.3 Types of Set

There are different types of set. They are:

1. Unit set or Singleton
2. Null or Empty set
3. Subset
4. Finite and infinite set
5. Universal set
6. Equal sets
7. Union of sets
8. Intersection of sets
9. Power set
10. Disjoin set
11. Difference of set
12. Compliment of a set.

1. **Unit set or Singleton:** The set which has only one element is called a unit set or singleton. A= $\{x: x$ are the months of the year with < 30 days$\}$
   A = {February
2. **Null or Empty set:** Any set that does not contain any element is called a null or empty set. Such a set is denoted by symbol $\emptyset$ or {}. E.g H= $\{x: x$ is a human-being with three eyes$\}$
3. **Subset:** Set A is said to be a proper subset of set B, if every element in A is an element in B, and B have at least one other element that is not in A. We donate this by ACB which reads " A is a proper subset of B" e.g. A = {a, e, u, w}, B = {a, e, u, r, s, w}

   Then A C B. If, however, every element in X is an element in Y and Y does not have any other element(s) that is not in x, then x is said to be equal to Y, XCY, XCY, YCX X=Y e.g. P= { f, o, e, w}, Q= {f, o, w, e}

   $\Longrightarrow$ Here, PCQ and QCP: P=Q
4. **Finite and infinite sets:** If the elements of a sect have a definite number like the days of the week, we term this set **finite,** otherwise it is **infinite** like the set of natural numbers or counting numbers. Examples of finite and infinite sets are as follows:-
   - D= {days in the week}, here set D is finite
   - E= $\{x: x$ is a multiple of 2$\}$ – E is infinite.

Activity: 2.1 Types of sets

Time Allowed: 1 hour

Practice what you have learnt, write five examples of finite and infinite sets in your study note

5. **Universal set:** Consider three sets, say x, y and z.

   Let X {2, 3, 5, 7, 11, 13, 17, 19}, Y= {2, 4, 6, 8, 10, 12, 14} and Z = {4, 8. 12} these three sets will now be used to identify a number of important ideas associated with sets generally.

   The entire set of elements under discussion in a given context is called the universal set. We donate this set by the symbol ∪ orϵ. Mathematically, ∪ = { 2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19}

6. **Equal sets**:- Two sets are equal if they have the same members. For example. If A= {4, 5, 6}, B = {5, 4, 6}, then A=B.

7. **Union of sets:** The set of all the elements in X or Y or both is called the union of x and y, written as X ∪ Y. i.e. X ∪ Y {2,3,4,5,6,7,8,9,10,11,12,13,14,17,19}

8. **Intersection of sets:** the sets X and Y both contain the element 2, which is the only element common to them. The set of all elements common to sets X and Y, written X ∩ Y. Thus X ∩ Y = {2}

9. **Power set:** The power set of a given set A is the set of all possible subsets of A. it is denoted by P (A). E.g. if B {2,3}Then the power set of B is P (B)= {(2,3), {2}, {3},∅}

   Similarly, if A= {a, b, c}, the power set of A is

   (A) = {(a, b, c), {a,b},{a,c},{b,c},{a},{b},{c},∅}

   Exercise: find the power set of the following sets:-

   (i) {a} (ii) {(b,c)²} (ii) {b, c} (iii) {3, 4, 5}

   Remarks: - (i) The number of subsets in a power set is 2^N where ^{N} is the number of elements, the number of subsets is 2^2=4, for set A which has (ii) ∅ and the set itself are automatic members of power set.

10. **Disjoin set:** Two sets A and B are said to be disjoint if they have no elements in common. In other words, two sets A and B are said to be disjoint if A ∩ B =∅.

    E. g if A = {2, 4, 6, 8}, B = {d, e, f, g} and C = {f, g, k, r}

    Finding (i) A ∪ B (ii) B ∩ C (iii) $\frac{n(A)\times n(B)}{n(C)+n(B)}$

11. **Difference of set:** The difference of sets A and B is the set of elements which belongs to B. i.e. A- B = $\{x(xEA, x\not{E}B)$. Where A-B is read "A different B" or A minus B.

    E.g. Let A = {P, q, r, s, t}

    B = {r, s, u, v}

    A-B = [p, q,t] and B-A = {u,v}

12. Compliment of a set: If A is a subset of the universal set U, then the set of elements in U that is not in A is called the compliment of A and is denoted by A^{I} or A^{c}.

Example: Let U = {a,b, c,d,e,f,g, h}

X= {a,b,e, f,g}

Y= {b,c,d,f,g,h}

Find (i) X^{I} (ii) Y^{I} (iii) $X^{I}-Y^{I}$ (iv) $(X \cup V)^{I}$

Solution: - (i) X^{I} = {c, d, h}, (ii) {a, e}

(iii) $X^{I}-Y^{I}$ = {c, d, h} (iv) $(X \cup Y)^{I}$ = {a, b, c, d, e, f, g, h} =∅.

In-Text Question

A = {2, 4, 6, 8}, B = {d, e, f, g}.This is true of these sets

A. Disjoint sets
B. Compliments of a set
C. Power Set
D. Subset

In-Text Answer

A. Disjoint set

Activity 2.2: Types of Set

Time Allowed: 20 Minutes

Practice this exercise and compare notes with your colleagues.

Given that U = {3, 4, 5, 6, 7, 8}

A= {5, 6, 8}, B= {3, 4, 5, 7}

Find (i) $(A \cup B)^{I}$ (ii) $(A \cap B)^{I}$ (iii) $A^{I} \cap B^{I}$

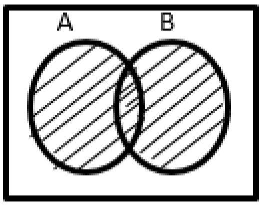
2.4 Venn diagram

A pictorial representation of sets is often useful in indicating and verifying relationship between two sets. One of such is the Venn diagram. It consists of a rectangle as universal set U and circles as subsets. By shading appropriate areas, all combinations of sets can be represented pictorially.

Example: Use the Venn diagrams to represent the following sets:

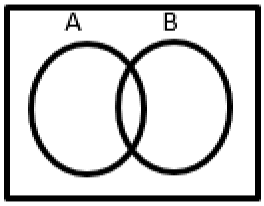
i. A ∪ B
ii. A ∩ B when A ∩ B ≠ ∅
iii. A ∩ B when A ∩ B = ∅
iv. A ∪ B when ACB
v. A ∩ B when ACB

i.



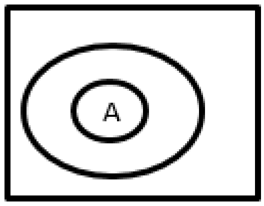
A U B

ii



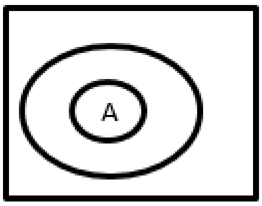
A ∩ B ≠ ∅

iv



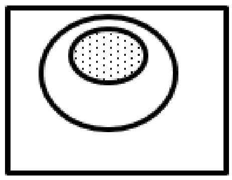
A ∪ B when ACB

iv



A ∪ B when ACB

v



A ∩ B when ACB

Cardinality of a set

The number of elements in a particular set is called its cardinality. E.g.

1) Let A = {2, 3, 5}, the number of element in A is 3. We write n (A) = 3, the cardinality of A is 3
2) Let B = { even no between 0 and 10} B= {2, 4, 6, 8, 10} n (B) = 5. Hence, the cardinality is 5
3) Let P = {x(x is an odd number divisible by 2} P =∅: n (P) = O. hence the cardinality of P is zero. Exercise: Given that ∅={x: $2 \leq x \leq 28$} where x is odd, find the cardinality of P

Application of Venn diagram

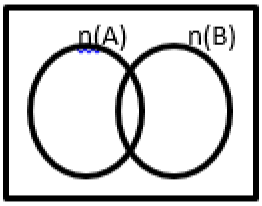


fig (i)

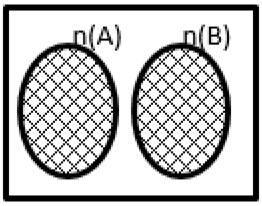


fig (ii)

If A and B are two different sets as shown in fig (i) such that A ∩ B ≠ 0, then: n (A ∪ B) = n (A)   n (B) – n (A ∩ B) …………….. (1)

i. If A ∩ B = ∅ as shown in fig (ii). Then n (A ∪ B) = n (A) + n (B) …………… (2)

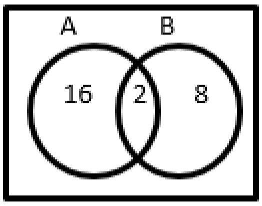
ii. N (U) = n (A ∪ B) + n (A ∪ B)$'$ ……………………….. (3)

Example 1:

Given that n (A ∩ B) =2, n (A ∩ B$'$) = 16 n (A$'$ ∩ B) = 8

U = (A U B). Draw a Venn diagram to illustrate the information above and use it to find   (i) n (A)   (ii) n (B) (iii) n (A U B).

Solution



The Venn diagram representing the given information

is as shown here from the diagram

n (A) = 16 +2 = 18

n (B) = 2 + 8 = 10

Since A ∩ B≠ ∅, apply eqn   (1)

i.e. n (A U B) = n (A) + n (B) – n (A ∩ B)

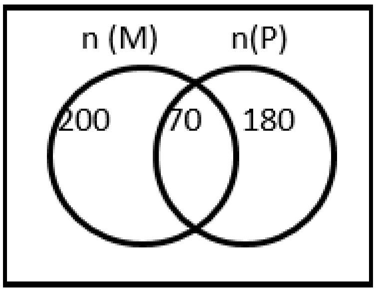
= 18 + 10 – 2

= 26

In a class of 500 students 200 took maths, 180 took physics and 70 took maths and physics. Find,

A. The number of students that did not take either of the two subject
B. The number of students that took maths only
C. The number of students that took physics only.
D. The number of students that took physics only

Solution



Since M ∩ P ≠ Ø then from eqn (3)

n (M U P)= n (M) + n(P) – n (M ∩ P)

= 200 + 180 – 70

= 310

1. This gives the number of students that took at least one of the two subjects

The number of the students who did not take either = n (M U P) ᴵ: From eqn (3)

n (U) – n (M U P) + n (M U P) ᴵ

I.e. n (M U P) ᴵ = n (U) – n (M U P)

500 – 310

=190

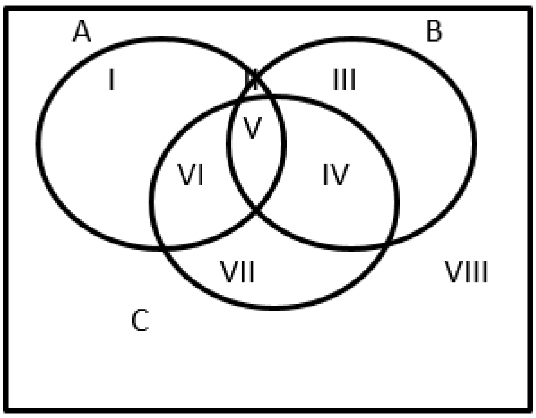
The number of students that took maths only

n (M) – n (M ∩ P) = 200 – 70 =130

The number of students that took physics only

n (P) – n (M ∩ P) = 180 – 70 = 110.

Three set problems: Here we are concerned with problems involving three sets. Unlike the two set problems, the three set problems have eight compartments or sectors as clearly indicated in the Venn diagram below:



Section I: - $A \cap B^{I} \cap C^{I} = A - B - C$ is elements of Aon

Section II: - $A \cap B \cap C^{I} = (A \cap B) - C$ i.e. elements that are common to both A and B only

Section III: - $A^{I} \cap B \cap C^{I} = B - A - C$ that is an element of B only

Section IV: - $A^{I} \cap B \cap C = (B \cap C) - A$ i.e. elements that C common to both B and C.

Section V: - $A \cap B \cap C$ that is elements that are common to the three sets A, B and C

Section VI: - $A \cap B \cap^{I} C = (A \cap C) - B$ that is elements that are common to both A and C only.

Section VII: - $A^{I} \cap B^{I} C = C - A - B$ that is elements of C only

Section VIII: - $A^{I} \cap B^{I} \cap C^{I} = U - (AUBUC)^{I}$ that is, the elements that are neither in A nor B in C.

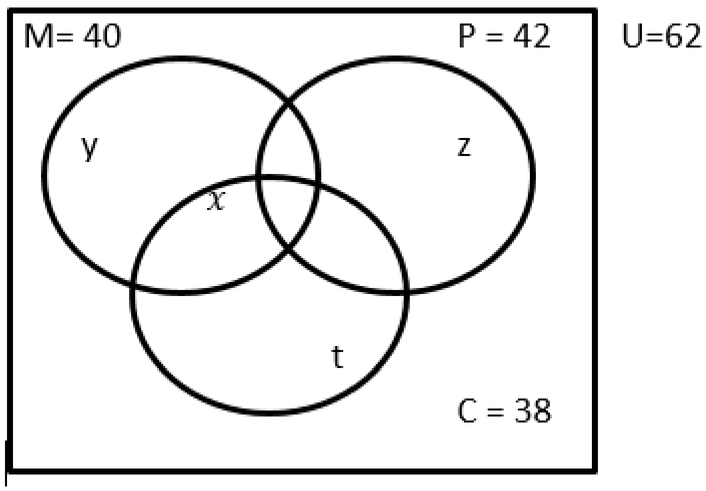
Hint: In solving problems involving three sets, we use that following rule:-

$n(AUBUC) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

Example 1: All the 62 students in SS3 of a named school take either maths (M) or physics (P) or chemistry (C). 40 take maths, 42 take physics, 38 take chemistry, 20 take maths and physics, 28 takes physics and chemistry, while 25 takes mathematics and chemistry. How many take

a) Mathematics, physics and chemistry (all 3 subjects)

b) Mathematics but neither physics nor chemistry.



n (MUPUC) = 62 i.e. total number of SS3 students

n (A) = 40 ⟹ students offering maths

n (P) = 42 ⟹ students offering physics

n(C) = 38 ⟹ students offering chemistry

n (M∩P) = 20, ⟹ no of students offering maths and physics

n(M∩C) = 25, no of students offering maths and chemistry

n (P∩C) = 28, no of students offering physics and chemistry

i Let n (M∩P∩C) =x, no of students offering all 3 subjects. Now apply the rule: i.e. n (MUPUC) = n (M) + n (P) + n(C) – n (M∩P) – n (M∩C) – n (P∩C) + n (M∩P∩C)

Substituting into the formula, we have:

62 = 40 + 42 + 38 – 20 – 25 – 28 +x

62 = 47 + x

x = 62 – 47

x = 15 i.e. 15 students offered the three subjects

ii. The number of students offering maths only, will be found by subtracting the sum of the other subject having intersection with maths. i.e. from the Venn diagram, the no offering 3 subjects = 15, this will be subtracted all the intersections with mathematics.

Hence, if n (M∩P) = 20, maths only will be (20 – 15) = 5

Similarly, if n (M∩C) = 25, maths only will be 25 – 15 = 10

So, the no offering maths only will be 25 – 25 = 10

40 – (15 + 10 + 5) = 40 – 30 = 10 students

Summary for Study Session 2

In this study, you have learnt that:

1. A set is a collection of elements having some common properties

2. Any set that does not contain any element is called a null or empty set
3. If the elements of a sect have a definite number like the days of the week, it is called finite set
4. The set which has only one element is called a unit set.
5. A pictorial representation of sets is often useful in indicating and verifying relationship between two sets

Self-Assessment Questions (SAQs) for Study Session 2

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study

SAQ 2.1

Define a set

SAQ 2.2

State whether set A is a proper or improper subset of B

i. A = $\{x: \frac{x}{3x^2} - 48 = 0\}$

B = {-4, 4}

ii. A = {3, 5, 7, 9}

B = $\{x: x$ is an odd number and $1 \leq x < 10\}$

SAQ 2.3

Given that U = {p, q, r, s, t, u, v}

R= {q, s, u, v}

T = {q, r, s, t, v}

W = {r, s, t, u, v}

Find

i. $(R \cap T^{I}) \cup W$

ii. $(R^{I} \cup T) \cap W^{I}$

iii. $T - R^{I}$

SAQ 2.4

In EME 208 class, there are fifty students, in EME 209 class, there are seventy students. Twenty enrolled for both courses. Find the number of students either in EME 208 class or EME 209 class if:

a) A student cannot take both courses
b) A student can take both courses

Notes for Study Session 2

SAQ 2.1

A set is a collection of elements having some common properties e.g. the set of all months with 30 days

SAQ 2.2

A is an improper subset of B

A is a proper subset of B

SAQ 2.3

(i) {r, s, t, u) (ii) {p, q} (iii) {q. r. s}

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Study Session 3: Algebraic process

Introduction

Educational planning encompasses the provision of mathematical information, necessary for framing and implementing appropriate policies in the short, medium and long terms with regard to the number of learners.

Learners in each level of education and the adjustments of the flow of persons from each level of education must be in accordance with economic and social needs. This study introduces you to the basic concept of mathematical tools used in management science, and economics of education, demographic data, etc. The tools are ratios, percentages, proportion, indices and logarithms.

Learning Outcomes for Study Session 3

At the end of this study, you should be able to:

3.1 Define the term 'Ratio'

3.2 Explain percentages

3. 3 Explain the laws of indices

3. 4 Discuss the laws of logarithms

3.1 Ratios

The Ratio is used to measure the value of quantity in relation to another. Such quantities may be of the same or different kinds. For example, if a student covers 6km from his house to the school in 1 ½ hrs, we say he travels at the ratio of $5\div\frac{3}{2}$ = 4km/hr. on the other hand, if a computer which sells for #250 depreciates by 50k then the depreciation rate is

$\frac{50}{250}\times\frac{100}{1} = 20k$ in the naira

Ratio is used to compare the relative magnitude of quantities of the same kind. For example, if a school 250 pupils has 150 boys, then the ratio of boys to girls is $\frac{150}{100}\ or\ \frac{3}{2}$ in its simplest form. The ratio $\frac{3}{2}$is commonly written as 3:2

Rules on ratio (a: b)

If K is a real and positive number, then

a: b= ka: kb

e.g. 2: 5 = b: 15 (k = 3) …………………………………….. (1)

ii. a: b = $\frac{a}{k}$: $\frac{b}{k}$

e.g. 8: 14 = 4: 7 i.e. k = 2 ………………………………………. (2)

iii. a: b ≠ a + k : b + k

E.g. 3: 5 ≠ 5: 7 where k = 2 ………………………………… (3)

iv. a: b ≠ a – k: b – k

E.g. 3: 5 ≠ 1: 3 (k=2) …………………………………………….(4)

Example

Find P if 5: P = 35: 21

Solution:

5: P = 35: 21 implies that

$\frac{5}{p} = \frac{35}{21}$

i.e. 35p = 21 X 5

∴ P = $\frac{21 \times 5}{35}$ = 3

The enrollment of 100 level students in the department of educational management was 49 in 2008/2009, later then was an increase in the new enrollment.

Solution

Let the new enrollment be 'y'

∴ 49: y = 7: 11

i.e. $\frac{49}{y} = \frac{7}{11}$

7y = 49 X 11

∴ $y = \frac{49 \times 11}{7}$

y = 27

Hence, the new intake is 77 students in 2009/10

In a country, if 185,000 live births were recorded in 2001 and the estimated mid – year population of the country in 2001 was 5 million, calculate the crude birth rate of the country in 2001.

Solution.

Hint: - The crude birth rate = $\frac{live\ birth\ in\ a\ year}{mid\ year\ total\ population} \times \frac{1000}{1}$

bt = $\frac{Bt}{Pt} \times \frac{1000}{1}$

∴ The crude birth rate b1 in 2001 in the country is given by

$b_{2001} = \frac{185000}{5000000} \times \frac{1000}{1} = 37$

Proportion: - proportion is the method used to divide a given quantity in a given ration

Example:

A grant of #260, 000 was divided among three departments in an institution in the ratio 6: 4: 3 according to their student enrollment. How much did each department receive?

Solution:-

The sum of the ratio = 6 + 4 + 3 = 13

Ist department received $\frac{6}{13} \times 260000 = 120{,}000$

2nd department received $\frac{4}{13} \times 260000 = \#80{,}000$

3rd department received $\frac{3}{13} \times 260000 = \#60000$

Example 2: An organization shared #150,000 to her best three rated workers in the ratio 2:3:5. Find the share of each.

Solution

Total share = 2 + 3 + 5 = 10

i.e. 10 shares = # 150,000

$\therefore$ 1 share = $\frac{150000}{10} = 15{,}000$

The worker with 2 shares had 2 X 15000= #30,000

The worker with 3 shares had 3 X #15,000= # 45,000

The worker with 5 shares had 5 X #15000= #75,000.00

In Text Questions

Find y if y: 3 = 35: 21

A. 10
B. 20
C. 5
D. 7

In-Text Answer

C. 5

3.2 Percentages

The ratio $x: y$ is expressed in percentage form when it is written as Z: 100. Thus, the percentage is simply a ratio that equates the second number to 100. The ratio 4:25 for example is equivalent to 16:100. Thus 4 is 16 percentages of 25. The Percentage is commonly used in scoring examination marks. We use the symbol % to represent percentages.

Example 1: the ratio of female to male in a faculty is 12:30. Express the number of females as a percentage of the number of males.

Solution: let x be the required percentage

$\therefore x$ Must be such that 12: 30 $x: 100$

$$\frac{12}{30} = \frac{x}{100}$$

$$30x = \frac{100 \times 12}{30} = 40\%$$

2. A man on an annual income of #480,000 spent #60,000 on children fees. What percentage of his income was spent on school fees?

Solution:

Annual income = # 480,000.00

Children school fees = #60,000

Required % = $\frac{6000}{480000} \times \frac{100}{1}$

12.5%

3. A trader bought a carton of chalk for #1000.00 and sold it for #1,200.00. Find his profit percentage.

Solution:

C.P = #1000.00

S P = #1,200

Profit = #1200 - # 1000

= #200.00

$\therefore$ Profit percentage = $\frac{200}{1000} \times \frac{100}{1} = 20\%$

In-Text Question

A shopkeeper loses 4% by selling an article at #480. At what price should he have sold the article to make a profit of 4%?

A. #520
B. #130
C. #450
D. #230

In-Text Answer

A. #520

3.3 Indices: - Laws of Indices

In any well organized establishment, there are laid-down rules for members to follow. Any misapplication of the rules may cause chaos. In the same way, there are set of laws in the indices. These laws are to be religiously applied to solve problems involving the application of indices in relation to management science education.

Consider the following

Simplify the following (i) 10^3 X 10^4 (ii) 5^3 X 5^2 X 5^5

Solution

i. 10^3 X 10^4 = (10 x 10 x 10) (10 x 10 x 10 x 10)

= 10 x 10 x 10 x 10 x 10 x 10 x 10

= 10^7

Or

10^3 X 10^4 = 10^{3+4}

= 10^7

ii 5^3 X 5^2 X 5^5 = (5 X 5 X 5) (5 X 5) (5 X 5 X 5 X 5 X 5)

5^{3+2+5}

5^{10}

Example 2

Simplify the following (i) a^2 X a^3 (ii) b^4 X b^6

a^2 X a^3 = (a X a) (a X a X a)

$=a^2 + 3$

$=a^5$

Generalising from the above examples

a^m X a^N = a^{m+N}

This is called the indices law of addition

Example 3

Simplify $3a^5$ X $4a^7$

$=3a^5$ X $4a^7$ = (3 X 4) (a^5 X a^7)

=12 X a^{5+7}

$=12a^{12}$

Example 4: evaluate the following

i. $a^5 \div a^5$

ii. $x^6 \div x^6$

iii. $10^6 \div 10^3$

i. $a^5 \div a^5 = \frac{a^5}{a^5} = \frac{a\times a\times a\times a\times a}{a\times a\times a\times a\times a} = \frac{1}{1} = 1$

ii. $x^6 \div x^6 \quad = \frac{x.x.x.x.x.x}{x.x.x.x.x.x} = \frac{1}{1} = .1$

$10^6 \div 10^3 = \frac{10^6}{10^3} = \frac{10.10.10.10.10.10}{10.10.10} = 10^3$ i.e. $10^{6-3} = 10^3$

These examples show that the index of the answer or quotient is the difference between the index of numerator and the index denominator.

$\therefore a^m \div a^N = a^{m-N}$ (this is called the indices law subtraction)

Multiplication law of indices $(a^N)^m = a^{mN}$

Example (i) simplify $(a^4)^2 = a^{4\text{x}2} = a^8$

$(2x^5)^3 \quad = 2^3$ X x^{5+3}

$= 8x^{15}$

Evaluate the following

i. $(\frac{27}{8})^{-1\ 3/}$

ii. $\frac{8^{1/3} \text{ X } 64^{-1\ 3/}}{27^{-1\ 3}}$

iii. $\left(\frac{625}{81}\right)^{1\ 4/} \times \frac{5^{-2}}{3^{-3}}$

Solution

i. $\left(\frac{27}{8}\right)^{-1\ 2\ /} = \left(\frac{3^3}{2^3}\right)^{-1\ 3} = \frac{3^{-1}}{3^{-1}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

ii. $\frac{8^{1/3} \times 64^{-1\ 3}}{27^{-1\ 3}} = \frac{(2^3)^{1\ 3}\ (4^3)^{-1\ 3}}{(3^3)^{-1\ 3}} \quad = \frac{2^1 \times 4^1}{3^{-1}}$

$$= \frac{2 \times \frac{1}{4}}{\frac{1}{3}}$$

$$= \frac{2}{1} \times \frac{1}{4} \times \frac{3}{1} = \frac{3}{2} = 1.5$$

iii. $\left(\frac{625}{81}\right)^{1\ 4}$ X $\frac{5^{-2}}{3^{-3}}$ $= \left(\frac{5^4}{3^4}\right)^{1\ 4} \times \frac{5^{-2}}{3^{-3}} = \frac{5}{13} \times \frac{3^3}{5^2}$

$$= \frac{15}{3} \times \frac{27}{25} = \frac{9}{5} = 1.8$$

In-Text Question

Simplify 2^3 X 8^2 X 32 .Leave your answer in index form.

A. ½

B. 4⅛

C. 7⅝

D. 2^{17}

In-Text Answer

D. 2^{17}

3.4 Logarithms

Laws of logarithms

(i) Log (xy) = log$x$ + log$y$

(ii) $\log\left(\frac{x}{y}\right) = logx - logy$

(iii) $\log x^m = mlogx$

Note

i. $log\frac{x}{y} \neq \frac{logx}{logy}$

ii. $loga^a = 1. for\ a \neq 0$

iii. $Log_a 1 = 0$ for $a \neq \frac{logx}{\log y}$

Example: 1 simplify the following logarithm

i. $\frac{\log 2\frac{1}{4}}{\log 1\frac{1}{2}}$ ii. $\frac{\text{Log}\, 8^{-1}}{\log 2}$ iii. $\left(\frac{\text{Log}\, 8^{-1}}{2}\right)$

Solution

i

$$\frac{\log 2\frac{1}{4}}{log 1\frac{1}{2}} = \frac{log\frac{9}{4}}{\log\frac{3}{2}} = \frac{\log\left(\frac{3}{2}\right)^2}{log\frac{3}{2}} = \frac{2log\frac{3}{2}}{log\frac{3}{2}} = 2$$

ii. $\frac{log8^{-1}}{log2} = \frac{log\frac{1}{8}}{log2} = \frac{log\frac{1}{2^3}}{log2} = \frac{log2^{-3}}{log2} = \frac{-3log2}{log2} = -3$

iii. $\log\left(\frac{8^{-1}}{2}\right) = log\frac{1}{8} \div log2 = log\frac{1}{2^3} - log2 = log2^{-3} - log2$

$=-3log - log2$

$=-4log2$

Example:-

Evaluate the following

i. $log\frac{1}{9^9}$ii. $\log_8 0.125$iii. $\log_3 72.9$

Solution

i. $\log_9\frac{1}{9}$ $=\log_9 9^{-1} = -1\frac{log9}{9} = -1$

ii. $\log_8 o.125 = \log_8\frac{1}{8} = \log_8 8^{-1} = -1\log_8 8 = -1$

iii. $\log_3 729 = \log_8 36 = 6\log_3 3 = 6$

Equations involving logarithm

Example 1:

Solve for x in the following

i $\log_x 81 = 4$

ii. $\log_x(\frac{1}{125}) = -3$

iii. Iii. $Log_3(x^2 + x - 5) = 0$

iv. $\log_{10}(x^2 - 15) = 1$

Solution

$$\log_x 81 = 4 \qquad 81 = x^4 = 3^4 = x^4, Hence\ x = 3$$

$$\log_x \frac{1}{125} = -3 \qquad \frac{1}{125} = x^{-3} \quad = \frac{1}{5^3} = x^{-3}$$

$5^{-3} = x^{-3}$

$$\therefore \quad x = 5$$

$$\log_3(x^2 + x - 5) = 0$$

$\therefore \quad x^2 + x - 5 = 3°$

$x^2 + x - 5 = 1$

$x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$

$x = 2\ or\ x = -3$

$\log_{10}(x^2 - 15)21 = 1 \qquad \therefore x^2 + x - 5 = 10^1$

i.e. $x^2 - 15 - 10 = 0$

$x^2 - 25 = 0$

$x^2 = 25$

$x = \sqrt{25}$

$x = \pm 5$

Change of base: The value of a certain logarithm cannot be found with aiding machine or four figure table. E.g.$\log_3 5$, $\log_9 7$, e.t.c. to find the value of such logarithms, we need to change its base to required base. The formula for change of base is $\log_a x = \frac{\log_b x}{\log_b a}$

This relation can also be used to change one base of a given logarithm to the other.

Example: change the base of the following to base of the following to base ten and evaluate your answer.

i $\log_3 6$ ii. $\log_8 5$

Solution:

$$\log_3 6 = \frac{\log_{10} 6}{\log_{10} 3} = \frac{0.7782}{0.4771} = 1.6311$$

$$\log_8 15 = \frac{\log_{10} 15}{\log_{10} 8} = \frac{1.1761}{0.9031} = 1.3023$$

Summary

In study session 3, you have learnt that:

1. Ratio is used to measure the value of quantity in relation to another
2. Percentage is simply a ratio that equates the second number to 100.
3. $a^m \times a^N = a^{m+N}$ under the law of addition
4. One of the laws of logarithms says that Log (xy) = log$x$ + log$y$

Self-Assessment Questions (SAQs) for Study Session 3

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study

SAQ 3.1

The student's enrolment in a department was 55 in 2009 and decreased in the ratio 11: 9 in 2010. Find the decrease in the enrolment.

SAQ 3.2

A man's salary is increased from #1850.00 to #2072.00 per month.

a) Find the percentage increase
b) If the increase is taxed at a rate of 25%, how much does he actually gain from the exercise?

SAQ 3.3

Simplify. $27^{2x} \times 9^{3x}$

SAQ 3.4

Show that $\log_{10}\sqrt{12} + \log_{10}\sqrt{15} - \log_{10}\sqrt{18} = \frac{1}{2}$

Notes for Study Session 3

SAQ 3.1

45

SAQ 3.2

(a) 12% (b) #165.50

SAQ 3.3

3^{12x}

SAQ 3.4

(a) 2 ½ (b) ½

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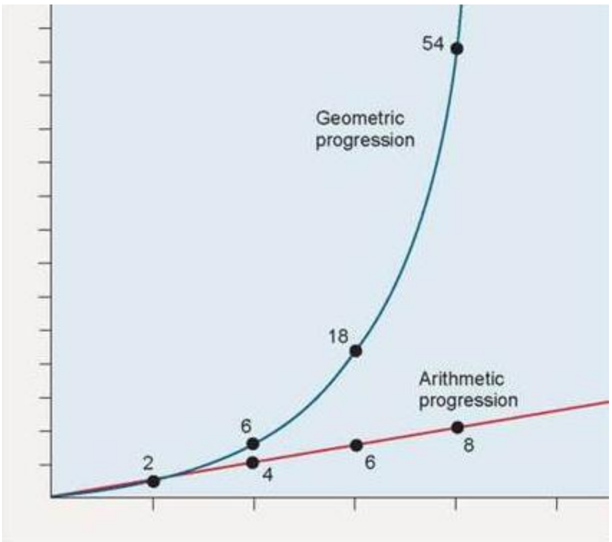
Algebraic process: sequences and series (AP and GP)



Study Session 4: Algebraic process: sequences and series (AP and GP)

Introduction

In the previous study, you were introduced to ratio, proportions, percentages, indices and logarithms as tools in management. In this study, you will be learning about sequence, series, Arithmetic and Geometrical Progression. You will also be guided with some worked exercises that will facilitate your understanding of the computation and calculation of AP and GP.

Learning Outcomes for Study 4

At the end of this study, you should be able to:

4.1 Discuss sequence and arithmetic progression

4.2 Explain geometric progression

4.1 Sequence and Arithmetic Progression

If a set of numbers is arranged consecutively, i.e. following one another and every member of the set is obtained from the previous member by a certain rule, you call the set of numbers a sequence. The sequence is called an Arithmetic Progression if and only if the difference between any number and member immediately before it is constant.

1. Sequence: - A sequence is a set of ordered numbers formed according to some definite rules, e.g. 1, 5, 9, 13,

2, 4, 8, 16, 32, …………….. (i)

2. Series: - A series is the sum of the terms in a sequence, e.g.

1 + 5 + 9 + 13 ……………………….

2 + 4 + 8 + 16 + 32 ………………..

3. Arithmetic Progression: - if the consecutive terms of a sequence differ by a constant number. It is called an Arithmetic Sequence or Arithmetic progression.

The nth term of an Arithmetic Progression (AP) is given by $Tn=a+(n-1)d$. Where $T\,n$ is the nth term or last term of the sequence.

'a' is the 1st term

'n' is the number of terms

d is the common difference

The common difference (d) is obtained by the relationship:-

Common difference =succeeding term – The proceeding term.

Hence for the Arithmetic Progression: 1, 5, 9, and 13

The common difference 'd' = 5 – 1 = 4

9 – 5 = 4

13 – 9 =4

The sum of n term of an A.P is given by

$Sn = \frac{n}{2}\{2a + (n-1)d\}$……………… (i)

OR

$Sn = \frac{n}{2}(a + l)$…………………… (ii)

Apply equation (ii) when the last term (l) is given

Apply equation (ii) when the last term (l) is given

Example 1

Find the 16th term of the sequence 7, 10, 13…..

The fifth term of an arithmetic progression is 15 and the common difference is 3, find the 7th term.

Solution (i) a = 7, d = 3, n = 15

The T n = $a + (n-1)d$

T_{15} = 7 + (15-1)3

= 7 + 14 x 3

= 7 + 42

= 49. I.e. the 15th term of the sequence is 49

5th term i.e. (T_5) = 15, d=3, n=5

But 5th term (T_5) = $a + (5-1)d$

= $a + 4d$

$\therefore a + 4d = 15$

$a + 4 \times 3 = 15$

i.e. $a + 12 = 15$

$a = 15 - 12$

$a = 3$

The seventh term = $a + 6d$

$3 + (6 \times 3)$

$3 + 18$

$= 21$

Example 2:

The 10th term of an AP is 47, the 14th term is 17. Write down the first 3 terms of the sequence.

$Tn = \{a(n-1)d\}$

Solution: the 10th term = T_{10} = $\{a + (10-1)d\}$

T_{10} = $a + 9d = 47$ …………………. (i)

And the 4th term (T_4) = $a + (4-1)d = 17$

i.e. $a + 3d = 17$……………………..(ii)

Solving the two equations simultaneously for 'a and d' we have

$a + 9d = 47 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (i)$

$a + 3d = 17 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$

By elimination method, we have

$6d = 30$

$\therefore d = 5. \quad from\ equation\ (ii) subtract \quad a + 3d = 17$

i.e. $a + 15 = 17$

This gives $a = 2$

∴ The first three terms of the sequence are 2, 7, 12

Remarks:

An AP can be either finite or infinite since the common difference $'d'$ can be added to obtain the next number finitely or finitely, but usually, a certain finite number of the terms is involved in most questions.

Note

The mean term of an AP is usually the middle term of the AP, if n, the number of terms is odd. The value of the arithmetic mean of any series is $\frac{1}{2}\{a + l\}n$. This is obtained by $\frac{3x}{x}$

The general or linear formula of an AP is usually the nth term and by putting n = 1, 2, 3, 4 ……. We get out all the required terms of the series.

The sigma rotation$\Sigma \frac{8}{n=1}\{38 - 3(12-1)\}$

Solution

There is a general form.

∴ Put n= 1, 2, 3 … to get the sequence

T n = $38 - 3(n-1)$ i.e.$n = 38, \quad d = -3$

T_1 = 38 – 3 (1 – 1) = 38

T_2 = 38 – 2 (2 – 1) = 35

T_3 = 38 - 3 (3 – 1) = 32 and so on.

The difference between two consecutive terms is – 3. For this series, $a = 38, d = -3, n = 8$

Hence, 4th term i.e. T_4 = 38 – 9 = 29

T_5 = 29 – 3 = 26

T_6 = 26 – 3 = 23

T_7 = 23 – 3 = 20

And T_8 = 20 – 3 = 17

$\therefore \Sigma_{n=1}^{8} \{38 - 3\,(n - 1)\} = 38 + 35 + 32 + 29 + 26 + 23 + 20 + 17 = 220.$

In-Text Question

Find the sum of the 16th terms of an A P 5, 13, 21 …………………………

A. 1045
B. 2045
C. 1234
D. 1040

In-Text Answer

D.1040

4.2 Geometric Progression

Example:-

3, 6, 12, 24…………………………………………………

$$r = \frac{6}{3} = \frac{12}{6} = \frac{24}{12} = \ldots\ldots\ldots\ldots\ldots\ldots = 2$$

81, - 27, 9, - 3

$$r = \frac{-27}{18} = \frac{9}{-27} = \frac{-3}{9} = \ldots\ldots\ldots\ldots\ldots\ldots = -\frac{1}{3}$$

2, 6, 18, 54

$$r = \frac{6}{2} = \frac{18}{6} = \frac{54}{18} = \ldots\ldots\ldots\ldots\ldots\ldots 3$$

Remark:

The common ratio 'r'' in each example above is obtained by dividing the succeeding term by the preceding term.

The nth term of a Geometric Progression is given by $T_n = ar^{n-1}$ where 'a' is the 1st term, r = common ratio and 'n' is the number of terms.

The sum of a geometric progression is given by:

$Sn = \frac{a(r^n-1)}{r-1}$ Where $r > 1$ OR $Sn = \frac{a(1-r^N)}{1-r}$ $where\ r < 1$

Also $Sn = a\,n$ $where\ r = 1$

Example

What is the 12th term of the GP 2, 14, 98 …………………………

Find the sum of 10 term of the GP 4, 8, 16…………

Solution: (i) $a = 2, r = 7, n = 12$

$\therefore Tn = ar^{n-1} T_{12} = 2 \text{ X } 7^{11}$

$= 2\,(7)^{11}$. Or $2 \text{ X } 7^{11}$

Ii $a = 4, r = 2,\ i.e\ r > 1, n = 10$

$Sn = \frac{a(r^n-1)}{r-1} = S_{10} = \frac{4(2^{10}-1)}{2-1} = \frac{4\times 2^{10}-4}{1} = 2^2 \text{ X } 2^{10} - 2^2$

$= 2^{12} - 2^2$

In some questions like the one above, it is simpler to leave the answer in its index form unless otherwise demanded.

Summary

In this study, you have learnt that

1. A sequence is a set of ordered numbers formed according to some definite rules
2. The n th term of an AP $= a + (n-1)d$
3. The sum of an AP = S $n = \frac{n}{2}\{2a + (n-1)d\}$
4. The sum of a geometrical progression is given by $Sn = \frac{a(r^n-1)}{r-1}\ when\ r > 1$ OR
5. The nth term of an Arithmetic Progression (AP) is given by Tn=$a + (n-1)d$

Self-Assessment Questions (SAQs) for Study Session 4

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study

SAQ 4.1

Three consecutive terms of an exponential sequence are $x-1, 2x\ and\ 5x+3$ respectively. Find (a) x (b) common ratio (c) sum of the first term

SAQ 4.2

What is the common ratio of the GP $\frac{5}{2}$, $\frac{5}{4}$, $\frac{5}{8}$…………………..?

References

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Study Session 5: Mathematics of finance

Introduction

Mathematical finance deals with quantitative finance. It is a field of applied mathematics, concerned with the financial markets. In this study, you will be introduced to the arithmetic of finance such as; simple interest, compound interest, depreciation, discount commission or share and hire purchase.

Learning Outcomes for Study Session 5

At the end of this study, you should be able to:

5.1 Explain in simple interest
5.2 Explain compound interest
5.3 Explain the term ‘commission and depreciation
5.4 Discuss the term ‘discount and hire purchase’

5.1 Simple Interest

Simple interest (I) is the additional money calculated at a given rate per cent per annum. (R) % p.a) on the original amount deposited or borrowed, (P) called this Principal remains constant throughout the period or time (T) of year the money is deposited or borrowed.

Simple interest formula.

If a principal amount of money (P) is invested over a period of time (T) yielding a simple interest (SI) or I, then the S.I given by $I = \frac{P \times R \times T}{100}$ ... ... (i) where R is the rate of interest every year. The amount (A) at the end of the investment period is A = P + I.

i.e. Amount = Principal + Interest

Example 1:

A car costing #10,500 was hired at 5% interest. Find the interest the car would have yielded in 8 years.

Solution:

$$I = \frac{PRT}{100} = \frac{10500 \times 5 \times 8}{100} = 105 \times 40$$

= #4,200

Many financial transactions are done on the basis of simple interest, but banks and big finance houses transact most businesses at compound interest.

Example 2:

Calculate the simple interest on #450.00 for 6 years 9 months at 4%.

Solution

$P = \#450, \quad R = 4\%\, p.a, \quad T = 6\frac{3}{4}\, yr$

$\therefore I = \frac{P \times R \times T}{100} = \frac{450 \times 4 \times 6.75}{100} = \#\, 121.50$

Example 3: calculate the principal which will amount to #949.00 in 4 years at 7 ½ % p. a

Solution: Amount $A = P + I, \quad I = \frac{P \times R \times T}{100}$

$\therefore A = P + \frac{P \times R \times T}{100}$

$A = P\left(\frac{I \times R \times T}{100}\right)$ From the data, $A = \#949.00, \; R = 7.5\%, \quad T = 4\, year$

$\therefore 949 = P + \frac{(P \times R \times T)}{100}$

$= \frac{100P + P \times R \times T}{100} = \frac{100P + P \times 7.5 \times 4}{100}$

$= 949 = \frac{100P + 30P}{100} = \frac{13P}{10} = 1.3P$

i.e. $949 = 1.3P$

$P = \frac{949}{1.3} = \#730$

In-Text Question

Find the rate per cent if # 4,500 yields #180.00 simple interest in 5 years.

A. 8%
B. 10%
C. 7%
D. 4%

In-Text Answer

A. 8%

5.2 Compound Interest

In simple interest, the principal remains the same year after year, but in compound interest, the interest earned in the first year of investment is added to the principal and the amount thus obtained becomes the second principal for the second year.

This is repeated year after year. When this happens, we say that the investment is made at compound interest. To obtain the interest after a period of investment, we calculate the interest year by year.

Unlike simple interest, compound interest changes year by year even though the rate is the same. Compound interest is calculated like simple interest as long as it is remembered that the amount at the first year becomes the principal for the second year and so on.

Calculating Compound Interest.

Example 1: what is the compound interest on #5000 for 4 years at 6% annually?

Solution:

Apply the formula $A = P\left(1 + \frac{r}{100}\right)^{N}$. Where $A = amount\ (\#)$ $P = principal\ (\#),$ $r = rate\ (\%)$ $N = Time\ (year)$

$P = \#500, \quad r = 6, \quad n = 4$

Hence $A = 5000\ (1 + \frac{6}{100})^4$

$= 5000\ (1 + 0.6)^4$

$= 5000 \times (1.06)^4$

$5000\ \times 1.26247$

$A = \#6312.3845$

The compound interest = Amount – Principal

= # 6312.38 - # 5000

= # 1312.38

Example 2: Find the compound interest on #400 for 3 years at 10 % p.a. also calculate the amount at the end of the third year.

Solution:

1st method

Principal (P) = 400

Interest in the first year $= \frac{400}{1} \times \frac{10}{100} \times \frac{1}{1} = \#40.00$

Principal for the 2nd year = 400 + 40 = 440

Interest for the 2nd year $= \frac{440}{1} \times \frac{10}{100} \times \frac{1}{1} = 44$

Principal for the 3rd year = 440 + 44 = 484

Interest for the 3rd year $= \frac{484}{1} \times \frac{10}{100} \times \frac{1}{1} = 48.40$

∴ Amount after 3 years = 484 $+ 48.40 = \#532.40$

Original principal = 400

$\therefore$ Compound interest = 532.40

$$\begin{array}{r} 532.40 \\ -\ 400.00 \\ \hline \#132.40 \\ \hline \end{array}$$

/

Alternative method

Amount$P\left(1+\frac{r}{100}\right)^{N}$, where $P = 400,\quad r = 10,\quad n = 3$

Substituting

$$\therefore A = 400\left(1+\frac{10}{100}\right)^{3} = 400\left(\frac{110}{100}\right)^{3} = 400\left(\frac{11\times 11\times 11}{10\times 10\times 10}\right) = \frac{121\times 11}{10}$$

$$= \frac{2662}{5} = 532.4$$

$\therefore$ Compound interest = (# 532.4 – 400) = #132.40

5.3 Commission and Depreciation

Commission is the remuneration, money or fee obtain by an agent for transacting a business or job for another person, or an organization. Usually, commission is a certain proportion of the amount involved in the business

2k in the # or 2%

7 ½ in# or 7.5%

10k in the # or 10%

Example 1:

A salesman receives $2\frac{3}{4}$ % commission on the money collected for a named company.

a) Calculate his commission in a month, when he collects #5,360
b) How much does he collect in a month when he receives #105.60 as commission?

Solution:

Commission on #5,360.00 at $2\frac{3}{4}$ %

$$\frac{2.75}{100} \times \frac{5360}{1} = \#147.40$$

If $2\frac{3}{4}$ % = $\frac{11}{400}$ of the money calculated = #105.60

Then, total money collected = 100%

$= 105.60 \text{ X } \frac{400}{11}$

= # 3,840.00

Depreciation

As the age of an article increases, its purchasing value decreases, i.e. an article depreciates in value with age

Example:

A machine bought for #10,700.00 depreciates at its rate of 15% per annum. Calculate its value at the end of the fourth year, correct to the nearest kobo.

Solution:-

Yearly calculation

Value at the end of the fourth year = 85% × #10,700.00 = #9065.00

Value at the end of 2^{nd} year = 85% X #9065 = #7730.75

Value at the end of 3^{rd} year = 85% X #7730.75 = #6571.14

Value at the end of the 4^{th} year = 85% X #6571.14 = #5585.47

∴The value of the car at the end of the fourth year = # 5585.47

In-Text Question

Find the approximate number of years that a sum of money invested at compound interest at 4% p.a will double itself. {Hint: if P is the principal, then amount will be $2P$.

A. 56
B. 10
C. 18
D. 11

In-Text Question

A. 18

5.4 Discount and Hire Purchase

Cash discount refers to the allowance given by a seller to a buyer of an article for immediate payment of cash. The discount is usually a percentage of the market price of the article

Example:

After giving a discount of 20% on an article, Adam paid #768.00

i Calculate the market price on the article

ii How much would Adam have paid if the discount was 16k in the # of the market price?

Solution

Adam paid 80% of the market price

i.e. 1% = $\frac{768}{80} = \#9{,}60$

100% which is the market price of the article

= 100 X #9.6

= #960.00

If the discount had been 16k in the #, the discount on #960 = 16k X #960

= 0.16 X 960

= #153.60

Hire purchase

In the hire arrangement, the buyer cannot pay cash for the article immediately. He reaches an agreement with the seller on the terms of payment. The buyer usually deposits part of the cost price (a given percentage) of the article, and pays the rest on a specific number of times. The hire purchase price is usually higher than the cash price. Example: the cost of a machine can be paid either by #24,000.00 cash or by a deposit of #5,000 plus 24 monthly instalments of #875.00 per month. Calculate the difference between the cash and instalment payment.

Solution: Payment by instalment = #5000 + #875 X 24 = # 26,000.00

Cash receipt = #24,000.00

Difference = # (26,000 –24,000)

= 2000.00

Summary

In this study you have learnt that:

1. Simple interest is the additional money calculated at a given rate per cent per annum (R% p.a) on the original amount deposited (P) called the principal remains constant throughout the period (T).$I = \frac{PRT}{100}$

2. Compound interest is calculated like simple interest as long as it is remembered that the amount at the first year becomes the principal for the second year and so on.
3. Commission is the remuneration, money or fee obtain by an agent for transacting a business or job for another person, or an organization.
4. Cash discount refers to the allowance given by a seller to a buyer of an article for immediate payment of cash.

Self-Assessment Questions (SAQs) for Study Session 5

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 5.1

Find the simple interest on #750 for 2 ½ years at 8% per annum

SAQ 5.2

Find the compound interest on #400 years at 10% per annum. Find also the amount at end of the 3 years

Note: A = $P\left(I + \frac{r}{100}\right)^{N}$

SAQ 5.3

Calculate the compound interest on #2578 for 4 years at 5% p.a. correct to the nearest #.

Find the principal which amounts at simple interest to #840 in 3 years at 4% P/A

How much money will you have to lend to get #24.00 interest at 6%, if you lend it for 1 year?

References

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Study Session 6: Graphical Solution of Equations

Introduction

A graph is simply a diagram showing a relationship between two variables. From your experience in elementary mathematics, you are familiar with the Cartesian coordinate system with the horizontal and vertical axes. More of these shall be examined in this study. Furthermore, you will examine the graphical method for solving some algebraic equations.

Learning Outcomes for Study Session 6

At the end of this study, you should be able to:

6.1 Explain the meaning of a graph

6.2 Discuss the graphs of linear functions

6.1 Graph

A graph is simply a diagram showing relationship between two variables. From your work in elementary Mathematics, you are familiar with the Cartesian coordinal system with the horizontal axis (called *x*-axis) and the vertical axis (called *y*-axis) at right angles to each other. The graph of $f(x)$ is the graph of the equation $y = f(x)$.

In this study you will learn the graphical method for solving some algebraic equations. You will learn about the graphs of:

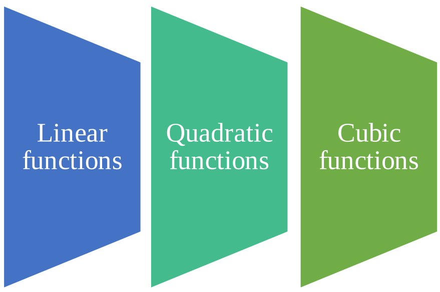
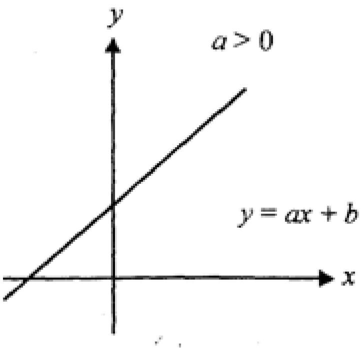
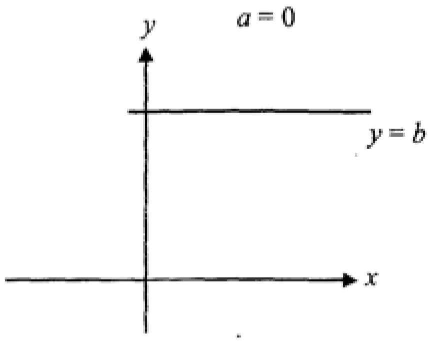


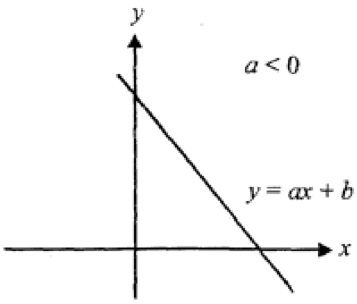
***Figure 6. 1**: Equations*

6.2 Graphs of Linear Functions

A Linear function is a function of the form $f(x) = ax + b$ where a and b are constants. The graph of the linear function $f(x) = ax + b$ is the graph of the equation y = ax + b and this graph is called a straight line. The constant *a* is called the slope or gradient of the line and indicates whether the line rises or falls.







The graph of y ax + b can easily be drawn from a table of values of *x* and *y*.

Example 1

Draw the graph of y = 3x + 2 taking values from -4 to +3.

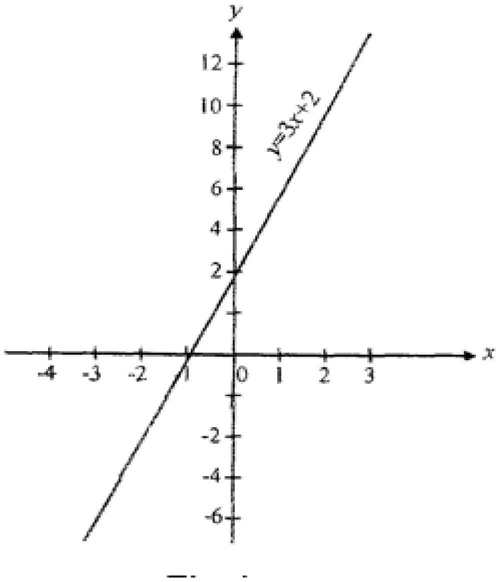
Solution

Construct a table of values of y and x as shown in the table below:

Y = 3x+2

x	–3	–2	–1	0	1	2	3	4
$3x$	–9	–6	–3	0	3	6	9	12
+2	2	2	2	2	2	2	2	2
y	–7	–4	–1	2	5	8	11	14

The graph is shown below:



The scale used is

2 cm: 1unit on *x*-axis

2 cm: 2 units on the *y*-axis

Simultaneous Equations

Two linear equations in two unknowns can be solved simultaneously using the graphical method.

Example 2

Use the graphical method to solve simultaneously the equations

$2y - 3x = 1$

y+2x = 11

Similarly, re-write y + 2x = 11 as

y = 11 - 2x

Next, construct two tables of values one for $y = \frac{3}{2}x - \frac{1}{2}$

and the other for y =11 - 2x

$y = \frac{3}{2}x - \frac{1}{2}$

x	–3	–2	–1	0	1	2	3
$\frac{3}{2}x$	$-4\frac{1}{2}$	–3	$-1\frac{1}{2}$	0	$1\frac{1}{2}$	3	$4\frac{1}{2}$
$+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
y	–4	$-2\frac{1}{2}$	–1	$\frac{1}{2}$	2	$3\frac{1}{2}$	5

Table 6.2

$y = 11 - 2x$

x	–3	–2	–1	0	1	2	3
11	11	11	11	11	11	11	11
$-2x$	6	4	2	0	–2	–4	–6
y	17	15	13	11	9	7	5

Table 6.3

Draw the two graphs on the same axes using the scale of:
2 cm to 1 unit on x-axis
2 cm to 2 units on y-axis

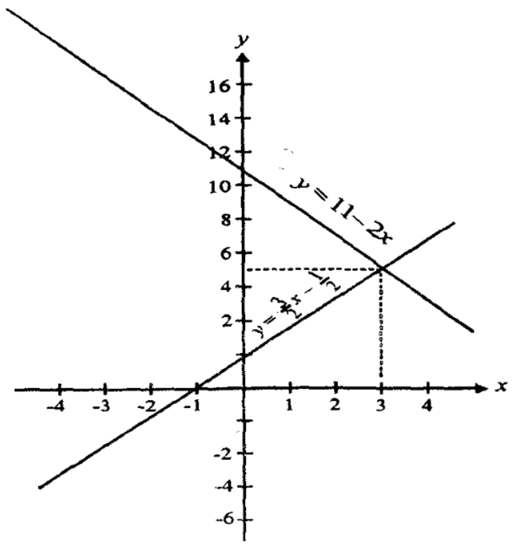


Fig. 6.5

Locate the point of intersection of the two lines. From this point drop a perpendicular to the x-axis to find the value of x.
Similarly, drop a perpendicular to the y-axis to find the value of y from the graph.

$X = 3, y = 5t$

6.3 Graphs of Quadratic Functions

A quadratic function as we have seen in the earlier part of our work is a function of the form $f(x)\ ax^2 \pm bx + c$ where a, b and c are constants such that $a \neq 0$.

The graph of the quadratic function $f(x)\ ax^2 + bx + c$ is the graph of the equation y $ax^2 + bx + c$. This graph is called a parabola. The configuration of the curve is dependent upon a, the coefficient of x^2 and $\Delta = b^2 - 4ac$, the discriminant. You will recall that we came across this expression on the chapter on Algebraic Equations.

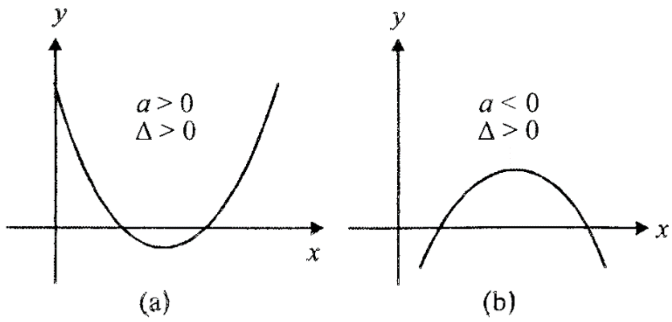


Fig. 6.6

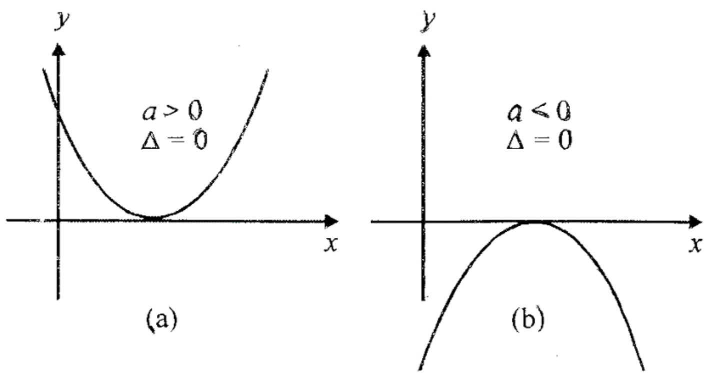


Fig. 6.7

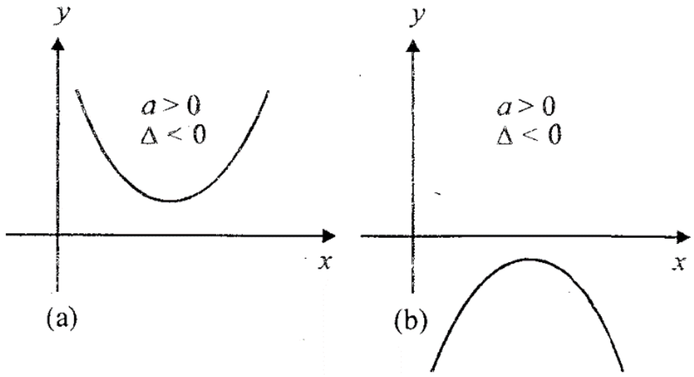


Fig. 6.8

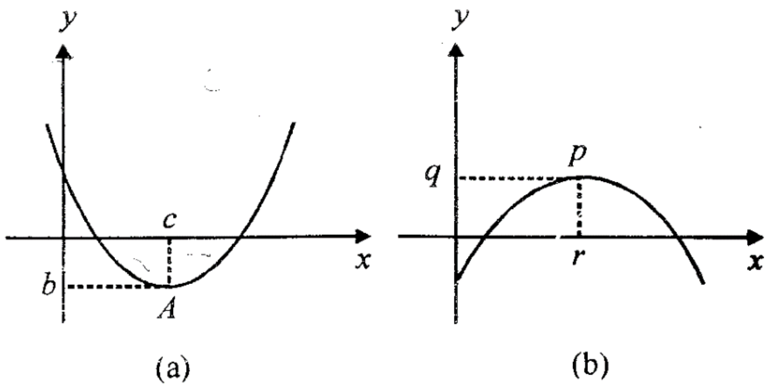


Fig. 6.9

In Fig. 6.9(a) the point *A* is called a minimum point of the curve. The minimum value of $y = b$ and the corresponding value of $x = c$.

Similarly, in Fig. 6.9 (b) the point *P* is called a maximum point of the curve. The maximum value of $y = q$, and the corresponding value of $x = r$.

Example 3

(a) Draw the graph of $y = 4 + 2x - x^2$ for values of x from - 2 to 4. using a scale of 2cm to represent 1 unit on both axes.

(b) Use the graph to obtain the roots of the equation, correct to one decimal place.

(c) (i) Using the same axes as in (a) above, draw the line graph which is required to solve the equation $7 + 2x - x^2 = 0$.

(ii) Use your graphs to obtain the roots of the equation $7 + 2x - x^2 = 0$.

Solution

$Y = 4+2x-x^2$

x	−2	−1	0	1	2	3	4
4	4	4	4	4	4	4	4
$+2x$	−4	−2	0	2	4	6	8
$-x^2$	−4	−1	0	−1	−4	−9	−16
y	−4	1	4	5	4	1	−4

Table 6.4

(a)

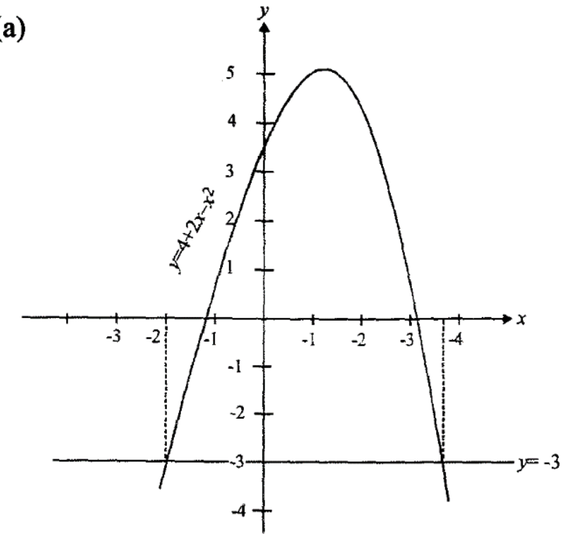


Fig 6.10

(b) The roots of the equation are x = 1.2 or$x$=3.2

(c) $7+ 2x —x^2 = a$

Subtracting 3 from both sides of equation (1)

$7+2x—x^2—3 = 0 - 3$

$4+2x—x^2 = - 3$

Consider the values of x where the parabola$y = 4+2x—x^2$ and$y = - 3$intersect, these values give us the root of the equation$7+2x—x2 = 0$

(i) From the two graphs, the roots of the equation $7+2x—x^2=0$ *are* $x= - l.8$ *or* $x =3.8$.

Example 4

(a) Draw the graph of $y =x2 - 5$ for values of x from - 4 to +4, using a scale of 2cm to 1 unit on the x-axis and 2cm to 2 units on the y-axis.

(b) Use your graph to estimate correct to one decimal place the roots of the equations:

(i) $x^2 - 5 = 0;$

(ii) $x^2 - 3 = 0.$

(c) Use your graph to find the square root of 7 correct to one decimal place.

Solution

(a)

$y = x^2 - 5$

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
–5	–5	–5	–5	–5	–5	–5	–5	–5	–5
y	11	4	–1	–4	–5	–4	–1	4	11

Table 6.5

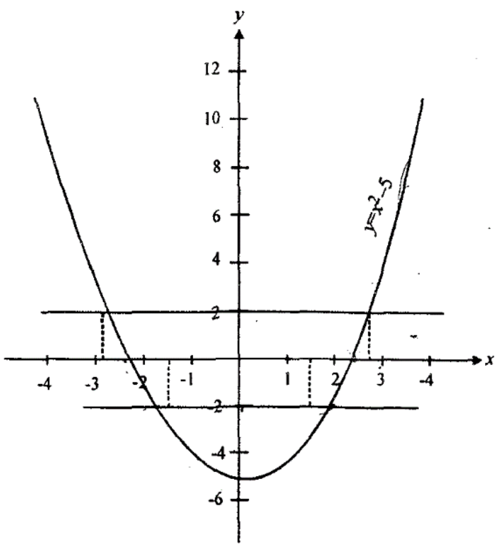


Fig. 6.11

(b) (i) From the graph, the roots of the equation $x^2 - 5 = 0$ are

$x \pm 2.2$

(ii) Given $x^2 - 3 = 0$

$\Rightarrow \quad x^2 = 3$

$\Rightarrow \quad x^2 - 5 = 3 - 5$

$x^2 - 5 = -2$

Consider the graphs of

$y = x^2 - 5$ *and* $y = -2$

The x-values of the point of intersection of the two graphs give the roots of the equation. From the graph

x

$x—5 =2$

(c) Let$x$ $= \sqrt{7}$

$x^2 = 7$

$x^2 – 5 = 2$

The values of x at the point of intersection of $y - x^2 -5$ and$y= 2$ give the square root 7. Hence the square roots of 7 are ± 2.6.

Example 6

(a) Copy and complete the following table of values for the relation

$y = x2 - 2x - 1$ for $-2 \leq x \leq 4$

x	–2	–1	0	1	2	3	4
y			–1			2	7

Table 6.6

(b) Draw the graph of the relation using a scale of 2cm to 1 unit on both axes.

(c) Using your graph, find the:

(i) The roots of the equation $x^2 — 2x — 1 = 0$;

(ii) The minimum value of y.

(d) (i) Using the same axes, draw the graph of $y = 2x —3$

(ii) From your graphs, determine the roots of the equation $x^2 — 2x — 1 =2x —$

3.

Solution

(a)

x	–2	–1	0	1	2	3	4	–1.5	3.5
x^2	4	1	0	1	4	9	16	2.25	12.25
$-2x$	4	2	0	–2	–4	–6	–8	3	–7
–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
y	7	2	–1	–2	–1	2	7	4.25	4.25

Table 6.7

The complete table of values is shown below:

$y — x^2 — 2x — 1$

x	–2	–1	0	1	2	3	4
y	7	2	–1	–2	–1	2	7

Table 6.8

(b)

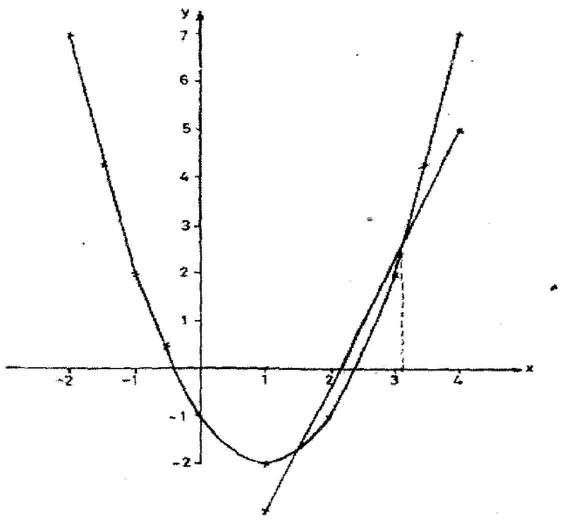


Fig. 6.12

(c) (i)From the graph the roots of the equation $x^2 — 2x — 1 = 0$ are — 0.4 and2.3.

(ii)Minimum value of $y = —2$.

(d) (i)

x	–1	0	4
$2x$	–2	0	8
–3	–3	–3	–3
y	–5	–3	5

Table 6.9

(ii) From the graphs, the roots of the equation $x^2—2x—1 = 2x—3$ are 1.5and 3.1.

Example 7

(i) Copy and complete the following table of values for the relation

$y = 3 — 4x — 2x^2$

x	–4	–3	–2	–1	0	1	2
y	–13				3		

Table 6.10

(ii) Draw the graph of $y = 3 — 4x — 2x^2$ for $— 4 \leq x \leq 2$, taking 2cm to represent 1 unit on the x-axis and 1cm to represent 1 unit on the y-axis.

(iii) Using the same axes, draw the graph of $y=3x+4$.

(iv) Use your graphs to find the values of x, correct to one decimal place for which:

(a) $3—4x—2x^2 = 0,$

(b) $3—4x—2x^2=3x+4$

(v) State the maximum value of y and the corresponding value of x.

Solution

(i)

x	–4	–3	–2	–1	0	1	2	–3.5	–1.5
3	3	3	3	3	3	3	3	3	3
$-4x$	16	12	8	4	0	–4	–8	14	–6
$-2x^2$	–32	–18	–8	–2	0	–2	–8	–24.5	–4.5
y	–13	–3	3	5	3	–3	–13	–7.5	–7.5

Table 6.11

It may sometimes be necessary to take some extra points.

The complete table is $y = 3 — 4x — 2x^2$

x	–4	–3	–2	–1	0	1	2
y	–13	–3	3	5	3	–3	–13

Table 6.12

(ii)

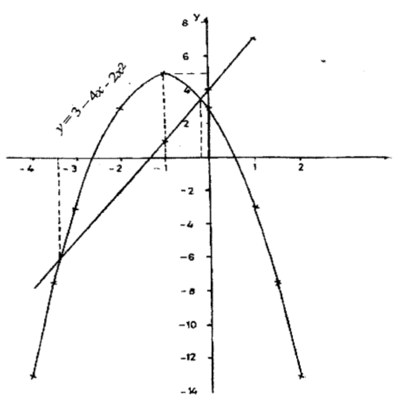


Fig. 6.13

(iii) $y — 3x + 4$

x	–1	0	1
$3x$	–3	0	3
+4	4	4	4
y	1	4	7

Table 6.13

(iv) (a) The values of x for which $3 — 4x — 2x = 0$ are —2.6 and 0.5.

(b) The values of x for which $3 — 4x — 2x^2 = 3x+4$ are —3.4 and —0.2.

(v) The maximum value of y = 5 and the corresponding value of x = - l.

Example 8

The height hem of a stone thrown vertically upwards is given, after time Is, by the relation

$h = 4t(8—t)^2$

(i) copy and complete the following table of values for this relation:

t	0	1	2	3	4	5	6	7	8
h	0	196	288						

Table 6.14

(ii) Draw a graph of the relation between h and t, using a scale of 2cm to 1 unit on the t axis and 2cm to 50 units on the h axis.

(iii) From your graph, find:

(a) the height of the stone when t = 3.5

(b) the values of t for which the height is 200cm.

(c) the greatest height reached by the stone and the time it attains this height.

Solution

t	0	1	2	3	4	5	6	7	8
$4t$	0	4	8	12	16	20	24	28	32
$(8-t)^2$	64	49	36	25	16	9	4	1	0
h	0	196	288	300	256	180	96	28	0

Table 6.15

The complete table for $h = 4t(8 — t)^2$ is

(i)

t	0	1	2	3	4	5	6	7	8
h	0	196	288	300	256	180	96	28	0

Table 6.16

(ii)

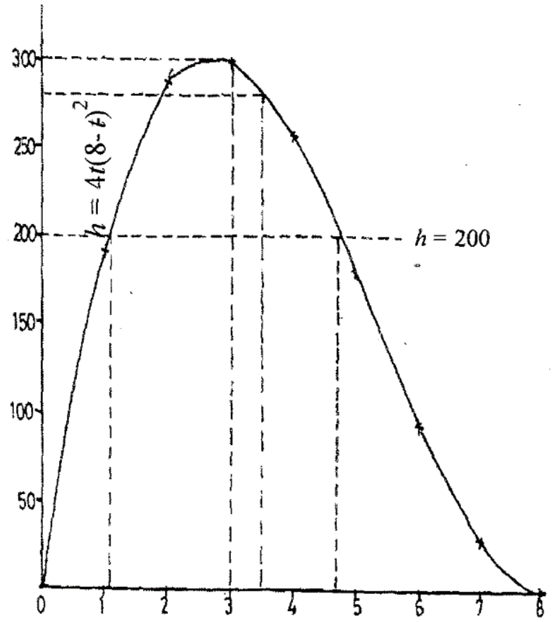


Fig. 6.14

(iii) (a) The height of the stone when t= 3.5 is 280cm.

(b) The values of t for which the height is200cm are 1.ls and 4.7s.

(c) The greatest height of the stone is 300cm and the time it attains this height is 3s.

Note that $h = 4t(8 — t)^2$ is not a quadratic function. It is an example of a cubic function.

Example 9

(a) Copy and complete the data below for the relation $y = 2 + 3x — x^2$

x	−1	0	1	2	3	4
y		2		4		

Table 6.17

(b) Using a scale of 2cm to 1 unit on both axes, draw the graph of $y =2 + 3x — x^2$ for$—1 \leq x \leq 4$.

(c) Use your graph to find the:

(i) maximum value of $2 + 3x — x^2$

(ii) the value of x for which $2 + 3x — x^2$ is maximum.

(d) Using the same axes, draw the graph of $2y—x = 2$.

(e) Use your graphs to find the value of x for which $2+5x—2x^2$ - 0.

Solution

(a)

x	−1	0	1	2	3	4	−0.5	0.5	1.5	2.5
2	2	2	2	2	2	2	2	2	2	2
$+3x$	−3	0	3	6	9	12	−1.5	1.5	4.5	7.5
$-x^2$	−1	0	−1	−4	−9	−16	−0.25	−0.25	−2.25	−6.25
y	−2	2	4	4	2	−2	0.25	3.25	4.25	3.25

Table 6.18

The required table is

x	−1	0	1	2	3	4
y	−2	2	4	4	2	−2

Table 6.19

(b)

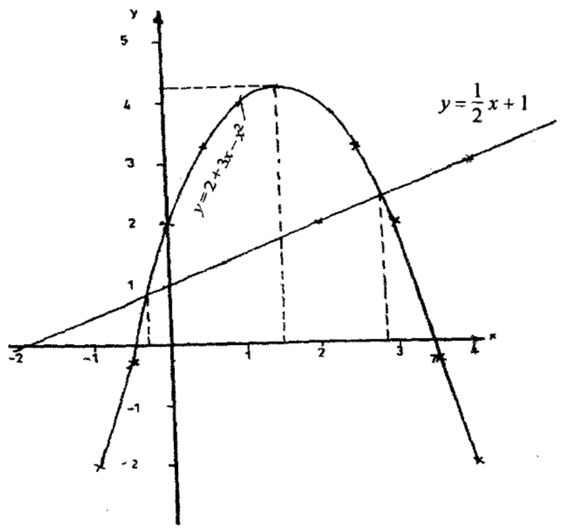


Fig. 6.15

(c) (i) Maximum value of $2 + 3x — x^2$ is 4.3.

(ii) The value of x for which $2 + 3x —x^2$ is maximum is 1.5.

(d) Given $2y—x = 2$

$$2y = 2+x$$

$$y = 1 + \frac{1}{2}x$$

$$y = \frac{1}{2}x + 1$$

x	0	2	4
$\frac{1}{2}x$	0	1	2
+1	1	1	1
y	1	2	3

Table 6.20

(f) Equation of the parabola is

$y = 2+3x—x^2$ ....(1)

Equation of the straight line is

$y = \frac{1}{2}x + 1$ ....(2)

By equating (1) and (2) we have

$$2 + 3x - x^2 = \frac{1}{2}x + 1$$

$$1 + \frac{5}{2}x - x^2 = 0$$

$\therefore 2+5x—2x^2 = 0$

Hence the values of x for which $2+ 5x—2x^2=0$ are the values of x for which $y = 2 + 3x - x^2$ and $y = \frac{1}{2}x + 1$ intersect. These values are —0.3 and 2.8.

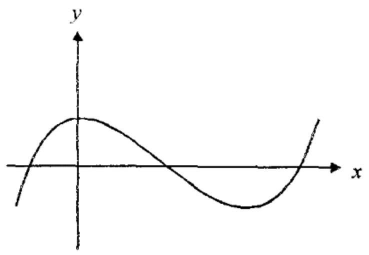
6.4 Graphs of Cubic Functions

A cubic function is a function of the form$f(x)ax^3+bx^2+cx+d$where a, b, c, and d are constants such that a ≠ 0.

The graph of the cubic function$f(x)$ — ax^3+bx^3+cx+d

is the graph of the equation

$y = ax3 + bx2 + cx + d.$



This graph is called cubic parabola. The configuration of the graph is dependent upon a, the coefficient of x^3

The graph of $y = ax^3 + bx^2 + cx + d$

for a > 0

The graph of $y = ax^3 + bx + cx + d$

for a < 0

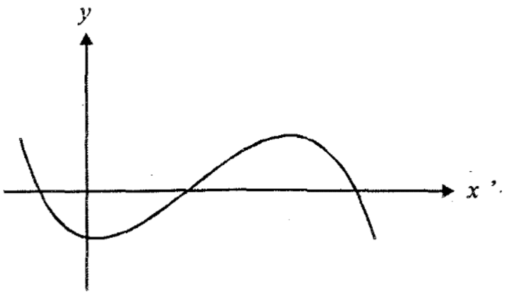


Fig. 6.17

Example 10

An open rectangular box is (10 — 2*x*)cm, (6 — 2*x*)cm wide and *x*cm high.

(a) Copy and complete the following table connecting *V* and *x*.

x	0	0.5	1	15	2	2.5	3
V	0		32		24		0

Table 6.21

(b) Using a scale of 4cm to represent 1 unit on the *x*-axis and 2cm to represent 5 units on the *V* axis, draw the graph of $V=4x(5 - x)(3 - x)$.

(d) Use your graph to find:

(i) the volume of the box when $x = 2.25$

(ii) the values of *x* when $V = 25$.

(iii) the maximum volume of the box and the corresponding value of x.

Solution

(a)

x	0	0.5	1	1.5	2	2.5	3
4*x*	0	2	4	6	8	10	12
5 − *x*	5	4.5	4	3.5	3	2.5	2
3 − *x*	3	2.5	2	1.5	1	0.5	0
V	0	22.5	32	31.5	24	12.5	0

Table 6.22

The complete table is

x	0	0.5	1	1.5	2	2.5	3
V	0	22.5	32	31.5	24	12.5	0

Table 6.23

(b)

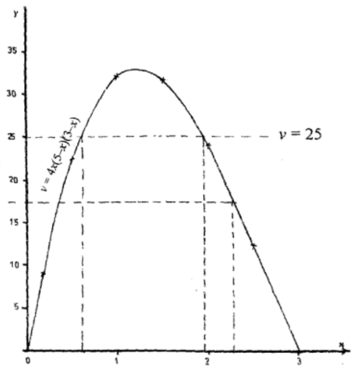


Fig. 6.18

(d) (i) Volume of the box when $x = 2.25$ is17.5 cm^3.

(ii) The values of x when $V = 25$ are 0.6cmand 1.95cm.

Maximum volume of box is 32.5 cm^3 and the corresponding value of x is 1.25cm.

Summary

In this study, the following were treated,

- The graph of the linear function $f(x) = ax+b$ is the graph of the equation $y=ax+b$ and it is called a straight line.
- The graph of the quadratic functions $f(x) = ax^2+bx+c$ is the graph of equation $y=ax^2+bx+c$ and it is called a parabola.
- The graph of the cubic function $f(x) = ax^3+bx2+ax+d$ is the graph of the equation $y = ax^3+bx2+ax+d$ and it is called a cubical parabola

Self-Assessment Questions (SAQs) for Study Session 6

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 6.1

Using a scale of 2cm to I unit on the x-axis and 1cm to 1 unit on the y-axis, draw on the same axes the graph of $y=3+2x-x^2$ ____ *(1)*

$y=2x+2x-3$

For $-3 \leq x \leq 4$___*(2)*

Using your graph

a) Solve the equation $6-x^2 = 0$
b) Find the maximum value of $3+2x-x^2$

SAQ 6.2

A bomb is released from an aeroplane when it is 1,000m above a certain military target. The height hm of the bomb above the target at time ts is given by the relation

$h = 1000 - 5t^2$

Copy and complete the following table for the above relation

t	0	1	3	5	7	9	11	13	15
h				875			395		-125

Using a scale of 1cm to 1sec on t-axis and 1cm to 50m on the h-axis, draw a graph of the relation between h and t.

Use your graph to find

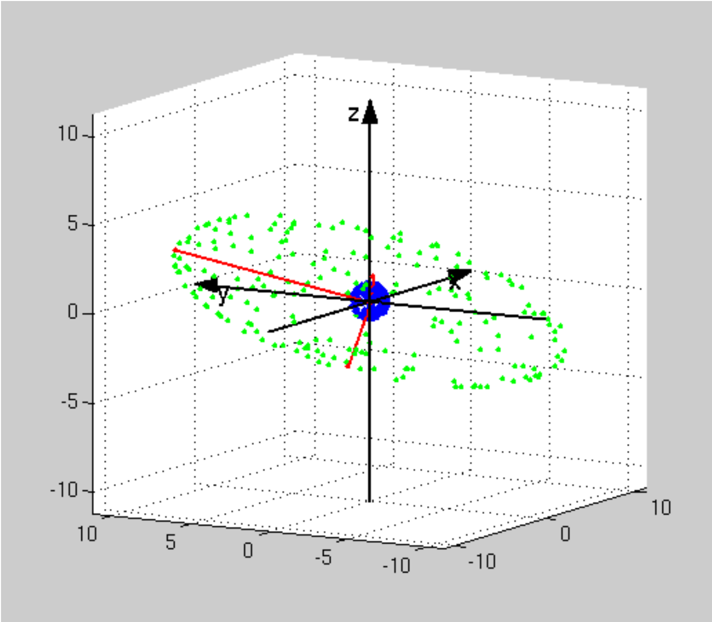
The time the bomb takes to reach the target correct to decimal place

The time the bomb takes to drop through the first 650m, correct to one decimal place.

Reference

T.R. Moses: Further Mathematic Scholastic Series

Matrix Algebra (1)



Study Session 7: Matrix Algebra (1)

Introduction

In previous study you learnt matrix algebra, the meaning, types and application of matrices. In this study however, the focus is on determinant and matrix inverse and its application to system equations.

Learning Outcomes for Study Session 7

At the end of this study, you should be able to:

7.1 Define a matrix
7.2 List types of Matrices
7.3 Acquire the calculative skill necessary in solving problems relating to matrices particularly the addition, subtraction and multiplication.

7.1 Matrices

A Matrix is a rectangular array of elements arranged in rows and columns enclosed within brackets. Example of a matrix (i) (2, 3, 4) (ii) $\begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$ (iii) $\begin{pmatrix} a & b & c \\ e & f & g \end{pmatrix}$

The numerals or variables (letters) contained in each of the matrices above are called elements of the matrix.

Matrices are often described by the number of rows and columns they have. For instance, a matrix A, where A = $\begin{pmatrix} 1 & 2 \\ 6 & 7 \end{pmatrix}$ has two rows and two columns.

The first row has 1 and 2 as its elements, while the 2^{nd} has elements 6 and 7. The 1^{st} column has elements 1 and six, while the 2^{nd} column has elements 2 and 7.

NOTE: All the horizontal components form the rows, while the vertical components form the columns. Also each element in a matrix has its own location. This location is described by the position of row and column the element occupies.

For example:

Consider Matrix X = $\begin{bmatrix} A11 & a12 \\ A11 & a12 \\ A11 & a12 \end{bmatrix}$ Row (2) Col 3
Row (3) Col 3
Row (4) Col 3

The first number subscript stands for row position and the second number subscript stands for column position.

Example 1: describe the position of the element 9 and 11

Matrix Y = $\begin{pmatrix} 2 & 5 & 9 \\ 7 & 8 & 11 \end{pmatrix}$

Order of a matrix: A matrix is described by its order. The order of a matrix is the number of rows and columns its contains.

For example $\begin{pmatrix} 2 & 3 \\ 5 & 8 \end{pmatrix}$ has two rows and two columns. It is said to be (2x2) matrix. Similarly a matrix A = $\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$ has three rows and two columns. This is (3x2) matrix.

In general, a matrix with 'm' rows and 'n' columns is said to be m x n matrix. The order is not however commutative, that is m x n matrix is not the same as n x m matrix.

7.2 Types of Matrices

1) Line Matrix: this is a matrix with only one row. It is also called a row matrix e.g. (1 3 6 7)
2) Column Matrix: A column matrix is a matrix with only one column e.g. $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$
3) Square Matrix: It is a matrix with the same number of rows and columns e.g A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, B = $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ where A is a 2 by 2 square matrix and B is a 3 by 3 square matrix.
4) A zero or null matrix: this is a matrix whose elements are zero the order notwithstanding e.g. X = $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, Y = $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
5) Diagonal Matrix: This is a square matrix whose elements are Zero except the elements in the leading diagonal e.g. (i) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$

6) Unit or identity Matrix: this is a square matrix with all the elements in the leading diagonal equal to 1 and every other element is zero e.g. I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, I_2 = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. I = 2x2 unit matrix and I_2 = 3x3 unit matrix

Remark:

Two Matrices are equal if and only if they are of the same order and the corresponding elements are equal e.g. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, = $\begin{pmatrix} r & s \\ t & u \end{pmatrix}$ if and only if a = r, b = s, c = t, d = 4.

Matrix Operation

Addition: Given two matrices A and B of the same order, then the sum of A and B is A+B is obtained by adding the corresponding elements in each matrix.

Example (i) $\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix}$, = $\begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$

(ii) Given that A= $\begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, B = $\begin{pmatrix} 3 & 5 & 6 \\ 7 & 8 & 9 \\ 1 & 2 & 3 \end{pmatrix}$ find A+B

Solution:

$$A+B = \begin{pmatrix} 2+3 & 3+5 & 4+6 \\ 5+7 & 6+8 & 7+9 \\ 8+1 & 9+2 & 1+3 \end{pmatrix} = \begin{pmatrix} 5 & 8 & 10 \\ 12 & 14 & 16 \\ 9 & 11 & 4 \end{pmatrix}$$

Exercise: find B+A

Subtraction: Given two matrices A and B of the same order, the difference between A and B i.e. A-B is obtained by subtracting corresponding elements in each matrix, e.g.

Given that X = $\begin{pmatrix} 5 & 6 & 7 \\ 1 & 4 & 3 \end{pmatrix}$, Y = $\begin{pmatrix} 2 & 6 & 4 \\ 3 & 2 & 1 \end{pmatrix}$

Find X-Y

Solution: X-Y = $\begin{pmatrix} 5-2 & 6-6 & 7-4 \\ 1-3 & 4-2 & 3-1 \end{pmatrix}$ = $\begin{pmatrix} 2 & 0 & 3 \\ -2 & 2 & 2 \end{pmatrix}$

Example 2: give that A = $\begin{pmatrix} 5 & 6 & 7 \\ 1 & 3 & 4 \\ 8 & 2 & 9 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \\ 8 & 1 & 2 \end{pmatrix}$

Find A-B

Solution; A-B = = $\begin{pmatrix} 5-3 & 6-4 & 7-5 \\ 1-6 & 3-7 & 4-8 \\ 8-8 & 2-1 & 9-2 \end{pmatrix}$ = = $\begin{pmatrix} 2 & 2 & 2 \\ -5 & -4 & -4 \\ 0 & 1 & 7 \end{pmatrix}$

Exercise: find (i) B-A (ii) is A-B equal to B-A.

Matrix Multiplication

Given two matrices A and B, the product is possible if and only if the number of column in A is equal to the number of rows in B, similarly, the product of BA is possible if and only if the number of columns in B is equal to the number of rows in

A.

Example: if A = $\begin{pmatrix} a & e \\ o & u \end{pmatrix}$ and B = $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

The product of AB is possible since the number of columns in A is 2 and the number of rows in B is also 2. BA is not possible because the number of columns in B is 1 and the number of rows in A is 2

For example R = (a b c), Q = $\begin{pmatrix} r \\ s \\ t \end{pmatrix}$

Since R has three columns and q has three rows, the product RQ is possible.

Then RQ = (a b c) $\begin{pmatrix} r \\ s \\ t \end{pmatrix}$ = (ar + bs = et)

Example: given that A =$\begin{pmatrix} x & e \\ w & t \end{pmatrix}$, D = $\begin{pmatrix} u & v \\ r & s \end{pmatrix}$

Then AD = $\begin{pmatrix} x & y \\ w & t \end{pmatrix}\begin{pmatrix} u & v \\ r & s \end{pmatrix}$ = $\begin{pmatrix} xu + yr & xv & ys \\ wu + tr & wv & ts \end{pmatrix}$

To obtain AD, the following steps are taken:

Multiply the <u>first row</u>of A by the first column of D. the result gives the first element (xu + yr) of the first row of the product.

Multiply the first row of A by the second column of D, the result gives the second element (xu + ys) of the first row of the product.

Multiply the second row of A by the first column of D. the result gives the first element (wu + tr) of the second column of D. the result gives the second element (wr + ts) of the second row of the product.

Example: if A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 0 & 4 \\ 3 & 1 & 5 \end{pmatrix}$ find AB and BA

Solution (i) AB = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & 0 & 4 \\ 3 & 1 & 5 \end{pmatrix}$

AB = = $\begin{pmatrix} (1x2) + (2x3) & (1x0) + (2x1) & (1x4) + (2x5) \\ (3x2) + (4x3) & (3x0) + (4x1) & (3x4) + (4x5) \end{pmatrix}$ = = $\begin{pmatrix} 6 & 2 & 14 \\ 18 & 4 & 32 \end{pmatrix}$

The number of columns in B is 3, while the number of rows in A is 2. Since the number of columns in B is not the same as the number of rows in A, it is therefore not possible to find product BA. Hence BA does not exist.

Application of Matrices

Transpose of a Matrix: this is the matrix obtained from a matrix A by interchanging rows and columns. In this case the first row of A becomes the first column of the transpose and so on. The transpose of matrix A is often denoted by A^T or A^I.

Example: (i) If B = $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ then B^T = $\begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix}$

(ii) Given that A = $\begin{pmatrix} 3 & 4 & 5 \\ 7 & 8 & 9 \\ 1 & 2 & 6 \end{pmatrix}$, A^T = $\begin{pmatrix} 5 & 7 & 1 \\ 4 & 8 & 2 \\ 5 & 9 & 6 \end{pmatrix}$

Symmetric Matrix: A matrix A is said to be symmetric if it is equal in all respect to its transpose i.e. if A = A^T e.g.

B = $\begin{pmatrix} 2 & 4 \\ 4 & 1 \end{pmatrix}$, B^T = $\begin{pmatrix} 2 & 4 \\ 4 & 1 \end{pmatrix}$ hence B = B^T, B is a symmetry matrix

Exercise: (i) Given that X = $\begin{pmatrix} 2 & 4 & 3 \\ 4 & 1 & 5 \\ 3 & 5 & 3 \end{pmatrix}$ is X a symmetric Matrix?

(ii) Find the transpose of the matrix $\begin{pmatrix} 3 & 5 & 6 \\ 2 & 2 & 1 \\ 5 & 4 & 3 \end{pmatrix}$

Trace of a Matrix: the trace of a matrix is the sum of the element in its principal diagonal. E.g. consider matrix A = $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}$ the element in the principal diagonal are a, e , and k. ∴ the trace = a + e + k

Example:

Find the trace of Matrix A.

A = $\begin{pmatrix} 3 & 1 & 5 \\ 2 & 3 & 5 \\ 5 & 1 & 6 \end{pmatrix}$ trace A = (3+3+6) = 12

Find the trace of matrix P = $\begin{pmatrix} 1 & 4 \\ 6 & 7 \end{pmatrix}$, trace P = 1+7 =8

Exercise: find the trace of the matrix

B = $\begin{pmatrix} 5 & 7 \\ 9 & 10 \end{pmatrix}$, (ii) C = $\begin{pmatrix} a & b & c \\ g & m & r \\ t & n & r \end{pmatrix}$ (iii) $\begin{pmatrix} 3 & 2 & 6 \\ 2 & 3 & 4 \\ 6 & 1 & 5 \end{pmatrix}$

In-Text Question

If A = $\begin{pmatrix} 7 & 3 \\ 4 & 2 \end{pmatrix}$, and B = $\begin{pmatrix} -3 & 1 \\ 2 & 4 \end{pmatrix}$ Find |A|

A. 2
B. 7
C. 8
D. 10

In-Text Answer

A. 2

Summary for Study Session 7

In study session 7, you have learnt:

1. The basic computational techniques in determinant and matrix inverse.
2. Some examples on how to find the determinant of a matrix.
3. The identification of minor and cofactor of a matrix
4. The determinant of a 3x3 matrix
5. The inverse of 2x2 matrix
6. The solution to simultaneous linear equation using crammer's rule.

Self-Assessment Questions (SAQs) for Study Session 7

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 7.1

Exercise: give the order of the following matrices:

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ (ii)$\begin{pmatrix} a & b & c \\ d & e & f \\ x & y & z \end{pmatrix}$ (iii) $(1\ 2\ 3\ 4)$ (iv) $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

SAQ 7.2

If X = =$\begin{pmatrix} 3 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix}$, Y = $\begin{pmatrix} 3 & 4 \\ 5 & 2 \\ 6 & 1 \end{pmatrix}$ find XY and YX

References

Odeyinka J.A.: Basic Mathematics and Statistics for undergraduates and prospective Undergraduates.

Maria N. David-Osuagwu et al (2000): New School Mathematics for Senior Secondary Schools.

Matrix Algebra (ii)

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Study Session 8: Matrix Algebra (ii)

Introduction

In the previous study, you learnt about functions, graph, equation and algebraic expressions. In this study, however you will learn matrix algebra. Emphasis will be in addition, subtraction and multiplication of matrices and some of the examples will be related to educational planning and administration.

Learning Outcomes for Study Session 8

At the end of this study, you should be able to:

- 8.1 Define the determinant of a square matrix
- 8.2 Explain the Minor and Cofactor of a Matrix
- 8.3 Explain simultaneous linear Equations using Cramer's Rule

8.1 Determinant of a Matrix

The determinant of a square matrix denoted as det (A) or (A) is the numerical valve of that matrix. Determinant of a 2 by 2 matrix

Given that A =$\begin{pmatrix} a & \text{b} \\ c & d \end{pmatrix}$ (A) = $\begin{pmatrix} a & \text{b} \\ c & d \end{pmatrix}$ = ad – bc

Example 1: Find the determinant of the matrix.

B=$\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$, (ii) C= $\begin{pmatrix} 6 & -2 \\ -5 & 4 \end{pmatrix}$

Solution

B=$\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$= (B) = $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$ = (3x5) – (4x2) = 15 – 8=7

C=$\begin{pmatrix} 6 & -2 \\ -5 & 4 \end{pmatrix}$, (C) = $\begin{pmatrix} 6 & -2 \\ -5 & 4 \end{pmatrix}$ = (6x4) – (2x – 5)

=24 – (10)

=24 – 10 =14

8.2 Minor and Cofactor of a Matrix

Minor:

Consider a 3 by 3 matrix

$$A= \begin{pmatrix} a11 & a12 & a13 \\ a2 & a22 & a23 \\ a3 & a32 & a33 \end{pmatrix}$$

The determinant obtained when the row and column containing a11 are deleted is the minor ofa11. It is always donated by

A11 $\begin{pmatrix} a22 & a23 \\ a32 & a33 \end{pmatrix}$ Similarly the minor of A12, a12 is $\begin{pmatrix} a21 & a23 \\ a31 & a33 \end{pmatrix}$

And the minor of A13 $\begin{pmatrix} a21 & a22 \\ a31 & a32 \end{pmatrix}$

Cofactor

This is obtained by associating appropriate sign to each of the minor obtained. This is the determination of the sign to be associated. The sign associated to each of the minor is determined by finding the sum of the subscript attached to each of them e.g. for minor A is, the sum of the subscript is I + s is even, the sign associated will be (+ve). If otherwise the sign associated will be (-ve)

e.g. A11, the sum of 1 + 1 = 2 = even (+ve)

A12, the sum of 1 + 2 = 3 = odd = (-ve)

Hence we have +A11 = + $\begin{pmatrix} a22 & a23 \\ a31 & a32 \end{pmatrix}$

-A12 = - $\begin{pmatrix} a21 & a23 \\ a31 & a33 \end{pmatrix}$

In general, the signs associated with the minor of each of the dements in 3x3 matrix depending on the location are given below:-

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

Example 2: - If A = $\begin{pmatrix} 2 & 5 & -3 \\ 3 & 6 & 4 \\ 7 & 8 & -2 \end{pmatrix}$ Find the minor cofactor of the following elements on A (i) 5 (ii) (7).

Solution:

The minor of 5 = $\begin{pmatrix} 3 & 4 \\ 7 & -2 \end{pmatrix}$ = (3x -2) – (4 x 7)

= - 6 – (28) = - 34

Since this is a 3 x 3 matrix, the corresponding cofactor of 5 is – (-34) = +34

(ii) The minor of 7 =$\begin{pmatrix} 5 & -3 \\ 6 & 4 \end{pmatrix}$ = (5 x 4) – (-3 x 6)

= 20 – (-18)

= 20 + 18 = +38

The corresponding cofactor of 7 is + (+38) = 38

Exercise:

Find the minor and cofactor of the elements 6 and 2 in matrix A above.

Determinant of a 3 by 3 matrix:-Given that

X=$\begin{pmatrix} p & q & r \\ s & t & u \\ v & w & y \end{pmatrix}$

Det X = 1 X 1 = $\begin{pmatrix} p & q & r \\ s & t & u \\ v & w & y \end{pmatrix}$ => p$\begin{pmatrix} t & u \\ w & y \end{pmatrix}$ - q $\begin{pmatrix} s & u \\ v & y \end{pmatrix}$ +r $\begin{pmatrix} s & t \\ v & w \end{pmatrix}$

P(ty – wu) –q (sy – uv) +r (sw – vt)

Example 3:-

If Z = $\begin{pmatrix} 5 & 2 & 3 \\ 1 & -2 & 4 \\ -3 & 5 & 7 \end{pmatrix}$ find (Z)

(Z)= $\begin{pmatrix} 5 & 2 & 3 \\ 1 & -2 & 4 \\ -3 & 5 & 7 \end{pmatrix}$ => 5$\begin{pmatrix} -2 & 4 \\ 5 & 7 \end{pmatrix}$ -2 $\begin{pmatrix} 1 & 4 \\ -3 & 7 \end{pmatrix}$ +3 $\begin{pmatrix} 1 & -2 \\ -3 & 5 \end{pmatrix}$

= 5(-2 x 7) – (5 x 4) -2 (1 x 7) – (3 x 4) +3 (1 x 5) – (-3 x 2)

=[5(-14 – 20)] -2[7 – (12)} +3 [(5 – (6)]

= - 170 + 10 – 3

=- 173 + 10

=- 163

Exercice:-

Find the determinant of matrix

(i) P= $\begin{pmatrix} 2 & 1 & -3 \\ 1 & 2 & -2 \\ -2 & 3 & 4 \end{pmatrix}$ (ii) A= $\begin{pmatrix} 3 & 7 \\ 4 & 8 \end{pmatrix}$ C= $\begin{pmatrix} 5 & 1 & 2 \\ 3 & 2 & 4 \\ 4 & 6 & 6 \end{pmatrix}$

The inverse of a 2 by 2 matrix.

Given a matrix A= $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ which is a 2 x2 matrix, the inverse of a 'A' is given by

$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ where ad – bc = (A)

If (A) = O, the increase does not exist. Matrix A is then said to be singular.

Example 4: find the inverse of matrix B= $\begin{pmatrix} 3 & -5 \\ 2 & 7 \end{pmatrix}$

Solution:

Here a=3, b=-5

C=2, d=7.

But $A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

:- $\frac{1}{(3\,x\,7) - (-5\,x\,2)}\begin{pmatrix} 7 & -(-5) \\ -2 & 3 \end{pmatrix}$

$\frac{1}{21\,x\,10}\begin{pmatrix} 7 & 5 \\ -2 & 3 \end{pmatrix} = \frac{1}{31}\begin{pmatrix} 7 & 5 \\ -2 & 3 \end{pmatrix}$

Exercise:

Find the inverse of the matrix y=$\begin{pmatrix} 5 & 5 \\ 6 & 7 \end{pmatrix}$

Solution of Simultaneous equation.

------------ ax + by = r } These can be broken into

------------ cx + dy = (} Three matrices such as:-

i. The matrix of the coefficients of the variables represented by A $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

ii The column matrix of the unknown donated by

X=$\begin{pmatrix} x \\ y \end{pmatrix}$, while the constants donated by B=$\begin{pmatrix} p \\ q \end{pmatrix}$

Thus, the simultaneous linear equations given above can be expressed in matrix form.

AX= B or $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$ ---------- (3)

Where A =$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, X=$\begin{pmatrix} x \\ y \end{pmatrix}$ and B = $\begin{pmatrix} p \\ q \end{pmatrix}$

From ---- (3) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}\begin{pmatrix} p \\ q \end{pmatrix}$

I.e. $\begin{pmatrix} x \\ y \end{pmatrix}= \frac{1}{ad- bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$ -----(4)

Where ad – bc = (A), from (4) numerical values of x and y are obtained

Example 5: X + Y = 5 ---------(1)

2x – y =1---------(2)

Find the valve of (i) x and y

3x + y.

Solution:-

Rewrite the equation in matrix form. i.e

$$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

Hence A= $\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ = (A)= $\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ = (1x -1) – (2x1) =-3

$\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$:- a=1, c=2, d =-1

$\begin{pmatrix} x \\ y \end{pmatrix}= \frac{1}{(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$

= $\frac{1}{-3}\begin{pmatrix} -1 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix}$

= $\frac{1}{3}\begin{pmatrix} -5 & -1 \\ -10 & +1 \end{pmatrix} = \frac{-1}{3}\begin{pmatrix} -6 \\ -9 \end{pmatrix}=\frac{2}{3}$

Ie $\begin{pmatrix} x \\ y \end{pmatrix}= \frac{2}{3}$

i.e x=2, y=3.

(ii) 3x + y = 6+3 = 9

<u>Exercise:-</u>

Solve for x and y in the equation: (2 – 1) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 9 \end{pmatrix}$

In-Text Question

Find (i) A + B

A. $\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & -1 \end{pmatrix}$

B. $\begin{pmatrix} 1 & -8 & 5 \\ 8 & -2 & 1 \end{pmatrix}$

C. 7 9 1

D. $\begin{pmatrix} -1 & 8 & -5 \\ -8 & 2 & -1 \end{pmatrix}$

In-Text Answer

$\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & -1 \end{pmatrix}$

8.3 Solution of simultaneous linear Equations using Cramer's Rule

Given that ax + by =p

By Cramer's rule, $X=\frac{\begin{pmatrix} p & b \\ q & d \end{pmatrix}}{(A)}$ $y=\frac{\begin{pmatrix} a & p \\ c & q \end{pmatrix}}{(A)}$

Where (A) = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ which is the determinant of A.

Example: - use Cramer's rule to solve the simultaneous equation

X + y=5

2x – y=1

Solution:- Coefficient matrix A=$\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$, where a=1, b=1, c=2, d=-1

(A) $\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$= -3: $\begin{pmatrix} p \\ q \end{pmatrix}$ = $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$

By Cramer's rule x= $\frac{\begin{pmatrix} p & b \\ q & d \end{pmatrix}}{(A)}$ = $\frac{\begin{pmatrix} 5 & 1 \\ 5 & -1 \end{pmatrix}}{(A)}$ = $\frac{(5x – 1) – (1 x 1)}{- 3}$

= $\begin{pmatrix} -6 \\ -3 \end{pmatrix}$ = 2. ie x=2

Y = $\frac{\begin{pmatrix} a & p \\ c & q \end{pmatrix}}{(A)}$ = $\frac{\begin{pmatrix} 1 & 5 \\ 2 & 1 \end{pmatrix}}{(-3)}$ = (ivi) – 5x2 - $\frac{1-9}{-3}$

= $\frac{-9}{-3}$ = 3

Y =3, Hence x=2, y =3

In-Text Question

Find (i) A + B

$A, \begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & 0 \end{pmatrix}$

B $\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & -1 \end{pmatrix}$

C $\begin{pmatrix} 3 & 0 & 1 \\ 2 & 33 & -1 \end{pmatrix}$

D $\begin{pmatrix} 3 & 0 & 1 \\ 9 & 4 & -1 \end{pmatrix}$

In-Text Answer

$\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & -1 \end{pmatrix}$

Summary

In study session 8, you have learnt:

1. Cofactoring which is obtained by associating appropriate sign to each of the minor obtained.
2. Simultaneous linear equation solution using Cramer's rule.

Post – Test:-

Ans:- u=5, v=3, w=1, x=2, y=0, z=4

(2) If A= $\begin{pmatrix} 2 & -4 & 3 \\ 5 & 1 & 0 \end{pmatrix}$ and B= $\begin{pmatrix} 1 & 4 & -2 \\ -3 & 3 & -1 \end{pmatrix}$

Find (i) A + B (ii) A – B (iii) B – A

Ans:- (i) $\begin{pmatrix} 3 & 0 & 1 \\ 2 & 4 & -1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & -8 & 5 \\ 8 & -2 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} -1 & 8 & -5 \\ -8 & 2 & -1 \end{pmatrix}$

Find (i) 2A- 3B + c (ii) 4(B -2C)

Ans:- $\begin{pmatrix} 3 & 19 \\ -5 & 2 \end{pmatrix}$ (ii) $\begin{pmatrix} -8 & -28 \\ -28 & 12 \end{pmatrix}$.

(i) AB

Ans:- $\begin{pmatrix} 11 & -6 & 14 \\ 1 & 2 & -14 \end{pmatrix}$

Self-Assessment Questions (SAQs) for Study Session 8

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 8.1

(1) Given that A= $\begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix}$ B= $\begin{pmatrix} 5 & 3 & 1 \\ -2 & 0 & 4 \end{pmatrix}$

Find the value of u v w, x y and z if A =B.

SAQ 8.2

(1) Given that A= $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, B= $\begin{pmatrix} 0 & -3 \\ 5 & 1 \end{pmatrix}$ and c= $\begin{pmatrix} 1 & 2 \\ 6 & -1 \end{pmatrix}$

SAQ 8.3

Use Cramer's rule to solve the equation

(2) 5x – 2y =14

2x + 2y =14

(3) Solve the equation $\begin{pmatrix} 2 & -3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

References:

Monia N. David – Osuagwa et al (2000) New School Mathematics for Senior Secondary Schools.

Odeyinka J.A:- Basic Mathematics and Statistics for undergraduates and prospective undergraduates.

Study Session 9: Probability Theory

Introduction

In everyday life, there are chances of certain events occurring. Such events could be in form of travelling abroad, winning a football match to mention a few.
All these forms of activities are referred to as experiments. This study focuses on probability concept, relative frequency of probability and probability laws.

Learning Outcomes for Study Session 9

At the end of this unit, you should be able to:
9.1 Define some probability concept
9.2 Explain Probability as a Relative Frequency
9.3 Discuss Probability Laws

9.1 Probability Concept

To understand probability concept, the following definition are necessary:

1. An Experiment
2. Sample space
3. Sample Point
4. Event
5. Number of Elements
6. Probability of an Element
7. Sure Event

(1) **An Experiment:** This is any process that generates raw data. E.g. throwing a coin or a die, number of students to be considered for admission to the department etc.

(2) **Sample space:** This is the set whose elements represents all possible outcomes of an experiment. It is represented by the symbol S. e.g. when a die is tossed once, the sample space. S= { 1,2,3,4,5,6}

(3) **Sample Point: -** An element of a sample space is called a sample point, e.g. given that a sample space, S= {a, b, c, d}, each number of the sample space is a sample point. E.g. ‘b’ is a sample point.

(4) **Event:** An event is a subset of the sample space ‘S’ governed by a given rule, e.g. When a fair die is tossed, the sample space S= {2, 3, 5,}.

(5) **Number of Elements:** Let S represent a sample space, n(s) is defined as the number of elements in the sample spaces.

(6) **Probability of an Event:** In general sense, the chance of happening of an event when expressed, quantitatively is called probability.

Remarks

Let ‘g’ be the sample space and ‘E’ an event in ‘S’. Let n (S) be the number of points in S and n(E) the number of point in ‘E’, then the probability of event E is defined as Pr(E)=$\frac{No of points in E}{No of points in S}$ u $\frac{n(E)}{n(S)}$

(7) **Sure Event:** This is an event whose probability is one e.g. Given that S={2,4,6,8}:- n(S)=4. Let A represent the event that an even number occurs :- A={2,4,6,8}

n(A)=4 Hence Pr(A)= $\frac{n(A)}{n(S)} = \frac{4}{4}$ =1.

Properties of probability:- Let A be an event and S the sample space, (i) O≤P (A)≤/

(i) ∑P (A) = 1

(ii) If A and A^c are complementary events, then P(A) + P(A^c) = 1. Or P(A)=1 – P(A^c).

Example 1

A fair coin is tossed twice, find the probability that at least one head appears.

Solution:-

The sample space S = {HH, HT, TH, TT}:- n(S) =4. Let X represent the event that at least one head appears, X= {HH, HT, TH}:- n(X) =3

Pr (of at least one head occurs) = $\frac{n(x)}{n(S)} = \frac{3}{4}$.

Example 2:- A number is chosen at random from the set S={X: 5≤X<15} where x is a natural number. Find the Probability that the number is an even number.

Solution

5≤X<15 implies that 5 is included in the set but 15 is not included.

: - S={X: 5≤X<15}

:- S= {5,6,7,8,9,10,11,12,13,14} :- n(S) = 10

Let B represent the event that the number chosen is an even number:-B (6, 8, 10, 12, 14}, n (B) =5

:- Pr (B) =$\frac{n(B)}{n(S)}$= $\frac{5}{10}$ =$\frac{1}{2}$.

In-Text Question

Two dice are tossed together. Find the probability of getting a total of 7 or 8 or 9.

A. $\frac{5}{12}$
B. 7/7
C. 8/9
D. 1/2

In-Text Answer

A. $\frac{5}{12}$

9.2 Probability as a Relative Frequency.

Exercise 1

The table below shows the marks obtained by 15 students in a Mathematics test whose obtainable mark is 10.

Mark x	2	3	4	5	6	7	8
No of student (f)	2	1	2	3	5	1	1

Use the table to find:-

a) The probability that a student scores exactly 6 marks
b) If the pass mark is greater than 4, find the probability that a student fails the test.
c) Hence find the probability that a student passed the test.

Solution:

The total number of student = 15 i.e. n(S) = 15

(a) If T is the event that a student scores exactly 6 marks, from the table, n (T) =5. Hence the Probability. that a student scores exactly 6 marks is

Pr(T)= $\frac{n(T)}{n(S)}$= $\frac{5}{15}$ =$\frac{1}{3}$.

(b) Let A represent the event that a student fails the test. If the pass mark is greater than 4, from the table, number of students who fails the test n(A)

n(A)=2+1+2=5

Pr (a student fails the test)= $\frac{n(A)}{n(S)} = \frac{5}{15} = \frac{1}{3}$.

(c)The probability that a student passes the test is

$P(P^c) = 1 - P(A)$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

9.3 Probability Laws

Probability laws are the following:

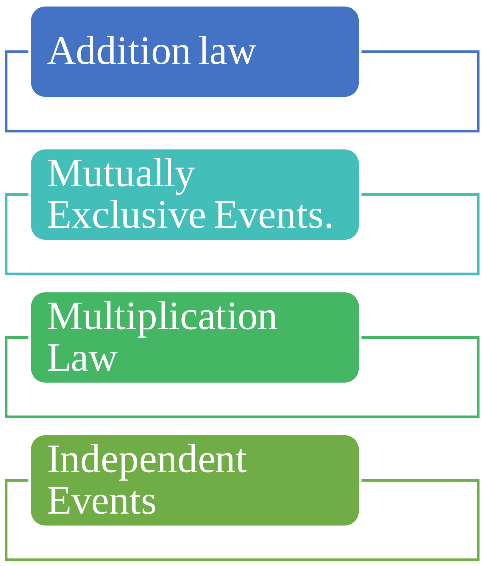


Figure 9.1: Probability laws

1. Addition law of probability

Given two events A and B, the addition law of probability states that

P (AUB)=P(A) + P(B) – P(AnB)

Example:- If A and B are two events such that P(AuB) = $\frac{3}{4}$, P(A^c) =$\frac{2}{3}$ and P(AnB) =$\frac{1}{8}$ find P(A) (ii) P(B).

Solution

Here, A and A^c are complementary events.

Hence, P (A) + (A^c) =1

: - P (A) = 1 – P (A^c)

$=1-\frac{2}{3}=\frac{1}{3}$. P (A) = $\frac{1}{3}$

By this addition law, of probability,

P (AuB) = P (A) + P (B) – P (AnB)

$\frac{4}{5}=\frac{1}{3}$ + P (B) - $\frac{1}{8}$

$\frac{4}{5}=\frac{1}{3}-\frac{1}{8}$ + P (B)

P (B) = $\frac{3}{4}-\frac{1}{3}+\frac{1}{8}=\frac{18-8-3}{24}=\frac{21-8}{24}=\frac{13}{24}$

P (B) = $\frac{13}{24}$.

Example

John set for an examination in English and Mathematics, the probability that he passes English is $\frac{3}{4}$ and the probability that he passes Mathematics is$\frac{4}{9}$. If the probability of passing at least one subject is$\frac{2}{3}$, what is the probability that he pass both subject.

Solution

Let E and M represent the event that John passes English and Mathematics respectively.

:- P(E)= $\frac{3}{4}$, P(M) + $\frac{4}{9}$, prob (of passing at least one) = P(EuM) = $\frac{2}{3}$

We are to find the probability that he will pass both subjects

i. e P (EnM), by addition law,

P (MuE) = P (M) + P (E) – P (MnE)

: - P (MnE) = P (M) + P (E) – P (MuE)

$\frac{4}{9}+\frac{3}{4}-\frac{2}{3}=\frac{19}{36}$.

Exercise: - Given that X and Y are two events such that P (XuY$\frac{7}{8}$, P (X^c) =$\frac{5}{8}$, P (XnY) =$\frac{1}{4}$, find P(Y)

2. Mutually Exclusive Events.

Two events A and B are said to be mutually exclusive if they cannot occur simultaneously. In other words, two evens A and B are mutually exclusive if AnB=Ø or P(AnB)=o.

Under this condition, for two mutually exclusive events AnB, the addition law of probability becomes; P(AnB)=P(A) + P(B).

Example1:

If 'H' is the event that an even number appears and 'I' is the event that an odd number greater than one occurs in a single throw of a die. (i) show that H and I are mutually exclusive

(i) Find the probability of obtaining either an even number or odd number greater than 1.

Solution 2:

The sample space S= {1, 2, 3, 4, 5, 6}:- n(S) =6

H= {2, 4, 6}; n (H) =3

: - P (H) = $\frac{n(I)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

I= {3, 5}:- n (I) =2.

Pr (I) = $\frac{n(I)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

Since H= {2, 4, 6} and B = {3, 5}

: - HnIØ, Hence H and I are mutually exclusive.

P (H3uI) =P (H) + P (I) = $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

Exercise:-

(1)

Ans:- = $\frac{9}{10}$ or 0.9

(2) If A and B are mutually exclusive events such that, $P(B^C)$=0.7, and $P(A^C)$= 0.8 find P(AuB).

Ans:-= $\frac{1}{2}$ or 0.5.

3. Multiplication Law.

Conditional Probability

Let A and B be two events, the probability that an event A occurs given that B has occurred is the conditional probability of A given B. it is define as:- P(A/B)= $\frac{P(AnB)}{P(B)}$ with P(B)>0 ----(1)

From (1) P (AnB) = P (B). P (A/B) ----- (2)

Equation (2) gives the multiplication law for conditional probability.

Example: A box of contains 15 identical balls. 7 of them are red and the remaining ones are black. If two balls are taken one after the other from the box without replacement, find:-

The prob. that the 1st drawn is red and the 2nd is black.

The two balls are of the same colour.

Solution:-

(a) Total Number of balls =15, Red =7, Black =8

Pr(R $\frac{7}{15}$, Pr(B) =$\frac{8}{14}$

:- Pr(AnB)= P(R). P (B/R)

= $\frac{7}{15}$ X $\frac{8}{14}$ = $\frac{4}{15}$

There are two possibilities, that

a) The first ball taken is R and the second one is also R or (b) the first ball taken is 'B' and the second ball taken is also 'B'.

Let R, represent the event that the first ball is Red and R_2, the first ball taken is Blue and B_2 the event. : - $P(R_1nB_2) = P(R_1). P (R_2/A_1)$

$= \frac{7}{15}$ X $\frac{6}{14} = \frac{1}{15}$

P (B_{1+}). P (B_2/B_1)

= $\frac{8}{15}$ X $\frac{7}{14}$ =$\frac{4}{15}$

Hence the probability that the two ball are of the same colour

=p (R_1nR_2) u P(B_1nB_2)

=p (R_1nR_2) + P(B_1nB_2)

= $\frac{1}{5} + \frac{4}{15} = \frac{7}{15}$.

4. Independent Events:

Two or more events are said to be independent if the probability of occurrence of any other.

In other words for two events X and Y to be independent,

P(X/Y) =P(X).

Recall that P(X/Y) = $\frac{P(XnY)}{P(Y)}$ ie

P (XnY)= P(X/Y). P(Y) ---------- (1)

If X and Y are independent, P(X/Y) = P(X)

Using this condition, (1) becomes

P(X/Y) = P(X). P(Y) ----------- (2)

(2) Gives the multiplication law for independent events.

Example (1).

A bag contains 6 oranges, 4 bananas and 2 mangoes, if three fruits are picked from the bag, find the probability that they are picked in the order: orange, banana and mango if the selection is done with replacement.

Solution:-

Let R represent the event that the fruit picked is orange, B the event that the fruit picked is banana and m event that the fruit picked is mango.

Total number of fruit = 6+4+2 =12

No of oranges =n(R) =6:- P(R) = $\frac{6}{12}$= $\frac{1}{2}$

No of bananas =n (B) =4:- P (B) = $\frac{4}{12}$ = $\frac{1}{3}$

No of mangoes n (M) =2:- P (M) = $\frac{2}{12}$ = $\frac{1}{6}$

: - P (RnBnM) = P (B). P (M) =$\frac{1}{2}$ X $\frac{1}{3}$ X $\frac{1}{6}$ =$\frac{1}{36}$.

Example: - (2).

If the probability of Mary solving a problem is$\frac{1}{6}$, and of Samuel solving is $\frac{3}{5}$, what is the probability that (i) at least one of them will solve it

(i) One will and one will not.

Solution:

(i) The probability of Mary solving is $\frac{1}{6}$

:- Prob. Of Mary not solving it = 1 - $\frac{1}{6}$ = $\frac{5}{6}$

The prob. Of Samuel solving it is $\frac{3}{5}$

Prob. Of Samuel not solving it = 1 - $\frac{3}{5}$ = $\frac{2}{5}$

The probability of at least one of them will solve it is the same as (i) the prob. Of either Mary solves the problem and Samuel does not or Samuel solves the problem and Mary does not or both of them solve the problem.

= ($\frac{1}{6}$,$\frac{2}{5}$) +($\frac{3}{5}$ X $\frac{3}{5}$) + ($\frac{1}{6}$ X $\frac{3}{5}$)

= $\frac{2}{3}$ X $\frac{15}{30}$ X $\frac{3}{30}$

$= \frac{20}{30} = \frac{2}{3}$.

ii The probability that one will and one will not is the same as either Mary solves X and Samuel does not or Samuel solves it and Mary does not.
Hence the probability of one will and one will not

$=(\frac{1}{6} X \frac{15}{30}) + (\frac{3}{5} X \frac{5}{6})$

$= \frac{2}{30} X \frac{15}{30} = \frac{17}{30}$.

In-Text Question

If A and B are mutually exclusive events such that, P (B^C) =0.7, and P (A^C) = 0.8 find P (AuB).

A. 0.1
B. 0.5
C. 0.7
D. 0.3

In-Text Answer

B. 0.5

Summary for study Session 9

In study session 9, you have learnt that:

1. An experiment is any process that generates raw data
2. In general sense, the chance of happening of an event when expressed, quantitatively is called probability
3. Properties of probability:- (i) $o \leq p(A) \leq 1$ (ii) $\sum p(A)=1$ (iii)If A and A_c are complementary, then $P(A) + P(A^C) = 1$ or $P(A) = 1 – P(A^C)$.

Self-Assessment Questions (SAQs) for Study Session 9

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 9.1

A fair die is thrown, find the probability of getting a prime number

A number is chosen at random from the set E= {P: 3≤P≤12}. Find the probability that the number chosen is a prime.

SAQ 9.2

The data below show the number of studys employed in various departments of a particular University.

Department	No of studys
Mathematics	20
Physics	15
Chemistry	9
Biology	8
Geography	25
Music	3

If a studyr is promoted, what is the probability that he is either from Mathematics department or Biology department?

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Study Session 10: Calculus I

Introduction

Change (i.e. increase or decrease) is constant in life. This may mean change in salary, change in cost of living, change in growth, change in population etc. The changes in these variables (i.e. Salary, height, cost, population etc.) could arise as a result of length of service, age, area, quantities available etc.

For instance, a government worker can have an increase in salary every year resulting in salary increase (or change in time). The salary(s) depends on the time (t) i.e. the variable 's'is a function of the variable't'. This is written symbolically as s=f (t).S is called the dependent variable and t, the independent variable. This study focuses on differentiation and direct differential laws

Learning Outcomes for Study Session 10

At the end this study, you should be able to:

10.1 Explain differential coefficient of a function.

10.2 Identify the direct differential laws

10.1 Differentiation

The rate of change of y with respect to x is defined by

$\frac{dy}{dx}$ Or f^1 (x)

If y = f(x), $\frac{dy}{dx} = f^1$ (x)

$\frac{dy}{dx}$ and f^1 (x) can be used interchangeable depending on what is given.

Differentiation from first principles

This is the method for finding the derivative of a function from the consideration of the limiting value.

Example 1

Find the derivation of $f(x) = x^2 + 3$ from the first principle. If f(x) is replaced by y, then $y= x^2+3$

Let y be a small increase in y and$\triangle$ x the corresponding small increase in x

: $y + \Delta y = (x + \Delta x)^2$

$\Delta y = [(x + \Delta x)^2 + 3] - y$

$= [(x + \Delta x)^2 + 3] - (x^2 + 3$

$= x^2 + 2x\Delta x + \Delta x^2 + 3 - x^2 - 3$

$= 2x\Delta x + \Delta x^2$

: $\frac{\Delta y}{\Delta x} = \frac{2x\Delta x + \Delta x^2}{\Delta x}$

$2x + \Delta x$

$\frac{dy}{dx} \quad \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim 2x + \Delta x = 2x$

: $\frac{dy}{dx} = 2x$ (same as above)

Example 2

Find from the first principle the differential coefficient of the following function with respect to x.

(i) $y = x^2$ (ii) $y = \frac{1}{x}$

SOLUTION

(i) $y = x^2$ i.e $f(x) = x^2$. If x change by Δx then $f(x + \Delta x) = (x + \Delta x)^2$ Applying the above:

$\frac{dy}{dx} = \frac{\lim_{\Delta x \to 0} f(x+\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{(x\Delta x)^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - (x)^2}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x}$

$= \lim_{\Delta x \to 0} 2x + \Delta x$ Δx vanishes as $\Delta x \to 0$

:- $\frac{dy}{dx} = 2x$

(ii) $y = f(x) = \frac{1}{x}$: $f(x + \Delta x) = \frac{1}{x + \Delta(x)}$

:- $\frac{dy}{dx} = \frac{\lim_{\Delta x \to 0} f(x4\Delta x) - f(x)}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$: Simplifying

$$\lim_{\Delta x \to 0} \frac{(x - \Delta x)}{x(x + \Delta x)\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{x + (x - \Delta x)}{x(x + \Delta x)\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\Delta x}{x(x + \Delta x)\Delta x} = \lim_{\Delta x \to 0} \frac{-1}{x(x + \Delta x)}$$

$= \frac{-1}{x^2}$

Example 3.

Find the differential coefficients of the followings functions at the point x = 2.

(i) $y=3x^3$ (ii) y=c where c is a constant.

Solution

:- y=> $f(x) = 3x^2$; $f(x + \Delta x) = 3(x + \Delta x)^3$

:- $f(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

⇨ $\lim \frac{3(x + \Delta x)^3 - 3x^3}{\Delta x}$

⇨ $\lim_{\Delta x \to 0} \frac{3x^3 + 9x^2 \Delta x + 9x(\Delta x)^2 + 3(\Delta x)^3 - 3x^3}{\Delta x}$

⇨ $\lim_{\Delta x \to 0} 9x^2 + 9x\Delta x + 3(\Delta x)^3$

⇨ $9x^2$

i.e $f'(x) = 9x^2$

:- $f'(2) = f'(x)$ at $x = 2 = 9(2)^2 =$ 36.

(ii) $y = f(x) = c$:- $f(x + \Delta x) = c$

:- $f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

⇨ $\lim_{\Delta x \to 0} \frac{c - c}{\Delta x} = 0$

:- $f^1(x) = 0$

$F^1 (2) = \underline{0}$

In-Text Question

Differentiate $3x^4 + 7x^3 – 5x + 1$

A. => $\frac{dy}{dx}$ = -5 – 12

B. $\frac{dy}{dx}$ = -5 – 14

C. $\frac{dy}{dx}$ = -5 – 18

D. $\frac{dy}{dx}$ = -5 – 17

In-Text Answer

A. => $\frac{dy}{dx}$ = -5 – 12

10.2 Direct Differential laws.

Given that

(i) $Y=x^n, \frac{dy}{dx} = nx^{n-1}$

(ii) $y=x^n \frac{dy}{dx} = nx^{n-1}$

(iii) $y=c \frac{dy}{dx} = 0.$

Example 4.

Differentiate the following functions

(i) $3x^4 + 7x^3 – 5x + 1$

(ii) $3 + \frac{5}{x} + \frac{6}{x2}$

(iii) $\frac{3x^4 + 4x^3 – 2x}{3x^2}$

Solution

(i) Let $y = 3x^4 + 7x^3 – 5x + 1$

$\frac{dy}{dx} = 12X^3 + 21x^2 – 5$

(ii) let $y = 3 + \frac{5}{x} + \frac{6}{x2}$

$= 3 + 5x^{-2} + 6x^{-2}$

$\frac{dy}{dx} = -5x^{-2} – 12x^{-3}$

$= \underline{-5} \quad - \underline{12}$

$X^2 \quad x^3$

$:- \frac{dy}{dx} = \frac{-5}{x^2} - \frac{12}{x^3}$

Note: In differentiation all the terms are expressed in indicial form before differentiating. The example above explains this.

Let $y = \frac{3x^4 + 4x^3 - 2x}{3x^2}$

$= x^2 + \frac{4x}{3} - \frac{3}{3x}$

$= x^2 + \frac{4x}{3} - \frac{3}{3x} \ -1$

Differentiation

$:- \frac{dy}{dx} = 2x + \frac{4x}{3} - \frac{3}{3x} - 2$

$= 2x + \frac{4}{3x} + \frac{2}{3x^2}$

Remark: If the given expression has a common denominator. We divide each term by the common denominator before differentiating.

Exercise:

Methods of Differentiation

(i) Function of functions.

It is easy to expand a function of the form $(3x^3 + 3x^2 + 3)^2$. The task becomes more cumbersome if a function of the form $(3x^3 + 3x^2 + 3)^{100}$ has to be expanded and differentiated. To be able to differentiate this type of function, the method of differentiation of functions has to be employed.

Example:-

Differentiate the following.

(i) $(ax^2 + bx + c)^n$ (ii) $(5x^2 + 2x + 1)^{100}$ (iii) $\sqrt{(x + \frac{1}{x})}$

SOLUTION

(ii) let $y = (ax^2 + bx + c)^n$

and $T = (ax^2 + bx + c$. $:- \frac{dT}{dx} = 2ax + b$

$:- y = T^n$ $:- \frac{dT}{dx} = nT^{n-1}$

$:- \frac{dy}{dx} = \frac{dy}{dT} \times \frac{dT}{dx} \qquad = (nT^{n-1})(2ax+b)$

$= n(ax^2 + bx + c)^{n-1}(2ax + b)$

$= n(2ax + b)(ax^2 + bx + c)^{n-1}$

:- $\frac{dy}{dx} = n(2ax + b)(ax^2 + bx + c)^{n-1}$.

(iii) $y=(5x^3 + 2x + 1)^{100}$

let $R = 5x^3 + 2x + 1$, $y=R^{100}$

:- $\frac{dR}{dx} = 15x^2 + 2$; $\frac{dy}{dR} = 100R^{99}$

:- $\frac{dy}{dx} = \frac{dy}{dR} . \frac{dR}{dx} = (100R^{99})(15x^2 + 2)$

$= 100(5x^3 + 2x + 1)^{99}(15x^2 + 2)$

:- $\frac{dy}{dx} = 100(15x^2 + 2)(5x^3 + 2x + 1)^{99}$.

(iii) $y = \sqrt{x + \frac{1}{x}} = (x + \frac{1}{x})^{1/2}$

Let $p = x + \frac{1}{x}$

$=(x+x^{-1})$; $y = p^{1/2}$

: - $\frac{dy}{dx} = (1 - x^{-2})$; $\frac{dy}{dp} = (\frac{1}{2}p^{1/2-1}) = \frac{1}{2}p^{-1/2}$.

: - $\frac{dy}{dx} = \frac{dy}{dp} \times \frac{dp}{dx}$

⇨ $(\frac{1}{2}P^{-1/2})(1 - x^{-2})$

⇨ $\frac{1}{2}(x + \frac{1}{x})^{-1/2}(1 - \frac{1}{X^2})$

⇨ $\frac{1}{2\sqrt{x + \frac{1}{x}}} \times (1 - \frac{1}{x^2})$

⇨ $\frac{1 - \frac{1}{X^2}}{2\sqrt{x + \frac{1}{X^2}}}$

Differentiation

$\frac{X^2 - 1}{\sqrt{2x^2 \; x + \frac{1}{x}}}$

Product rule

Given that y=uv where u and v are functions of x.

By product rule, $\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{du}{dx}$

Example 5:

Find the derivations of

i. $y=(3x - 5)(2x^2 + 3x + 2)$

ii. $y= (8x - 3)^2 (5x^2 + 3)$.

Solution

$y= (3x - 5) (2x^2 + 3x + 2)$

This is in form y = UV where.

$U= 3x - 5$; $V =2x^2 + 3x + 2$.

: - $\frac{du}{dx}= 3$; $\frac{dv}{dx} = 4x + 3$.

:- By product rule,

$\frac{dy}{dx} = U \frac{dv}{dx} + V \frac{du}{dx}$

$= (3x - 5) (4x - 3) + (2x^2 + 3x + 9x) x 3$

$= (12x^2 + 9x - 20x - 15) + (6x^2 + 9x + 6)$

$= 18x^2 - 2x - 9$.

:- $\frac{dy}{dx} = 18x^2 - 2x - 9$.

(ii) $y=(8x - 3)^2 (5x^2 + 3)$.

Let $U = (8x - 3)^2$; $V = (5x^2 + 3)$

$\frac{du}{dx} = 2(8x - 3) \frac{d}{dx} (8x - 3)$ $\frac{dv}{dx} = 10x$

$=2X8 (2x - 3)$

$= 16(8x - 3)$

$\frac{dv}{dx} = 10x$.

Hence $\frac{dy}{dx} = U \frac{dv}{dx} + V \frac{du}{dx}$

$= (8x - 3)^2. 10x + (5x^2 + 3) 16(8x - 3)$

$= 10x (8x - 3)^2 + 16 (8x - 3) (5x^2 +3)$

Differentiation

$= (8x - 3) [10x (8x - 3) + (16) (5x^2 + 3)]$

$= (8x - 3) (80x^2 - 30x + 80x^2 + 48)$

$= (8x - 3) (160x^2 - 30x + 48)$

: - $\frac{dy}{dx} = 2 (8x - 3) (80x^2 - 15x + 24)$.

Exercise: Differentiate (i) $(2x^2 + 1) (3x + 2)$ (ii) $(x^2 +\frac{1}{x}) (2x^2 - 3)$

Quotient Rule.

Let $y= \frac{U}{V}$

By Quotient rule,

$$\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V_2}$$

Where U and V are functions of x.

Example: - Differentiate the following (i) $\frac{5x^2 + 2x}{x - 2}$ (ii) $\frac{(2x + 1)^2}{3x - 5}$

Solution

Let $y = \frac{5x^2 + 2x}{x - 2}$

Let $u = 5x^2 + 2x$ $v = x - 2$.

(i) :- $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V_2}$

$= \frac{(x - 2)(10x - 2) - (5x^2 + 2x) .1}{(x - 2)^2}$

$= \frac{(10x^2 + 2x - 20x - 4) - 5x^2 + 2x)}{(x - 2)^2}$

:- $\frac{dy}{dx} = \frac{5x^2 - 20x - 4}{(x - 2)^2}$

(ii) $y = \frac{(2x - 1)^2}{3x - 5}$

$U = (2x + 1)^2$; $V = 3x - 5$

:- $\frac{du}{dx} = 2(2x + 1)^1 \frac{d}{dx}(2x + 1)$

⇨ $4(2x + 1)$

$\frac{dv}{dx} = 3.$

:- $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V_2}$ $= \frac{[(3x - 5)(4)(2x + 1)^2 \text{ x } 3]}{(3x - 5)^2}$

$= \frac{(8x + 4)(3x - 5) - 3(2x + 1)^2}{(3x - 5)^2}$

$= \frac{24x^2 - 40x + 12x - 20 - 3(4x^2 + 4x + 1)}{(3x - 5)^2}$

:- $\frac{dy}{dx} = \frac{12x^2 - 40x - 23}{(3x - 5)^2}.$

In-Text Question

Differentiate w.r.t x $\frac{5x^2 + 2x}{X - 2}$

A. 3x+56/23-5
B. 44x-11x2/x-6
C. 1x2-4/4x-1
D. $\frac{5x^2 - 20x - 4}{(x - 2)^2}$

In-Text Answer

D $\frac{5x^2 - 20x - 4}{(x - 2)^2}$

Summary for Study Session 10

In study session 10, you have learnt:

1. The basic concepts which are computational techniques of differential calculus analysis in relation to educational planning and development.
2. Differential coefficient of a function.
3. The standard derivatives of some basic function were derived and applied with some worked example.

Self-Assessment Questions (SAQs) for Study Session 10

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 10.1

Differentiate the following (i) $2x^7 + 3x^4 - 2x$ (ii) $x^2 - \frac{1}{x}$ (iii) $\frac{4x^3 + 3x^2 + 4}{2x}$

SAQ 10.2

Find the derivatives of $y = (3x - 5)(2x^2 + 3x + 2)$

References:

Tutuh Adigun M.R: Further Maths Project.

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Study Session 11: Integral Calculus

Introduction

In differential calculus you learnt how to find the differential coefficient of y i.e. $\frac{dy}{dx}$ when given y as a function of x. In integral calculus, you will understand how to find back the function y when the differential coefficient $\frac{dy}{dx}$ is given, the reverse process of finding y when $\frac{dy}{dx}$ is given, and what it is called.

Learning Outcomes for Study Session 11

At the end of this study, you should be able to:

11.1 Explain integration

11.2 Discuss the application definite integral

11.1 Integration

Now suppose you are given that $\frac{dy}{dx} = 2x$ and you are required to find y, immediately you know that the solution is $y = x^2$. But a closer look reveals that there are in fact many other solutions such as $y = x^2 + 2$, $y = x^2 - 4$ $y = x^2 + 10$ e.t.c. Indeed, any general solution of the form $y = x^2 + c$ (where c is a constant) will serve as a solution. We call $x^2 + c$ the integral of $2x$. With a respect to x written $\int 2xdx = x^2 + c$.

The symbol for integration $\int$ is called integral. So in general, if $\frac{dy}{dx} = gcx)$ then $\int g(x)dx = y + c$.

Where ca constant is called the arbitrary constant of integration. It is arbitrary because it can take any numerical value which can be determined when additional information is given. We thus see that whenever we integrate any function, we have integrated (i.e. $2x + c$) is called the integral. The symbol dx attached beside integrand shows that the integration is "with respect to$x$"

Some standard Integrals

The following is a list of some integrals written beside their corresponding differential coefficients. You will notice that they are in reverse order since integration is the reverse of differentiation

1. $\frac{d}{dx}(ax^n) = anx^{n-1} \quad \therefore \int ax^n dx = \frac{ax^{n+1}+c}{n+1} \quad (provided\ n \neq -1$

2. $\frac{d}{dx}(sinx) = cosx \quad \therefore \int cos\ xdx = sinx + c$

3. $\frac{d}{dx}(\sin ax) = acosx \quad \therefore \int cosaxdx = \frac{sinax+c}{a}$

4. $\frac{d}{dx}(cosx) = -sinx \quad \therefore \int sinxdx \ = -\cos x + c$

5. $\frac{d}{dx}(cosax) = -asinax \quad \therefore \int \sin axdx = -\frac{cosax+c}{a}$

6. $\frac{d}{dx}(e^x) = e^x \quad \therefore \int e^x dx = e^x + c$

7. $\frac{d}{dx}(e^{ax}) = ae^{ax} \quad \therefore \int e^{ax}dx = \frac{e^{ax}}{a} + c$

8. $\frac{d}{dx}(a^x) = a^x\ in\ a \ \int a^x dx = \frac{a^x}{lna} + c \ \therefore$

9. $\frac{d}{dx}(lnx) = \frac{1}{x} \ \therefore \ \int \frac{1}{1+x} dx = lnx + c$

10. $\frac{d}{x}(\ln(x+a)) = \frac{1}{x+a} \quad \therefore \int \frac{1}{1+x} dx = \ln(x+a) + c$

11. $\frac{d}{dx}(tanx) = sec^2 x \quad \therefore \int sec^2 xdx = tanx + c$

12. $\frac{d}{dx}(co + x) = -cosec^2 x \ \therefore \int cosec^2 xdx = xdx = -cotx + c$

Example 1:

Find the integral of the following functions with respect to x

(i) $4x$ $\quad (ii) 3x^2 + 2x - 5$

Solution

(i) $\int 4xdx = 2x^2 + c$

i. $\int(3x^2 + 2x + 5)dx \;=\; \int(3x^2)dx + \int 2xdx - \int 5dx$

Integrate the following with respect to x

(i) $6x^4$ $\quad ii.\ \frac{5}{x} \quad iii.\ \frac{1}{(x-2)^2+25} \quad iv.$ $\sin 5x$

Solution

Using the standard integral, you have;

$$\int 6x^4dx = 6\int x^4dx = \frac{6}{4+1}x^{4+1} + c = \frac{6x^5}{5} + c$$

i. $\int \frac{5}{x}dx = 5\int \frac{1}{x}dx = 5lnx + c$

ii. $\int \frac{1}{(x-2)^2+25}dx = \int \frac{1}{(x-2)^2+5^2}dx = \frac{1}{5}tan^{-1}\left(\frac{x-2}{5}\right) + c$

iii. $\int sin5xdx = \frac{1}{5}\cos 5x + c$

iv. $\int \frac{7}{2}x^{\frac{3}{2}}\,dx = \frac{7}{2}\int x^{\frac{3}{2}}dx \quad = \frac{7}{2}\left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right) + c = \frac{7}{2}\left(\frac{x^{\frac{3}{2}}}{\frac{5}{2}}\right) + c$

$= \quad \frac{7}{2}\left(\frac{5}{2}x^{\frac{5}{2}}\right) + c$

$= \quad \frac{7}{2}x^{\frac{5}{2}} + c$

Example2

Given that $\frac{dy}{dx} = 3x^2 + 2x + 6, find\ y$

Solution

$$\frac{dy}{dx} = 3x^2 + 2x + 6$$

$dy = 3x^2 + 2x + 6)dx$

$\therefore y = \int(3x^2 + 2x + 6)dx$

$\int 3x^2dx + \int 2xdx + \int 6dx$

$= 3\left(\frac{x^2+1}{2+1}\right) + 2\left(\frac{x^{1+1}}{1+1}\right) + 6(\frac{x^{0+1}}{0+1} + c$

$= \quad \frac{3x^3}{3} + \frac{2x^2}{2} + 6x + c$

$= x^3 + x^2 + 6x + c$

Example 3: find $\left(\frac{3x^4+2x^3+5x^2}{2x^2}\right) dx$

Solution:

$$\int \frac{3x^4 + 2x^3 + 5x^2}{2x^2} dx \quad = \int \frac{3x^4}{2x^x} + \int \frac{2x^3}{2x^2} + \int \frac{5}{2} dx + c$$

$$= \int \frac{3}{2}x^2 + \int xdx + \int \frac{5}{2} dx + c$$

$$= \frac{3}{2}\left(\frac{x^3}{3}\right) + \frac{x^2}{2} + \frac{5}{2}x + c \quad = \frac{x^3}{2} + \frac{x^2}{2} + \frac{5}{2}x + c$$

$$\frac{1}{2}x^3 + \frac{1}{2}x^2 + \frac{5}{2}x + c$$

Method 2:

Substitution method

Integral of form $\int (bx + k)^n dx.\ where\ b\ and\ c\ are\ constants\ and\ bx + k\ is\ linear\ are$re always solved by substitution method

Example: Find $\int (bx + k)^n dx$

Solution: Put $u = bx + k$

$\frac{dy}{dx} = b, \therefore \quad du = bdx \quad \ggg dx = \frac{du}{b}$

Using the above transformation,

$$\therefore \int (bx + k)^n dx = \int u^n \frac{du}{b} = \frac{1}{b}\int u^n du \qquad = \frac{1}{b}\left(\frac{u^{n+1}}{n+1}\right) + c$$

$$= \frac{1}{b(n+1)}(bx + k)^{n+1} + c$$

$$= \frac{(bx+k)^{n+1}}{b(n+1)} + c$$

Example: Integrate the following

i. $(3x+5)^8$ $\quad ii. \quad \frac{1}{(3x-5)^7}$

Solution:

i. Let $y = \int (3x+5)^8 dx$

Put $U = 3x+5, \ \frac{du}{dx} = 3$

$$\therefore dx = \frac{du}{3}$$

$$hence, y = \int U^8 \frac{du}{3} = \frac{1}{3}\int U^8 du$$

$$= \frac{1}{3}\frac{U^9}{9} + c$$

$$= \frac{U^9}{27} + c$$

$$= \frac{1}{27}U^9 + c \quad = \frac{1}{27}(3x+5)^9 + c$$

$$y = \int \frac{1}{(3x-5)^7} dx \qquad = \int (3x-5)^{-7} dx$$

Put $U = 3x - 5 \ \therefore du = 3dx \ \ i.e. dx = \frac{du}{3}$

$$\gg y = \int U^{-7}\left(\frac{du}{3}\right) = \frac{1}{3}\int U^{-7} du \ = \frac{1}{3}\left(\frac{U^{-6}}{-6}\right) + c$$

$$= -\frac{1}{18}U^{-6} + c \quad = -\frac{1}{18}(3x-5)^{-6} + c$$

$$= \frac{-1}{18(3x-5)^6} + c$$

Definite Integral

An integral $\int_c^d (x)dx$ is called a definite integral with $'c'$ as the lower limit and ‘d’ as the upper limit. This represents geometrically the areas bounded by the curve $y = f(x)$, the line $x = c, \ x = d \ and \ the \ axis.$

Example: $\int_1^2 2x^2 dx = 2\left\{\frac{x^6}{6}\right\} = \frac{x^6}{3} \ = \frac{1}{3}\{2^6 - 1^6\} = \frac{1}{3}(64-1)\frac{63}{3} = 21$

Example 5:

Evaluate the following

i. $\int_1^3 \frac{x^3-x^2}{x^2}$ $\quad$ *ii.* $\int_0^{\frac{x}{4}} cos2xdx$

Solution: $\int_1^3 \frac{x^3-x^2}{x^2}dx \quad = \int_1^3 \left(\frac{x^3}{x^2} - \frac{x^2}{x^2}\right)dx \quad = \int_1^3 (x-1)dx$

$$= \frac{1}{2}x^2 - x|^3{}_1 = \frac{1}{2}x^2|^3{}_1 - \ x^2|^3{}_1$$

$$= \left\{\frac{1}{2}(3^1 - 1^2) - (3-1)\right\}$$

$$= \left\{\frac{1}{2}(8) - 2\right\}$$

$$= 4 - 2$$

$$= 2$$

ii. $\int_0^{\frac{x}{4}} 3cos2xdx \quad = \frac{3}{2}\Big| sin2x\, {}^{\pi}/_{4}$ $\circ$

$$= \frac{3}{2}\{sin2\left(\frac{\pi}{4}\right) - sin2(0)$$

$$= \frac{3}{2}\{\sin\ \frac{\pi}{2} - sin0$$

$$= \frac{3}{2}(1-0)$$

$$= \frac{3}{2} = 1.5$$

In-Text Question

The integral of $(x-1)(2x-1)$ is $\frac{1}{3}x^3 - \frac{3}{2}x^2 + x + c$ True or False

In-Text Answer

True

11.2 Application of definite integral

Equation curve

The gradient of a curve at any point (x, y) is given by $\frac{dy}{dx} = 3x + 1$. If the point (2,1) lies on the curve, find the equation of the curve.

Solution

Given that $\frac{dy}{dx} = 3x + 1$

$$\therefore y \int dy = \int (3x+1)dx$$

$$= \frac{3}{2}x^2 + x + c$$

$$\therefore y = \frac{3}{2}x^2 + x + c \ldots\ldots\ldots\ldots..(1)$$

At the point (2,1), equation (1) becomes $x = 2, y = 1$

$y = \frac{3}{2}(2)^2 + 2 + c$

$$1 = 6 + 2 + c$$

$$c = -7$$

Subt. -7 for c in (1), we have

$y = \frac{3}{2}x^2 + x - 7$ which is the equation of the curve.

Example 6:

The gradient of a curve at any point is given by $3(x^2 - 2)$. If the curve passes through the origin, find the equation of the curve.

Solution: Gradient of the curve is $\frac{dy}{dx} = 3(x^2 - 2)$

$$\therefore y = \int dy = \int 3(x^2 - 2)dx$$

$$= \int (3x^2 - 2)dx$$

$$= x^3 - 6x + c$$

$$\therefore y = x^3 - 6x + c \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

Since the curve passes through the origin, (0,0) equation (1) becomes $0 = 0{,}0 + c$

$\therefore c = 0$. Substitute 0, for c in equation (1)

$$y = x^3 + 6x. hence, the\ equation\ of\ the\ curve\ is\ y = x^3 + 6x$$

In-Text Question

Evaluate $\int_1^2 \frac{x^3 - x^2}{x^x}$

A. 1
B. 2
C. 2
D. 4

In-Text Answer

A. 1

Summary for Study Session 11

In this study, you have learnt:

1. The basic principles of integral calculus, such as standard results in integration,

2. The methods of integration, trigonometric functions of integration, definite integration ad its applications

Self-Assessment Questions (SAQs) for Study Session 11

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 11.1

Integrate $\int cos^5 x \sin xdx$

SAQ 11.2

Find the integral of the following functions with respect to x

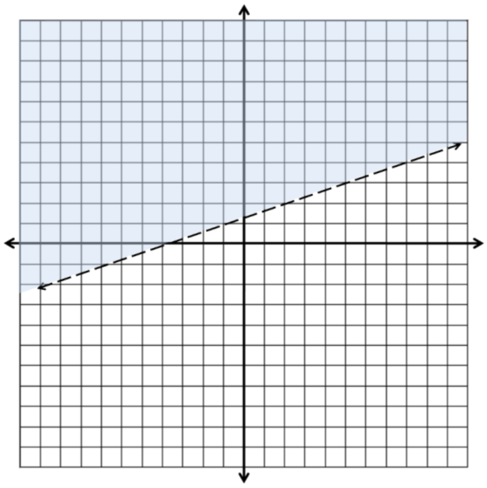
$\int 8xdx$ ii. $\int 4x^2 + 2x + 5$ iii.$4cosx - 4x^2$

References:

Maria N. David. Osuagwu: New Mathematics for Senior Secondary Schools. New Edition.

Odeyinka J.A:Basic Mathematics and Statistics for Undergraduates and prospective undergraduates.

Linear Inequalities



Study Session 12: Linear Inequalities

Introduction

One of the most interesting and important application of linear inequalities is in the area of linear programming. This usually occurs in industry, commerce, government, institutional management and other real life situations where you can decide on the most effective way of using available resources to solve a particular problem. This study will introduce you to linear inequalities in one and two variables and simultaneous linear inequalities.

Learning Outcomes for Study Session 12

At the end of this study, you should be able to:

12.1 Solve problems in linear inequalities in one and two variables.

12.2 Explain simultaneous linear inequalities.

12.1 Linear inequalities in one and two variables

Having learnt about linear inequalities in one variable whose graphical representation are seen as intervals on the real number line, you can now learn about linear inequalities involving two variables.

Some examples of such inequalities are $2x - y < 3$, $x + y \geq -1$, $2x - 3y > 4$, etc. The solution sets of such inequalities can be found to be ordered pairs (x, y) of real numbers whose graphical representations are found to be region on the x-y plane.

Now suppose you want to solve the inequality $3x - y < -2$.

You will start by drawing the straight line graph $3x - y = -2$. This line will partition the x-y plane into two regions (i.e. the region above the line and the one below the line) thereby forming three disjoint sets of points, namely; (1) those above the line *A* (2) those on the line *B* and (3) those below the line *C* (see Fig 12.1)

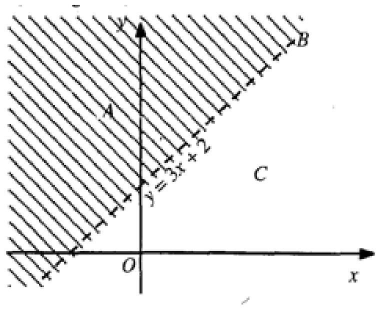


Fig. 12.1: Inequalities in graph

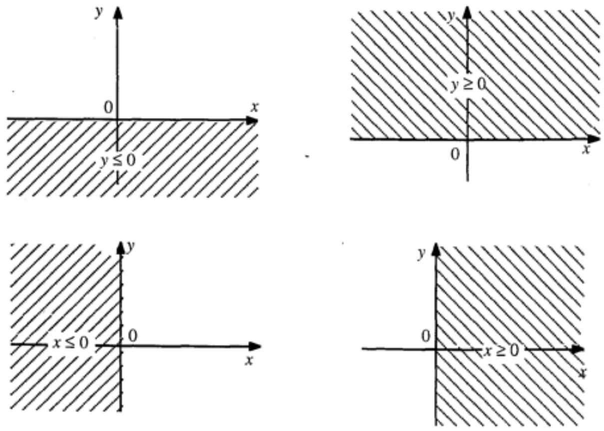
The solution set of the inequality $3x — y <—2$ is then the region either above or below the line $3x — y = —2$ that satisfies the inequality. This region is determined by picking a point in either of the regions and testing to see if the point is a solution of the inequality. If it is, then the portion of the plane containing the point is the solution set. If not then the other portion is the solution set. The difference in the solution sets of linear inequality in one variable and in two variables is that while the solution to the first is a set of numbers representable on a number line, the solution to the second is a set of ordered pairs or co-ordinates which can be represented on a plane, e.g from the graph in Fig 12.1 the origin (0, 0) is a point below the line.

Substituting (0, 0) in $3x — y <—2$, we have that 0 <—2 which is not true. The point (0, 0) is not, therefore, a solution of the inequality $3x — y < —2$. Hence the region A above the line $y = 3x + 2$ is the solution set of the inequality.

This solution (region) is usually shaded, as shown in Fig 12.1.

Note that the line $3x — y = —2$ is represented by a broken line which shows that the set of point on that line is not included in the solution set of the inequality. This can be checked by testing any point, say (0, 2), on that line in the inequality $3x — y <—2$ thus 0 — 2 < —2, ie —2 <—2 which is false.

It is also good to note the following different regions in the *x*-*y* plane. The shaded regions are described by their corresponding inequalities.



Example 1

Show graphically the region represented by the following inequalities

a. x+y < 4 and x+y ≥ 4

b. x—2y+ 1 ≤ 0 and x—2y+ 1 > 0.

Solution

x+y< 4 and x+y ≥ 4 Consider the line x + y = 4.

We then draw the graph of this line x + y = 4 shown in Fig 12.2 below by finding the x and y intercepts of the line as follows:

Substituting x = 0 in the equation x + y = 4, we have y = 4 and substituting y = 0 we have x = 4 ∴ the intercepts are (0, 4) and (4, 0) which are plotted to get the line drawn in Fig 12.2.

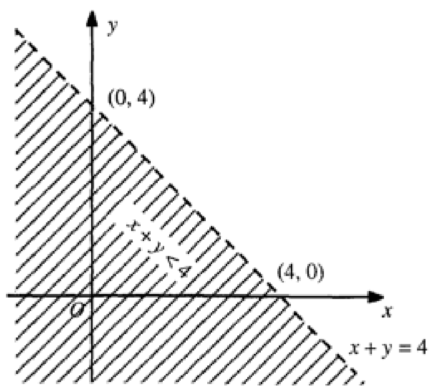


Fig 12.2

Note that the line is broken because of the strict inequality <showing that the points on the line are not included in the solution set of the inequality *x* + *v*< 4. Substituting the origin (0, 0) into *x* + *y*< 4, we have that 0 < 4 which is a true statement. The region containing the point (0, 0) is the solution set of the inequality *x* + *y*<4. This region is shaded as shown in Fig 12.2 above.

∴ The region on the other side of the line is represented by the inequality $x + y \geq 4$. The points on the line also satisfy $x + y \geq 4$.

$X—2y+1 \leq 0$ and $x—2y+1 > 0$. Consider the line $x — 2y + 1 = 0$

The values $x = 0$ and $y = 0$ are substituted into the equation $x — 2y + 1 = 0$ to get the x And$y$ intercepts (—1, 0) and $\left(0, \frac{1}{2}\right)$on the x and y axes respectively.

Then we have the graph in fig. 12.3 representing the line $x — 2y + 1 = 0$.

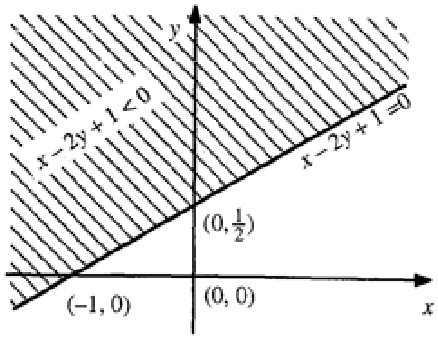


Fig 12.3: graph representing the line x — 2y + 1 = 0.

Substituting the origin (0, 0) into the given inequality $x — 2y + 1 \leq 0$ we have that $1 \leq 0$ which is a false statement and so the unshaded region is not the solution set. The region on the same side of the origin will not satisfy the inequality $x — 2y + 1 \leq 0$. Hence the shaded region on the other side of the line and all the points on the line will satisfy the inequality.

Notice that the line $x — 2y + 1 \leq 0$ is full (or continuous) i.e. not broken because of the inequality sign $\leq$ showing that the points on the line are included in the solution set.

In-Text Question

The solution sets of such inequalities can be found to be ordered pairs (x, y) of real numbers whose graphical representations are found to be region on the x-y plane. True or False

inequality sign $\leq$ showing that the points on the line are included in the solution set.

In-Text Question

True

12.2 Simultaneous Linear Inequalities

The solution set of a system of linear inequalities in two variables can be found by graphing the inequalities and shading the region of their intersection (if any) of the graphs of the solution sets of each inequality. It is illustrated with the following examples.

Example 1

Find the solution set of

$$x+y<2$$
$$3x-y \geq 6$$

Solution

We draw the graph of each inequality and shade as shown above the region that satisfy each inequality.

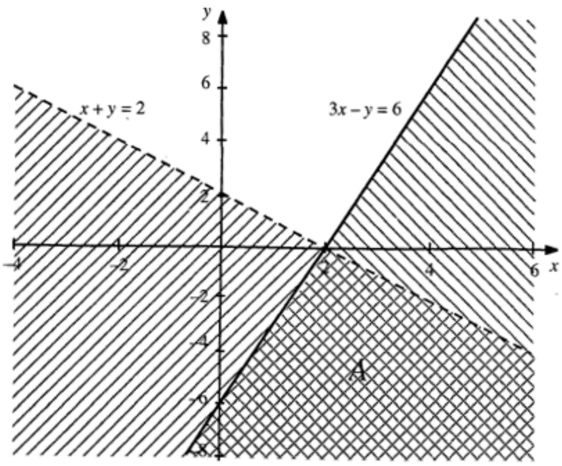


Fig. 12.4

The solution set of the two simultaneous inequalities is the intersection of the two shaded portions which is represented by the double shaded area A as shown in Fig. 12.4.

Example 2

Show the region which satisfies simultaneously the inequalities

$2x + 3y \leq 8$

$x - 2y \geq -3$

$x \geq 0$

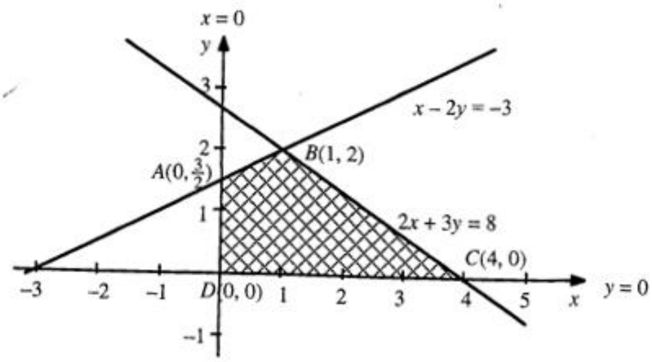
$y \geq 0.$

Solution

The region satisfying the inequalities simultaneously is the shaded region enclosed by the

Polygon ABCD as shown in Fig. 12.5.

Fig. 12.5



It can be verified that the vertices *A*, *B*, *C* and *D* are respectively $\left(0,\frac{3}{2}\right)$,(1, 2), (4, 0) and (0, 0) as follows:

$A\left(0,\frac{3}{2}\right)$ is obtained by solving *x — 2y = —3* and *x* = 0 simultaneously.

B (l, 2) is obtained by solving *x — 2y = —3* and *2x + 8y = 8* simultaneously.

C (4, 0) is obtained by solving simultaneously *2x + 8y = 8* and *y* = 0 and *D*(0, 0) is the origin. Note that any point in the shaded region satisfies all the inequalities.

Maximum and Minimum Values

Sometimes we may be required to find the maximum or minimum of a given linear function *f(x, y) = ax + by* of two variables which is defined under a given set of inequalities (*a* and *b* being constants).

It can be shown that this maximum or minimum value of the linear function occurs at the corner points of the region which satisfies the given set of the inequalities.

Example

a. Show the region satisfying the set of inequalities

$y+x \leq 3;\ y+x \geq l;\ y—x \leq 1;\ x \geq 0;\ y \geq 0.$

b. Find the maximum and minimum value of a function *f(x, y) = 3x + 4y* defined under the set of the inequalities.

Solution

The required region satisfying the inequalities simultaneously is the shaded region enclosed by the polygon *ABCD* in Fig 12.6.

At *A* (l, 0), the value of the function $f(x, y) = 3x + 4y$ is given by $f(1, 0) = 3(1) + 4(0) = 3$.

At *B* (3, 0) the value is $f(3, 0) = 3(3) + 4(0) = 9$

At *C* (l, 2), $f(1, 2) = 3(1) + 4(2) = 11$

At D (0, 1), $f(0, 1) = 3(0) + 4(1) = 4$

At any other point, say (1, 1), (2, 1), (2, 0), etc. on the boundary of or within the shaded region, the value of the function will be between 3 the smallest number) and 11 (the largest number) above.

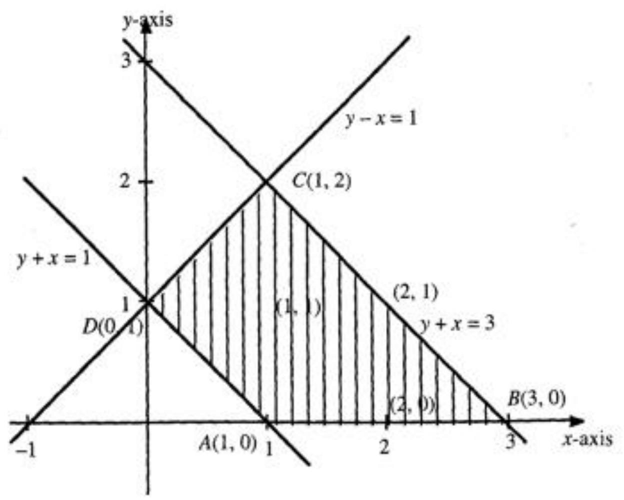


Fig. 12.6

This can be verified by substituting these values in the given function as follows:

$f(1, 1) = 3(1) + 4(1) = 7$

$f(2, 1) = 3(2) + 4(1) = 10$

$f(2, 0) = 3(2) + 4(0) = 6$, etc.

Hence the maximum value of *f* is 11 which occurs at *C* (l, 2) while the minimum value is 3which occurs at *A*(1,0).

Summary for Study Session 12

In study session 12, you have learnt:

1. The concepts of linear inequalities in one or two variable
2. The concepts of simultaneous linear inequalities and linear programming in relation to management science, industry and socio-sciences.

Self-Assessment Questions (SAQs) for Study Session 12

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 12.1

Show graphically the region represented by the following inequalities.

1. $2x + y + 1 \geq 0$
2. $X - 8 \leq 2y + 3x$

SAQ 12.2

A tailor makes two types of dresses X and Y by making use of two type of material U and V. The quantities of material U and V used for each type of dress X and Y available in metre squares and profit in naira on each dress are shown in the following table.

	U	V	Profit (#)
X	3	2	2
Y	4	5	3
Quality of material available in m^2	18	19	

References

Maria N. David. Osuagwu: New Mathematics for Senior Secondary Schools. New Edition.

Odeyinka J.A:Basic Mathematics and Statistics for Undergraduates and prospective undergraduates.

Study Session 13: Linear Programming

Introduction

One of the most interesting and important application of linear inequalities is in the area of linear programming. This usually occurs in industry, commerce, government, institutional management and other real life situations where you can decide on the most effective way of using available resources to solve a particular problem. In this study, you will learn about the use of linear programming to solve management problems and maximizing profit.

Learning Outcomes for Study Session

At the end of the study, you should be able to:

13.1 Use linear programming to solve some management problems

13. 2 Use linear programming to maximize profit and minimize cost.

13.1 linear programming in Management

One of the most interesting and important applications of linear inequalities is in the area of linear programming. This usually occurs in industry, commerce, government establishment and other real life situations where you want to decide on the most effective or best way of using available resources to solve a particular problem (for example maximizing the profit or minimizing the cost in a factory) taking into account the factors or conditions that will affect the decision.

Linear programming therefore attempts to maximize or minimize a given linear function called the objective function which is defined under a given set of inequalities called the constraints.

You have seen that this maximum or minimum value of the objective function occurs at the corner points or vertices of the polygon formed by the region which satisfies the constraints.

Example 1

A tailor makes two types of dresses *X* and *Y* by making use of two types of materials *U* and *V*.

The quantities of materials *U* and *V* used for each type of dress *X* and *Y* available in m^2 and the profit in ₦ on each dress are shown in the following table.

	U	V	Profit(₦)
X	3	2	2
Y	4	5	3
Quantity of material available (m^2)	18	19	

a. Assuming that the tailor makes *x* units of *X* and *y* units of *y*, write down the inequalities containing *x* and y.

b. Find the number of each type of dress the tailor should make in order to maximize his profit.

Solution

The quantity of material *U* used in making 3*x* units of dress *X* and 4*y* units of dress *Y* should not exceed 18 m^2

i.e. $3x + 4y \leq 18$ (1)

Similarly for material *V*

$2x + 5y \leq 19$ (2)

Also $x \geq 0$ (3)

and $y \geq 0$ (4)

If *P* is the profit then $P = 2x + 3y$ (from the last column of the above table)

The graph of the above inequalities (1) — (4) is shown in Fig. 12.1. The solution set of the inequalities is the shaded area.

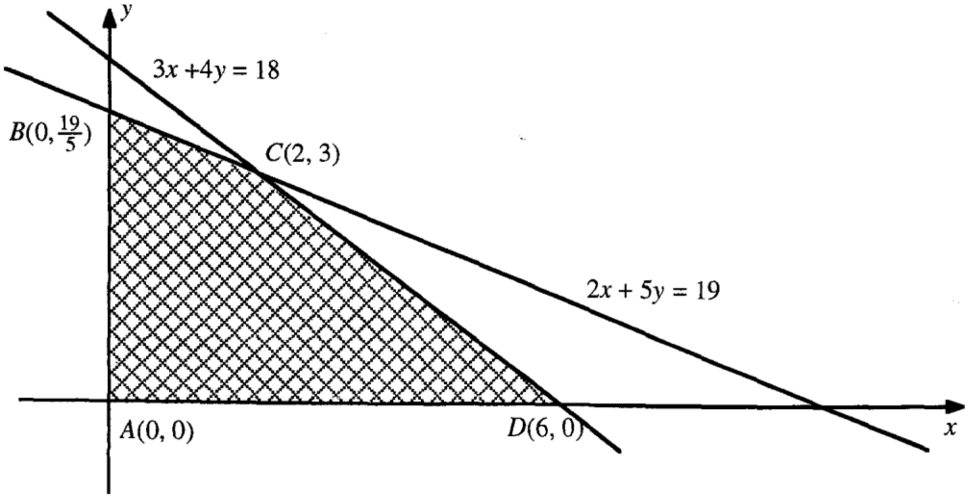


Fig. 12.1

The vertices of the polygon *ABCD* formed by the shaded region and the profit at each vertex is tabulated below.

Vertices	Profit (₦) $P = 2x + 3y$
$A(0, 0)$	$P = 2(0) + 3(0) = 0$
$B\left(0, \frac{19}{5}\right)$	$P = 2(0) + 3\left(\frac{19}{5}\right) = 11.4$
$C(2, 3)$	$P = 2(2) + 3(3) = 13$
$D(6, 0)$	$P = 2(6) + 3(0) = 12$

From the table, the largest value is 13 which corresponds to the point *C* (2, 3). Hence the tailor should make 2 dresses of type *X* and 3 dresses of type *Y*in order to make a maximum profit of₦13.00.

In-Text Question

Linear programming tends to maximize or minimize a given linear function. True or False

In-Text Answer

True

13.2 Linear programming profit maximization and cost minimization

Example 2

A soap producing factory produces a special type of detergent that must contain the following three ingredients X, Y and Z such that

X is not less than 10%, Y is not less than 30%, Z is not more than 25%.

The factory uses two basic materials P and Q which provide the % ingredient as shown in the table below.

	X	*Y*	*Z*
P	0.2	0.2	0.2
Q	0.05	0.5	0.3

If *P* costs ₦l.50 per litre and *Q* costs ₦2.25 per litre, what quantity of these two should be used to produce 100 litres of the special detergent so as to minimize cost?

Solution

Let *x* be the number of litre of *P* used and *y* be the number of litre of *Q* used.

From the information given, we have the following inequalities

$X+y \leq 100$ (i)

$0.2x+0.05y \geq .10$ (ii)

$0.2x+0.5y \geq 30$ (iii)

$0.2.\ x+0.3y \leq 25$ (IV)

The cost function K (₦) is given by

$K 1.50x+2.25y$ (v)

So the graphs of these inequalities (i)—(iv) are shown in Fig. 12.2. The shaded polygon is the region which satisfied the inequalities and where the cost function is defined.

The coordinates of the four vertices and the value of the cost function at each vertex are tabulated as follows:

Vertices	Cost function $K = 1.50x + 2.25y$
$A\left(\frac{350}{9}, \frac{400}{9}\right)$	$K = 1.50\left(\frac{350}{9}\right)2.25\left(\frac{400}{9}\right)$ = ₦158.33
$B\left(\frac{200}{9}, \frac{100}{9}\right)$	$K = 1.50\left(\frac{200}{9}\right) + 2.25\left(\frac{100}{9}\right)$ = ₦175.00
$C(50, 50)$	$K = 1.50(50) + 2.25(50)$ = ₦187.00
$D(35, 60)$	$K = 1.50(35) + 2.25(60)$ = ₦187.00

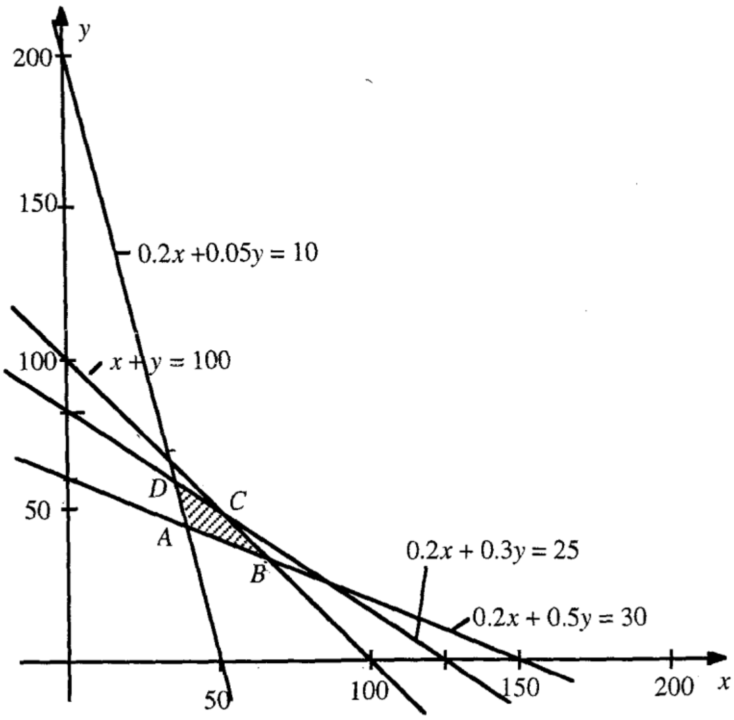


Fig. 12.2

Hence from the table, the vertex with the least amount is, $A\left(\frac{350}{9}, \frac{400}{9}\right)$

∴ $\frac{350}{9} \cong 39$ litres of P and $\frac{400}{9} \cong 45$ litres of Q shall be used for minimum cost of production of the detergent.

Summary

In study session 13, you have learnt the basic concepts of linear inequalities in two variables, simultaneous linear inequalities and linear programming in relation to management science, industry and socio-sciences. Some examples were worked out to guide you in understanding the concepts.

Self-Assessment Questions (SAQs) for Study Session 13

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 13.1

Show graphically the region represented by the following inequalities.

1. $2x + y + 1 \geq 0$
2. $X - 8 \leq 2y + 3x$

SAQ 13.2

A tailor makes two types of dresses X and Y by making use of two types of material U and V. The quantities of material U and V used for each type of dress X and Y available in metre squares and profit in naira on each dress are shown in the following table.

	U	V	Profit (#)
X	3	2	2
Y	4	5	3
Quality of material available in m^2	18	19	

References

Maria N. David. Osuagwu: New Mathematics for Senior Secondary Schools. New Edition.

Odeyinka J.A:Basic Mathematics and Statistics for Undergraduates and prospective undergraduates.

Study Session 14: Logical Reasoning

Introduction

In everyday life, Language is used in many different ways. You can ask questions like ('Where is the house?'); answers like ('It is over there.') can be made. You can make requests ('Please pass me that pen.') or give commands ('be quiet.') etc. If an expression has a verb, it is a sentence. All of the previous examples are sentences. But we often communicate without verbs: 'What a lovely picture'; 'a tall student.'
Poetry in motion.'; '600 mm.' None of these have a verb; therefore, they are not sentences. Individuals use language to develop a line of argument, or reasoning that convinces other people. It is often called logical reasoning. In logical reasoning, a statement is spoken. It is a written sentence that gives information about something. The statement may be true or false.

Learning Outcomes for Study Session 14

At the end of this study, you should be able to:
14.1 Explain the term 'Negation'
14.2 Explain logical reasoning by using a '/.Venn diagram

14.1 Negation

Example 1

Decide whether or not each of the following is a sentence and/or a statement. Also say whether it is true or false.
a) Lagos is a city in Nigeria.
b) Who is she?
c) They are lovely people.
d) 3 x 2 = 5
e) Submit your assignment.
f) God willing.
g) I saw them last week.
h) A square is a rectangle.
Answers are given in Table 14.1, where Y stands for Yes and N stands for No.
In Table 14.1, some statements were either true or false. You call those closed statements. Closed statements are about well-defined things. Other statements, such as c) and g), are not about well- defined situations. You don't know if they are true or

false. You can call those open statements. In mathematics you can use capital letters to show statements.

Expression	A sentence?	A statement?	True or false?
a)	Y	Y	True
b)	Y	N	Not applicable (a question)
c)	Y	Y	Impossible to say
d)	Y	Y	False
e)	Y	N	Not applicable (a command)
f)	N	N	Not applicable (a saying)
g)	Y	Y	Impossible to say
h)	Y	Y	False (Although a square is a member of the set of rectangles, it is not the same as a rectangle)

Example 2

For each of the following, say whether it is a closed or an open statement or neither, also whether it is true or false.

a: 3 is an odd number.

b. x is an even number.

c: What is your name?

d: The Niger is a famous European river.

e: Salt and pepper.

f: Follow me.

g: Chemistry is a science subject.

h. Chemistry is an easy subject.

Expression	A statement?			True or false?
	Closed?	Open?	Neither?	
A	✓			True
B		✓		Impossible to say
C			✓	Not applicable
D	✓			False
E			✓	Not applicable
F			✓	Not applicable
G	✓			True
H		✓		Impossible to say (The word *easy* in the statement is not well defined; also, this statement only has meaning with reference to particular people.)

In a logical context, questions, commands, sayings and exclamations are not considered to be statements. This accounts for the 'not applicable' answers in Example 2.

Negation

Think of negation as meaning not. Given a statement 5, we can write and say the negation of S in a number of ways:

~S 'not S'\

S 'bar S'

S' 'S prime'

If S is true, then ~S is false and if S is false, then ~S is true. For example, if the statement 'I am fifteen years old.' is true, then its negation, 'I am not fifteen years old.' will be false.

To form the negation of a statement, insert the word 'not' at an appropriate place or write 'It is false that...' before the statement.

Example 3

Write the negation of the following statements.

a: Bose is brilliant.

b: Bauchi has an international airport.

c: John is not a sailor

d: x=5.

~A: Bose is not brilliant. Orlt is false that Bose is brilliant.

~B: Bauchi does not have an international airport.

~C: John is a sailor. Orlt is not true that John is not a sailor.

~D: $x \neq 5$.

Notice in part C of Example 3 how two negatives result in a positive outcome.

Example 4

Write the negations of the following statements, without using the word 'not'.

a This word ends with a consonant.

b He ran at a constant speed.

c He was present at the party.

d This word ends with a vowel.

e He ran at a variable speed.

f He was absent from the party.

Very great care should be taken when using the opposite property to form a negation of a statement. This method does not always give the correct negation, especially if the language is vague.

Example 5

Consider whether B is the negation of A in the following. If not, give reasons.

a   A: James scored the lowest mark in the test.

B: James scored the highest mark in the test.

b   A: This is an easy exercise.

B: This is a difficult exercise.

c   A: The student was inside the room.

B: The student was outside the room.

a.   B is not the negation of A, because James could have scored any mark greater than the lowest.

b.   B is not the negation of A, because some exercises may be neither difficult nor easy.

c.   B is not the negation of A, because the student might have been standing at the doorway.

If 'not' had been used in the above statements, the negations would be correct.

a. A: James scored the lowest mark in the test.

~A: James did not score the lowest mark in the test.

b A: This is an easy exercise.

~A: This is not an easy exercise.

c) A: The boy was inside the room.

~A: The boy was not inside the room.

In-Text Question

To form the negation of a statement, insert the word 'not' at an appropriate place or write.

This statement is True or False

In-Text Answer

True

14.2 Using Venn Diagrams in logical reasoning

Venn diagrams can be used to solve problems in logical reasoning. Consider the statements.

A: Students in SS1 who offer physics.

~A: Students in SS1 who do not offer physics.

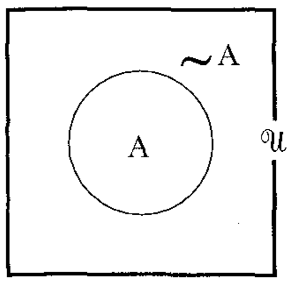


Fig. 14.1

Fig. 14.1 is the Venn diagram for the above statements where

U= {students in SS1}

A = {students who offer physics}

~A= {students who do not offer physics

Note that A and A' both mean the complement of set A.

Implication

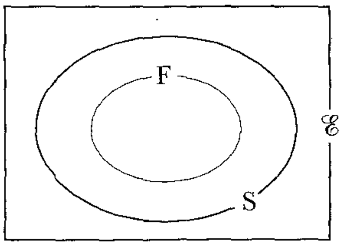


Fig. 14.2

Fig. 14.2 is a Venn diagram where

E= {students in City College}

S = {students who take part in sports)

F = {students who play football)

Notice that F is a subset of S, i.e. F CS. This means that if a student plays football, then the student also takes part in sports. We can write this as:

S: If a student plays football, then the student takes part in sports.
Statement S is really a combination of the following statements.
X: A student at City College plays football.
Y: A student at City College takes part in sports.

S states that if X is true, then Ywill also be true. A compound statement like this is called an **implication**. Symbolically, we write the implication S as X = which is read as 'X implies Y'. It can also be interpreted as

If X then Y or
Y is necessary for X or
Xis sufficient for Y or
X only If Y or
Follows from X or
Y if X

In the implication S: X $\Rightarrow$ Y, the sub statement X is called the antecedent, while the sub statement Y is called the consequent of X $\Rightarrow$ Y. The arrow shows that Y follows from X. X $\Rightarrow$ Yis not the same as Y $\Rightarrow$ X. The statement X $\Rightarrow$ Yis sometimes called a conditional statement.

Example 6

Given the Venn diagram in Fig. 14.3, write an implication about x, a student who is member of the Jet Club. Also write the implication using symbols.

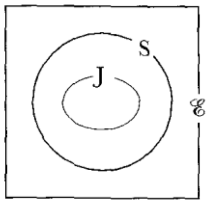


Fig. 14.3

E= students in Lantana School

S = science students in Lantana School

J = students in the Jet Club in Lantana School

or

P: If xis in the Jet Club, then he or she is a science student.

P: xis in the jet Club $\Rightarrow$ xis a science student.

In symbols:

Since CS, $x \in J \Rightarrow x \in S$

or, simply, P: $J \Rightarrow S$

Example 7

Given Fig. 14.4, use words and symbols to write three implications about a teacher t that relate to the three regions of the Venn diagram.

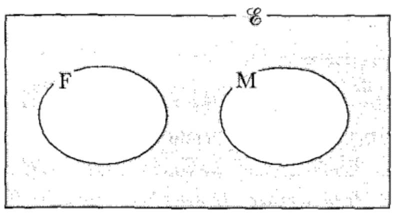


Fig. 14.4

E= {teachers at City College}

M = {teachers of mathematics)

F = {teachers of French)

The possibilities for t depend on where t appears in the Venn diagram.

1st case

If *t* is a teacher of French, then t is not a teacher of mathematics.

or

t is a French teacher $\Rightarrow$ t is not a mathematics teacher

In symbols:

Since F ∩ M=Ø, then t$\in$F $\Rightarrow$ t $\notin$ M

2nd case:

If t is a teacher of mathematics, then t is not a teacher of French.

Or

is a mathematics teacher = t is not a French teacher

In symbols:

Since F ∩ M=Ø, then t$\in$M $\Rightarrow$ t $\notin$ F

3rd case:

If t does not teach French or mathematics, then t $\in$ F' ∩ M', i.e. tis in the shaded region.

The other thing to notice about Fig. 9.4 is that no teacher in City College teaches both French and mathematics.

Example 8

Consider the following statements

F: A solid is a prism.

C: A solid is a cuboid.

R: A solid has a regular cross-section.

Use your knowledge of solids to write implications using the symbols P, C and R. State them in sentences.

a) R $\Rightarrow$ P: A solid with regular cross-section is a prism.

b) C $\Rightarrow$ R: A cuboid has a regular cross-section.

c) C $\Rightarrow$ P: A cuboid is a prism.

d) P $\Rightarrow$ R: A prism has a regular cross-section.

Notice in Example 8 that R $\Rightarrow$ P and P $\Rightarrow$ R, i.e. R implies P and P implies R. We can shorten this to R $\Rightarrow$ P. The symbol shows an equivalence relationship. This will be discussed later.

Equivalence

Example 9

Write down the converse of the implication S: Ade offers physics $\Rightarrow$ Ade is clever.

The converse of S is T

whereT: Ade is clever Ade offers physics.

These implications can be represented in Venn diagrams as shown in Fig. 14.8.

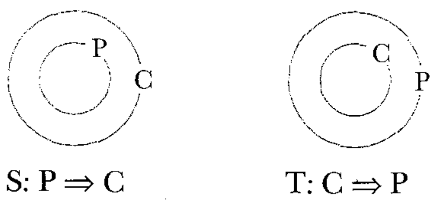


Fig. 14.8

Where P = {people who offer physics}

C = {people who are clever}

Example 10

Write down the converse of the implication S where

S: Peter fails his examination ⇒ Peter is lazy.

Show S and its converse in a Venn diagram.

The converse T: Peter is lazy Peter fails his examination. The Venn diagrams showing S and its converse are shown in Fig. 14.9.

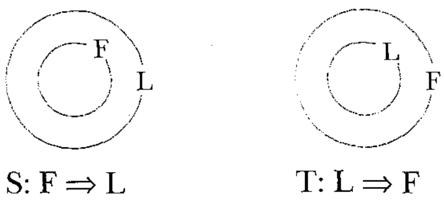


Fig. 14.9

Where

L = {lazy students)

F = {students who failed their examination)

Equivalent Statement

Consider the following implications

S_1: If a student passes the examination, then he or she has scored more than 44%.

S_2: If a student scores more than 44%, then he or she passes the examination.

If C and D are statements as follows,

C: A student passes the examination.

D: A student scores more than 44%.

then S_1: C ⇒ D

S_2: D ⇒ C

If C $\Rightarrow$ D and D $\Rightarrow$C, then we write C $\Leftrightarrow$D and say that C and D are equivalent statements.

C$\Leftrightarrow$ D is read 'If C then D and if D then C' or 'C if and only if D'.

From the implication A $\Rightarrow$B, we can deduce the equivalent implication

~ B $\Rightarrow$ ~ A

(Note the order.) Also see Example 12.

The statement 'neither A nor B' is equivalent to ~A and ~B.

Equivalent statements can be illustrated by use of Venn diagrams.

In the above, if

E= {students in Lantana school}

P = {students who pass the examination}

F = {students who score more than 44%}

then Fig. 14.10 shows P $\Rightarrow$F.

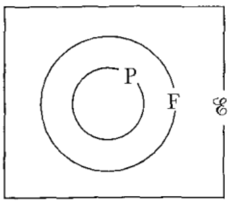


Fig. 14.10

And Fig. 14.11 shows F $\Rightarrow$P:

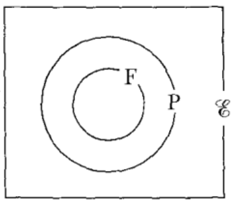


Fig. 14.11

IfS_1 and S_2 are both true, then F and P are the same set. We say that students pass the examination if and only if they score more than44%.

Note that in general a statement may be true or false. In mathematics, however, whenever we use the symbol we always mean that the implication is true.

Example 11

Investigate whether it is true that x^2 — 16 $\Rightarrow$ x = 4. If not true, correct the statement.

Ifx^2 = 16, then

X^2— 16 = 0

(x—4)(x+4) = 0 [difference of two squares]

x = +4orx=—4

The given statement, $x^2 = 16 \Leftrightarrow x = 4$, does not recognise that $x = -4$ is a possible implication from $x^2 = 16$.Therefore, $x^2 = 16 \Leftrightarrow x = 4$is nottrue.

Correct statements would be

$x^2 = 16 \Leftrightarrow x = \pm 4$

or $x^2 = 16 \Leftarrow x = 4$(i.e.$x=4 \Rightarrow x^2 = 16$)

where<= means ‘is implied by’.

Example 12

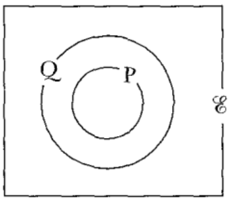


Fig. 14.12

Given Fig. 14.12 and statements A and B, where

A: x is inside P B: x is inside Q

Which of the following are true and which are false?

A => B, b) A => ~B, c) B => A, d) B => ~A, e) ~B =>A. f) ~B=>~A.

The results are set out in Table 14.3

implication	**true/false**
a) $A \Rightarrow B$	true
b) $A \Rightarrow \sim B$	false
c) $B \Rightarrow A$	false
d) $B \Rightarrow \sim A$	false
e) $\sim B \Rightarrow A$	false
f) $\sim B \Rightarrow \sim A$	true

Notice in Example 12 that, if A =>B is true, then ~B => ~A is also true.

Example 13

Write statements which are equivalent to the following.

a) If you press the switch the door will open.

b) No one who earns less than ₦40, 000 per annum pays income tax.

Equivalent

a) If you do not press the switch the door will not open.

b) Those who earn less than N40 000, do not pay income tax.

Valid Argument

An argument is a relationship between a set of propositions X_1, X_2... X_n, called premises and another proposition, S, called the conclusion.

An important application of logic is the determination of the validity (correctness) or otherwise of arguments. The validity of an argument can be determined by the following methods: i) Venn diagrams; ii) the chain rule. An argument is valid if the conclusion is true whenever all the premises X_1, X_2 X_n are true. An argument that is not valid is called a fallacy.

Example 14

Determine the validity of the argument below with premises X_1 and X_2 and conclusion S.

X_1: All doctors are intelligent.

X_2: Some Nigerians are doctors.

S: Some Nigerians are intelligent.

In the Venn diagram (Fig. 14.13):

E = {all people}

Let I = {intelligent people}

N = {Nigerians}

D = {doctors}

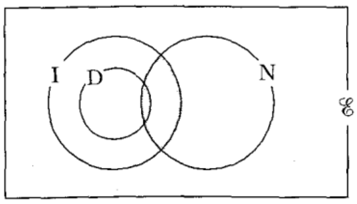


Fig. 14.13

The structure of the argument is shown in Fig. 9.13. The shaded region represents N fl I, those Nigerians who are intelligent. The conclusion that some Nigerians are intelligent therefore follows from the premises, and the argument is valid.

Example 15

In the following argument, find whether or not the conclusion necessarily follows from the premise. Draw an appropriate Venn diagram and support your answer with a reason.

London is in Nigeria.

Nigeria is in Africa.

Therefore London is in Africa.
Fig. 14.14 shows the data in a Venn diagram.

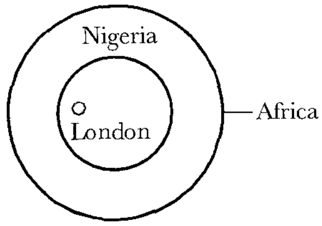


Fig. 14.14

In Fig. 9.14, the conclusion follows from the premises, L C N and N C A. The argument is therefore valid. Notice, however, that the conclusion is untrue because the first premise 'London is in Nigeria' is untrue. Therefore, we may have an argument that is valid but in which the conclusion is untrue.

The Chain Rule

The chain rule states that if X, Y and Z are statements such that X =>Yand Y=> Z, then

X => Z. A chain of statements can have as many 'links' as necessary. Example 15 is an example of the chain rule.

When using chain rule, it is essential that the implication arrows point in the same direction. Itis not of much value, for example, to have something like X => Q<= R because no useful deductions can be made from it.

Example 16

In the following argument, determine whether or not conclusion necessarily follows from the given premises.

All drivers are careful. (1st premise)

Careful people are patient. (2nd premise)

Therefore all drivers are patient. (Conclusion)

If D: people who are drivers

C: people who are careful

P: people who are patient

then D =>C (1st premise)

and C => P (2nd premise)

If D=>CandC=>P

then D =>P (chain rule) The conclusion follows from the premises.

In-Text Question

An argument is a relationship between a set of propositions X_1, X_2... X_n, called _____

a. Premises.
b. Angle
c. Chain Rule
d. Click

In-Text Answer

A. Premises.

Summary for Study Session 14

In study session 14, you have learnt the basic principles behind logic and reasoning, such as deductive method and many examples have been worked out to ensure the understanding of the topics and its application in everyday life.

Self-Assessment Questions (SAQs) for Study Session 14

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 14.1

Fig. 14.15 is a Venn diagram for the premises.

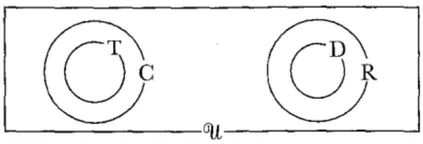


Fig. 14.15

From Fig. 14.15, the following conclusions can be deduced.

i) S_1 is true, i.e. no teacher is rich. ($T \cap R = 0$)

ii) S_2 is false, i.e. doctors are contented people is false. ($D \cap C= 0$)

iii) S_3 is true, i.e. no one can be a teacher and a doctor. ($T \cap D =0$)

SAQ 14.2

Write the negation of the following statements.

A: Bose is brilliant.

B: Bauchi has an international airport.

C: John is not a sailor

D: x=5.

~A: Bose is not brilliant. Orlt is false that Bose is brilliant.

~B: Bauchi does not have an international airport.

~C: John is a sailor. Or lt is not true that John is not a sailor.

~D:x≠5.

References

T.R.Moses: Further mathematics Schorlastic Series I

Operations on Number Bases

Number Bases ∞ →

Numbers ∞ ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	100	10	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	1000	11	10	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4		100	11	10	4	4	4	4	4	4	4	4	4	4	4	4	4
5		101	12	11	10	5	5	5	5	5	5	5	5	5	5	5	5
6		110	20	12	11	10	6	6	6	6	6	6	6	6	6	6	6
7		111	21	13	12	11	10	7	7	7	7	7	7	7	7	7	7
8		1000	22	20	13	12	11	10	8	8	8	8	8	8	8	8	8
9		1001	100	21	14	13	12	11	10	9	9	9	9	9	9	9	9
10		1010	101	22	20	14	13	12	11	10	A	A	A	A	A	A	A
11		1011	102	23	21	15	14	13	12	11	10	B	B	B	B	B	B
12		1100	110	30	22	20	15	14	13	12	11	10	C	C	C	C	C
13		1101	111	31	23	21	16	15	14	13	12	11	10	D	D	D	D
14		1110	112	32	24	22	20	16	15	14	13	12	11	10	E	E	E
15		1111	120	33	30	23	21	17	16	15	14	13	12	11	10	F	F
16			121	100	31	24	22	20	17	16	15	14	13	12	11	10	G
17			122	101	32	25	23	21	18	17	16	15	14	13	12	11	10

Study Session 15: Operations on Number Bases

Introduction

"Base" refers to a particular mathematical object that is used as a building block. It is most commonly used as related concepts of the number system whose digits are used to characterise numbers and the number system in which logarithms are defined. In this study, you will learn the history of operations on number bases, how to convert number in base to base 10 and how to solve problems in conversion of binaries to number bases.

Learning Outcomes for Study Session 15

At the end of this study, you should be able to:

15.1 Discuss the history of operations on number bases

15. 2 Convert numbers in base 3, 4, and 5 to base 10 and other higher bases

15. 3 Solve problems in conversion of binaries to other number bases

15.1 History of operations on number bases

Before the advent of Western education that came with the art of writing and symbolic expressions, different races had different systems of counting i.e. they had different number bases. The development of numeration in any particular society ultimately depends upon the economic development of that society.

The much celebrated number system synonymous with modern mathematics, was as old as life itself in Africa. The Igbos counted in quinary (base 5) and vigesimal scale (base 20). They also counted days in fours. In Sierra Leone, the two major ethnic groups, the Temne and the Mende have a "toes-and-fingers" counting system, which grouped objects in fives then in twenties.

This led to the definition of 20 by the Mendes as "one man finished". (Ohuche, 1976), that is, utilizing all toes and fingers. The Yoruba system of numeration reaches an advanced stage, for example, forty-five was expressed as "five from, ten from, and three twenties". Examples of some numbers in symbolic notations by the Yoruba are:

$$45 = (20 \times 3) - 10 - 5$$

106= (20 x 6) – 10 – 4

300=30 x (20 – 5)

525= (200 x 3) – (20 x 4) + 5. (Zaslavsky, 1979)

Since man's fingers and toes were conveniently used as a device, it was not surprising that 10 was ultimately chosen and internationally accepted more than the other bases.
The quinary scale – number system on 5 – was the first scale to be used extensively.
The vigesimal scale - the number bases on 20 – was widely used. This scale was used by African, American and Indian people.
The Hottentots were said to be the first who used the binary scale. They had only the two number words 'a' as one and "oa" as two.

They called four "Oa Oa" and five "Oa, Oa, a". Although, this is not a scale bases on two, but merely a way of expressing numbers. At first some people thought this scale is so primitive and only meant for those of low degree of mental development, or those who were incapable of the formation of any other number scale worthy to be called by the name system.

Nevertheless, with the invention of Computers, the arguments changed drastically and the binary scale has become most valuable.

In-Text Question

The quinary scale – number system on 5 – was the first scale to be used extensively. True or False

In-Text Answer

True

15.2 The Denary or Decimal Scale (Bases 10)

Since base 10 (the denary of decimal base) has been accepted by the international community, individuals now count in groups of 10s and make use of the digits 0,1,2,3,4,5,6,7,8, and 9 together with the concept of place values to represent any number whatsoever.

For example, 2574=2000 + 500 + 70 + 4

2000 = 2 x 10^3; 500 x 10^2; 70 x 10^1 and 4 = 4 x 1 = 4 x 10^0.

So in expanded form 2574 = 2000 + 500 + 70 + 4

=2 x 1000 + 5 x 100 + 7 x 10 + 4 x 1

$=2 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$

Using place values, 2574 can be written as follows:

	Thousand	Hundred	Ten	Ones
	2	5	7	4
I.e.	2 x 1000	5 x 100	7 x 10	4 x 1
I.e.	2×10^3	5×10^2	7×10^1	4×10^0

Similarly, 42865 = 4 x 10000 + 2 x 1000 + 8 x 100 + 6 x 10 + 5 x 1

$= 4 \times 10^4 + 2 \times 10^3 + 8 \times 10^2 + 6 \times 10^1 + 5 \times 10^0$

The indices are written starting from 0,1,2,3, and so on. Index 0 corresponds to the units or one place value, 1 to the tens, 2 to the hundreds, 3 to the thousands and so on. Hence the expanded form of any number can be written by merely counting the positions of the digits.

For example, the number 52865 can be expressed in Digit position like

10^4	10^3	10^2	10^1	10^0
5	2	8	6	5

Tth	Th	H	T	U
5	2	8	6	5

Which means the powers, i.e. 10^4, 10^3, 10^2, 10^1, and 10^0 here shows the place values of the digits.

Examples 1:

Write the following in expanded form by counting the position of the digits

(i) 2481 (ii) 60367 (iii) 700342

Solutions:

Here the position of the digits correspond to their respective place values as we have in Th, H, T, U. Thus

$2481 = 2 \times 10^3 + 4 \times 10^2 + 8 \times 10^1 + 1 \times 10^0$

$60367 = 6 \times 10^4 + 0 \times 10^3 + 3 \times 10^2 + 6 \times 10^1 + 7 \times 10^0$

$700342 = 7 \times 10^5 + 0 \times 10^4 + 0 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 2 \times 10^0$

Write the following expanded number numbers in ordinary form

$3 \times 10^3 + 4 \times 10^0$

$5 \times 10^5 + 7 \times 10^4 + 4 \times 10^2 + 5 \times 10^0$

$6 \times 10^6 + 5 \times 10^5 + 4 \times 10^4 + 3 \times 10^3 + 2 \times 10^2 + 4 \times 10^0$

Solution: - Hence we expand the powers as in indices and multiply with the corresponding digit and sum up thus:

$3 \times 10^3 + 4 \times 10^0 = 3 \times 1000 + 4 \times 1 = 3000 + 4$

$=3004$

$5 \times 10^5 + 7 \times 10^4 + 4 \times 10^2 + 5 \times 10^0 = 5 \times 100000 + 7 \times 10000 + 4 \times 100 + 5 \times 1$

$= 500000 + 70000 + 400 + 5$

$= 570405$

$6 \times 10^6 + 5 \times 10^5 + 4 \times 10^4 + 3 \times 10^3 + 2 \times 10^2 + 4 \times 10^0$

$= 6 \times 1000000 + 5 \times 100000 + 4 \times 10000 + 3 \times 1000 + 2 \times 100 + 4 \times 1$

$= 6000000 + 500000 + 40000 + 3000 + 200 + 4$

$= 6543204.$

Conversion from Other Bases to Base 10 (Denary)

It is possible to use any number as a building or a numeral system. The number of digit used in the system is always equal to the base.

Decimal/Denary base 10-Digits	Quinary Base 5-Digits	Binary Base 2-Digits	Octal Base 8-Digits	Hexadecimal Base 16-Digits
0	0	0	0	0
1	1	1	1	1
2	2		2	2
3	3		3	3
4	4		4	4
5			5	5
6			6	6
7			7	7
8				8
9				9
				10 (A) These are
				11(B) the current
				12(C) conven-
				13(d) tional
				14(E) notations
				15(F)

In the hexadecimal (base 16) system the letter A represents 10, B represents 11, C for 12, D for 13, E for 14 and F for 15, since these numbers are regarded as single digits in the hexadecimal scale. These are the current conventional notations.

In other to differentiate the numbers in different bases we usually use subscript notation. For example, 25_8readas two, five base 8 meaning $2 \infty 8^1 + 5 \infty 8^0$that is 2 eights ones. This equals $16_{10} + 5_{10} = 21_{10}$.

21_3 read as two, one to base three (not twenty one because this is the language in the denary system) can also be read as “two one to base 3”. 1220_3 is read as one, two, zero base three.

To obtain the value of the number12203 in the expanded form, we have

(1 x 33) + (2 x 32) + (2 x 31) + (0 x 30)

The powers correspond to the place value of the number and they show the order of the digits.

ie $1220_3 = 27 + 18 + 9 + 0$ (all in denary) $= 54_{10}$.

Hence, $1220_3 = 541_{10}$.

Example2:- Express 637_8 in the expanded form and find the value in base 10.

Solution $637_8 = (6 \text{ x } 8^2) + (3 \text{ x } 8^1) + (7 \text{ x } 8^0)$

= (6 x 64) + (3 x 8) + (7 x 1) which in base 10.

$= 384_{10} + 24_{10} + 7_{10} = 415_{10}$

: - $637_8 = 415_{10}$

Hence, it is easier to express numbers in any base in their expanded form. So to convert the numbers from other bases to the denary system (base 10), we express such numbers in their expanded form or in powers of the base of the given number and find the values in base 10.

Example3: Change the following numbers to denary numbers (base 10):

1246_7 b. 134_{16} c. 382_{12} d. 10111_2

$10111_2 = (1 \text{ x } 2^4) + (0 \text{ x } 2^3) + (1 \text{ x } 2^2) + (1 \text{ x } 2^1) + (1 \text{ x } 2^0)$

= (1 x 16) + (0 x 8) + (1 x 4) + (1 x 2) + (1 x 1)

$= 16_{10} + 0_{10} + 4_{10} + 2_{10} + 1_{10} = 23_{10}$

: - $10111_2 = 23_{10}$

It is conventional to leave the suffix 10 when we write a number in the decimal system, ie $23_{10} = 23$. We leave the suffix 10.

Fractional Parts

Sometimes we are faced with numbers which are not whole numbers. Hence, it is very necessary to study also the conversion of fractional parts of numbers. The following example can be in used in our study of the conversion of fractional parts of other bases to decimal system.

Note that this fractional part must be expressed in decimal form before conversion is possible. This is because it is important to represent the place value of the number before conversion can be done.

Example 4

6.4_7 to denary number.

	7^0	7^{-1}	7^{-2}
Number	6	4	

In expanded form $6.4_7 = (6 \text{ x } 7^0) + (4 \text{ x } 7^{-1})$

$= (6 \text{ x } 1) + (4\, x\, \frac{1}{7}) = 6 + \frac{4}{7} = (6\frac{4}{7})_{10}$.

: - $6.4_7 = (6\frac{4}{7})_{10} = 6.57$ (2 places of decimal)

32.51_6

	6^1	6^0	6^{-1}	6^{-2}
Number	3	2	5	1

$32.51_6 = (3 \text{ x } 6^1) + (2 \text{ x } 6^0) + (5 \text{ x } 6^{-1}) + (1 \text{ x } 6^{-2})$

$= (3 \text{ x } 6) + (2 \text{ x } 1) + (5\, x\, \frac{1}{6}) + (1 \text{ x } \frac{1}{6^2}$

$=18 + 2 + \frac{5}{6} + \frac{1}{36}$

$=20 + \frac{30 + 1}{36} = (20\frac{31}{36})_{10}$

: - $32.51_6 = (20\frac{31}{36})_{10} = 20.86$ (2 places of decimal)

Example 5: Convert $(11011.1001)_2$to decimal base.

Solution: The integral part is 11011_2 while the fractional part of this number is 1001_2

: - 110112 = (1 x 24) + (1 x 23) + (0 x 22) + (1 x 21) + (1 x 20)

$=16 + 8 + 0 + 2 + 1 = 27_{10}$

And $0.1001_2 = (1 \text{ x } 2^{-1}) + (0 \text{ x } 2^{-2}) + (0 \text{ x } 2^{-3}) + (1 \text{ x } 2^{-4})$

$=\frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16} = \frac{8+1}{16} = \frac{9}{16}$

$= 0.5 + 0 + 0.0625 = 0.5625.$

So $\frac{9}{16} = 0.5625$

Hence $11011.1001_2 = 27_{10} + 0.5625_{10} = 27.5625_{10}$

So $11011.1001_2 = 27.5625_{10}$

Conversion from One base to another

Here you will learn the conversion numbers from one base say binary to quinary (base 5).

To achieve this, convert the given number first to denary (base 10), then to the required base.

Example 6: Express 4104_{five} to Octal.

Solution: 4104_{five} to denary.

5^3	5^2	5^1	5^0
4	1	0	4

$(4 \times 5^3) + (1 \times 5^2) + (0 \times 5^1) + (4 \times 5^0)$

$(4 \times 125) + (1 \times 25) + (0 \times 5) + (4 \times 1)$

$500 + 25 + 0 + 4 = 529_{10}$

Then convert 529_{10} to Octal

8	529
8	66 + 1
8	8 + 2
8	1 + 0
	0 + 1

$529_{10} = 1021_{eight}$

So $4104_{five} = \underline{1021_{eight}}$

Binary Numbers

The binary numbers are numbers written in base 2. They have only two digits 0 and 1 which are very importance in this computer age. The computer machine which is important today makes an extensive use of the of-on process which translates to mean the 0, 1 digits respectively.

The binary numbers are used to express responses to statements or questions which have only two possible responses – "Yes" and "No". Learners who play games can make extensive use of the binary numbers. Having learnt that the binary numbers are made up of the two digits, 0 and 1, they can code their messages and give responses to the questions.

Example 7: The answers to the following questions can be coded as follows:

Yes | No

1	Have you paid your school fees?	1	0	
2	Is $50 = 2 \times 5^2$ in indices?	1	0	
3	Is $\log_{10} 100 = 2$?	1	0	
4	Is $\sin 45^0 = \frac{1}{\sqrt{5}}$?		1	0
5	Is $\tan 60^0 = \infty$?		0	1
6	Are you attending tonight's party?		0	1

Since nobody outside their group of friends is meant to understand the replies to the above questions, the respondent used 0 for No and 1 for Yes in his/her responses as shown in the table above. Also, having learnt the conversion of decimal numbers to binary they can communicate with each other. For example they can assign values to the English alphabet thus.

Letter	Numerical Code	Binary Equivalent/ codes
A	01	00001
B	02	00010
C	03	00011
D	04	00100
E	05	00101
F	06	00110
G	07	00111
H	08	01000
I	09	01001
J	10	01010
K	11	01011
L	12	01100
M	13	01101
N	14	01110
O	15	01111
P	16	10000
Q	17	10001
R	18	10010
S	19	10011
T	20	10100

U	21	10101
V	22	10110
W	23	10111
X	24	11000
Y	25	11001
Z	26	11010

In-Text Question

Write in ordinary from $4 \times 10^3 + 1x\ 10^0$

A. 4001
B. 4010
C. 3400
D. 4000

In-Text Question

A. 4001

15.3 Operation with Bases

Operation with bases could be with Addition and Subtraction

Addition and Subtraction.

Addition and Subtraction in any base other than ten is the same, the only exception is that the number/digit would not exceed the highest digit of the base. This is better illustrated in the following examples.

Example 8:

Add the following $72_8 + 643_8$.

Solution: Set out the working as if the numbers are in base 10 thus:

$$\begin{array}{r} 72 \\ +\ 643 \\ \hline 745 \end{array}$$

From the right hand column you add 2 + 3 = 5, five is a digit and does not exceed 8, so the digit 5 is left like that. Then you add the second column 7 + 4 = 11 containing a big bundle of 8 and 3 small bundle of 8, then you write 13 instead of 11, hence you write 3 in the second column and add the 1 left to the next column on the left.

Thus 6 + 1 = 7. The answer is then = 735_8 .

2 Add 301_8, 407_8, 170_8.

Solution: The numbers can be placed at their rightful place values thus:

	8^3	8^2	8^1	8^0	since we are in base 8
		3	0	1	
+		4	0	7	
		1	7	0	
	1	1	0	0_8	

You can see straight away that when each column is added the sum equals 8 the base since the place values are clearly spelt out the large bundles move to the next left column like addition in base 10 base 10 and the first 1 is written at the next place value, so the sum of 3018, 4078 and 1708 is 11008.

3 Simplify 23114 - 2134.

Solution:

Placing the numbers correctly under their place values we have:

	4^3	4^2	4^1	4^0
	2	3	1	1
-		2	1	3
	2	0	3	2_4

You can observe here that in the extreme right hand column the digit above 3, i.e. 1 is less than the digit below. Quickly rename the 1 under the ones by adding it to the one taken from the four column, i.e. 1 + 4 = 5 and hence we subtract 3 from 5, i.e. 5 – 3 = 2 so 2 is written. Do the same for the subsequence columns where the digits below are greater than the ones above them.

Hence since 0 is left under 4^1 after the first exercise we now have 0 + 4 (taken from the 4^2 column) so we have 4 – 1 = 3 and continuing we have 2 – 2 = 0 and finally 2 – 0 = 2.

: - 23114 - 2134 = 20324.

4 If 3014five = 2112five - xfive, what is the value of x?

Solution:

This type of equation is solves as in the case of simple equations.

So we have $3014_{five} - 2112_{five} = x_{five}$

Simplifying as in example 3 above

  3014
- 2112

402_{five}

: - xfive = 402five

Summary for study session 15

In this study session, you have learnt :

1. The expansion of numbers to their various value system
2. The conversion of number from one base to another addition,subraction, multiplication and division of various number bases
3. The application of binary numbers in real life situation.

Self-Assessment Questions (SAQs) for Study Session 15

Now that you have completed this study session, you can assess how well you have achieved its Learning outcomes by answering the following questions. . You can check your answers with the Notes on the Self-Assessment questions at the end of this study.

SAQ 15.1

Express 4104_{five} to Octal.

SAQ 15.2

Covert 6.4_{7} to denary number

References

T.R. Moses: Further mathematics Schorlastic Series I