Basic Statistics in Educational Planning

GCE 203

University of Ibadan Distance Learning Centre

Open and Distance Learning Course Series Development

ISBN 978—2828—04—1

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Vice-Chancellor's Message

The Distance Learning Centre is building on a solid tradition of over two decades of service in the provision of External Studies Programme and now Distance Learning Education in Nigeria and beyond. The Distance Learning mode to which we are committed is providing access to many deserving Nigerians in having access to higher education especially those who by the nature of their engagement do not have the luxury of full time education. Recently, it is contributing in no small measure to providing places for teeming Nigerian youths who for one reason or the other could not get admission into the conventional universities.

These course materials have been written by writers specially trained in ODL course delivery. The writers have made great efforts to provide up to date information, knowledge and skills in the different disciplines and ensure that the materials are user-friendly.

In addition to provision of course materials in print and e-format, a lot of Information Technology input has also gone into the deployment of course materials. Most of them can be downloaded from the DLC website and are available in audio format which you can also download into your mobile phones, IPod, MP3 among other devices to allow you listen to the audio study sessions. Some of the study session materials have been scripted and are being broadcast on the university's Diamond Radio FM 101.1, while others have been delivered and captured in audio-visual format in a classroom environment for use by our students. Detailed information on availability and access is available on the website. We will continue in our efforts to provide and review course materials for our courses.

However, for you to take advantage of these formats, you will need to improve on your I.T. skills and develop requisite distance learning Culture. It is well known that, for efficient and effective provision of Distance learning education, availability of appropriate and relevant course materials is a *sine qua non.* So also, is the availability of multiple plat form for the convenience of our students. It is in fulfilment of this, that series of course materials are being written to enable our students study at their own pace and convenience.

It is our hope that you will put these course materials to the best use.

Prof. Abel Idowu Olayinka
Vice-Chancellor

Foreword

As part of its vision of providing education for "Liberty and Development" for Nigerians and the International Community, the University of Ibadan, Distance Learning Centre has recently embarked on a vigorous repositioning agenda which aimed at embracing a holistic and all encompassing approach to the delivery of its Open Distance Learning (ODL) programmes. Thus we are committed to global best practices in distance learning provision. Apart from providing an efficient administrative and academic support for our students, we are committed to providing educational resource materials for the use of our students. We are convinced that, without an up-to-date, learner-friendly and distance learning compliant course materials, there cannot be any basis to lay claim to being a provider of distance learning education. Indeed, availability of appropriate course materials in multiple formats is the hub of any distance learning provision worldwide.

In view of the above, we are vigorously pursuing as a matter of priority, the provision of credible, learner-friendly and interactive course materials for all our courses. We commissioned the authoring of, and review of course materials to teams of experts and their outputs were subjected to rigorous peer review to ensure standard. The approach not only emphasizes cognitive knowledge, but also skills and humane values which are at the core of education, even in an ICT age.

The development of the materials which is on-going also had input from experienced editors and illustrators who have ensured that they are accurate, current and learner-friendly. They are specially written with distance learners in mind. This is very important because, distance learning involves non-residential students who can often feel isolated from the community of learners.

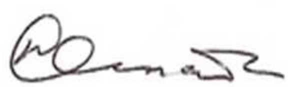
It is important to note that, for a distance learner to excel there is the need to source and read relevant materials apart from this course material. Therefore, adequate supplementary reading materials as well as other information sources are suggested in the course materials.

Apart from the responsibility for you to read this course material with others, you are also advised to seek assistance from your course facilitators especially academic advisors during your study even before the interactive session which is by design for revision. Your academic advisors will assist you using convenient technology including Google Hang Out, You Tube, Talk Fusion, etc. but you have to take advantage of these. It is also going to be of immense advantage if you complete assignments as at when due so as to have necessary feedbacks as a guide.

The implication of the above is that, a distance learner has a responsibility to develop requisite distance learning culture which includes diligent and disciplined self-study, seeking available administrative and academic support and acquisition of basic information technology skills. This is why you are encouraged to develop your computer skills by availing yourself the opportunity of training that the Centre's provide and put these into use.

In conclusion, it is envisaged that the course materials would also be useful for the regular students of tertiary institutions in Nigeria who are faced with a dearth of high quality textbooks. We are therefore, delighted to present these titles to both our distance learning students and the university's regular students. We are confident that the materials will be an invaluable resource to all.
We would like to thank all our authors, reviewers and production staff for the high quality of work.

Best wishes.



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Course Introduction

This course is designed to introduce and acquaint you with the basic ideas and principles in statistics, as they are applied in educational practice. The main objective of this course is to make you comprehend and appreciate fully the scope, purposes and applications of statistics in educational research. More specifically, however, having studied all the lectures (that is, lecture 1 to 15) you should be able to:

- discuss the meaning and definitions of various terminologies in statistics;
- explain the meaning and purpose of statistics in education;
- explain the different kinds of statistics;
- discuss the frequency distributions and know how to construct frequency distributions and histograms;
- discuss the different measures of central tendency, such as the mean, the median, and the mode, how they are obtained and their differences;
- discuss the various measures of variability and how they could be obtained from either ungrouped or grouped data;
- explain the meaning of correlation and its application to educational research;
- explain the principles of probability and know why it is important in education;
- appreciate fully the set theory and binominal probability;
- discuss what the chi square is all about;
- discuss how to compute the chi square;
- discuss the essence of measurement in education; discuss the four levels of measurement;
- differentiate between parametric and nonparametric tests; and
- discuss that basic statistics is sine qua non in educational research, and finally solve some statistical problems in educational practice.

Study Session 1: Definition of Terms

Expected duration: 1 week or 2 contact hours

Introduction

There are concepts used in statistics, which may not have the same meaning as used in everyday conversation or communication. In this study, you will be exposed to some of these words, in order to facilitate your better understanding of this course.

Learning Outcomes for Session 1

When you have studied this session, you should be able to define the following term as used in statistics:

1.1. Population and Sample (SAQ 1.1)
1.2. Parameters and Statistic (SAQ 1.2)
1.3. Summation Notation (SAQ 1.3)
1.4. Questionnaire (SAQ 1.4)
1.5. Data (SAQ 1.5)
1.6. Variable (SAQ 1.6)
1.7. Sampling Frame (SAQ 1.7)
1.8. Sample space (SAQ 1.8)
1.9. Random Experiment (SAQ 1.9)
1.10. Equally likely (SAQ 1.10)

1.1 Definition of Terms

1.1.1 Population

In everyday language, the term population is used to refer to groups or aggregates of people who occupy defined geographical regions at specified times; such as the population of Nigeria, Oyo state, etc. The statistician employs the term in a more general sense to refer not only to defined groups or aggregates of people, but also to defined groups or aggregates of animals, objects, materials, measurements, or things or events of any kind.

Types of population are: finite and infinite populations. While finite population can be counted and finite number obtained (e.g. students); infinite occurs when the total number may not be obtainable (e.g. stars, etc.). Statistics is concerned with the numerical properties of populations, that is, with properties to which numerals can in some manner be assigned. For example, in any population of psychiatric hospital patients, some may be classed as depressed, drug addicts, schizophrenics, epileptics, organic disorders, etc.

- ○ Take a moment to reflect on this question and note whether it is true or false: population is the total number of people living in a place at a given time.

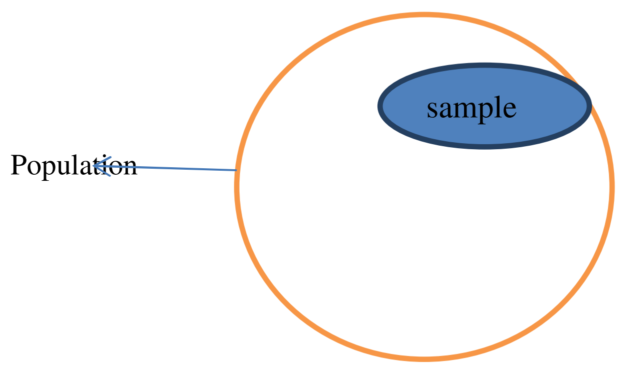
- It is false, because in statistics population cover both animate and inanimate beings.

Further, some patients may come from broken homes, while others may have a normal healthy home background. Thus one can say that statistics is a methodology for the exploration of and the making of statements about the properties of groups or aggregates called populations.

1.2 Samples and sampling techniques

A sample is a subset of the population. It is any subgroup or sub-aggregate drawn by some appropriate method from a population. For example, if a population is indefinitely large, it is of course impossible to produce complete population statistics. Therefore under circumstances such as these, the researcher or investigator draws what is spoken of as a sample. Having drawn his sample, the investigator utilizes appropriate statistical methods to describe its properties. The researcher will then proceed to make statements about the properties of the population from his knowledge of the properties of the sample, that is, he proceeds to generalize from the sample to the population.

Figure 1.1



- ○ One of your classmates is confused on classifying the children attending school in the city of Ibadan or the inmates of the Neuropsychiatry hospital in Aro, Abeokuta, and the cards in a deck.

- They are finite populations.

1.3. Parameters and Statistic

A clear distinction is usually drawn between parameters and statistic. A parameter is a property descriptive of the population. It is the numerical value describing a characteristic of a population that is called a parameter. The term statistic refers to the property of a sample drawn at random from a population. Thus the statistic value is presumed to be an estimate of a corresponding population parameter. For example, Suppose, that a sample of 1,000 adult male Nigerians of a given age range is drawn from the total population, the height of the members of the sample measured, and a mean value, 68.972in., obtained. This value is an estimate of the population parameter which would have been obtained had it been possible to measure all the members in the population. Usually parameters or population values are unknown. It is estimated from sample characteristics. The distinction between parameter and statistic reflect itself in statistical notation. For example, the sample mean could be obtained by using the formula below.

Activity 1.1

Take a moment to reflect on the meaning of parameter and statistic, and write your opinion.

Activity 1.1 Feedback:

Parameter is the property characteristics of the population; while statistic refers to the property of a sample drawn at random from a population

1.4. Summation Notation

One of the more frequently used forms of notation is spoken of as summation notation (∑). It means addition of scores or values.

Consider the group of five scores below:

Subject	Score symbol	Score value
1	X_1	4
2	X_2	6
3	X_3	8
4	X_4	4
5	X_5	3

The summation (∑) = 4+6+8+4+3 = 25

The calculation for the average is obtained by using this formula below:

$$\text{Average} = \frac{\text{sum of all the scores}}{\text{The number of scores}} = 25/5 = 5$$

- Assuming in a test, the scores obtained by participants are as follow: 6, 5, 9, 1, 4. Calculate the summation and its average

- Summation (∑) = 25; Average ∑ = 25/5 = 5.

1.5. Questionnaire

Questionnaire is one of the major tools that statistician make use of in the course of discharging their duty. Questionnaire could be set of stimuli, questions or statements prepared to elicit response from the respondents. For example an item in a questionnaire could read Obasanjo is the best president Nigeria ever had: (a) Strongly agree (b) Agree (c) Disagree (d) Strongly disagree etc. Questionnaire is a means of data collection in the course of carrying out research. It can also be used to sample the opinion of individuals on a given phenomenon.

1.6. Data

Data is a piece of information about a given phenomenon and the feeling of individual about a particular occurrence. The score of individual students in GCE 203 is a data that speaks bundle about each testee. Data can be in a verbal form or numerical form. However, the verbal form of data can be converted to numerical form and the numerical form of data can also be converted to verbal form. Also data can be referred to as continuous or discrete data. Therefore, that can be said to be of two types, the discrete and the continuous data. Data is said to be discrete when it can assume only a whole number, for example number of students in GCE 203 class and number of students from each state in Nigeria etc while continuous data is a kind of data that can assume fractional numbers, example of such data is a data that has to do with height of persons or things.

1.7. Variable

Variable is any given phenomenon that differs from one hypothetical population to another. Variable can be characteristic, number, or quantity that increases or decreases over time, or takes different values in different situations. Two basic types are (1) Independent variable: that can take different values and can cause corresponding changes in other variables, and (2) Dependent variable: that can take different values only in response to an independent variable. Others include moderating variable, etc.

Activity 1.2

Take a moment to reflect on the variable and mention its types.

Activity 1.2 Feedback:

Independent variable, dependent variable, moderating variable

1.8. Sampling Frame

Sampling frame is a systematic way of assigning number to all the elements in the target population such that all the elements in the target population have identifiable number. This term is useful in the cause of selecting sample from the population.

1.9 Random Number Digits

Random numbers are numbers that occur in a sequence such that two conditions are met: (1) the values are uniformly distributed over a defined interval or set, and (2) it is impossible to predict future values based on past or present ones. Random numbers are important in statistical analysis and probability theory. It is used mainly in selecting sample from population such that element of bias is eliminated in the course of selecting sample from the population.

1.10. Sample space

Sample space simply means when an experiment is performed all the possible outcome is referred to as sample space. When a die is tossed, the possible outcomes are {1, 2, 3, 4, 5 and 6}

1.11. Random Experiment

An experiment is said to be random when all possible outcomes (sample space) is known in advance that is prior performing the experiment.

Activity 1.3

Take a moment to reflect on the sampling space of a coin.

Activity 1.3 Feedback:

Sampling space of a coin is Head and Tail

1.12. Equally likely

If there are two possible outcomes, the probability would be 50% or 1/2 (AN EVEN CHANCE). "Equally likely events" refers to the chances of each possible outcome among many being equal. For example, using a six-sided die in a dice game yields a 1/6 chance for any one of the numbers to appear on top of the cube. Assuming that the die is not loaded, all six numbers are presumed to have an equal likelihood to end up on top in a given throw.

1.13. At Most and At Least

When talking about **"at least"** and **"at most"** probabilities, it is necessary to consider, in addition to the given probability,

- all probabilities larger than the given probability ("at least")
- all probabilities smaller than the given probability ("at most")

  - Attempt this quiz: write out the numbers that are at most 20 and at least 13?

- 13, 14, 15, 16, 17, 18, 19, 20.

Box 1.1: Definition of Terms

It is important to note the following:

- Commonly use terminologies in statistics are population, sample, data and its types-discrete and continuous, parameter, statistic, questionnaire and its types-open and close, variable and its types-independent, dependent, moderator, confounding, etc. A sample is a subset of the population. A parameter is a property descriptive of the population. Others are random number digits, sampling frame, sample space, random experiment, equally likely, at least and at most etc.

Summary of Study Session 1

In Session 1, you have learned that:

1. Population is total number of members of a group.
2. Sample is selected representative of a population. It could be probabilistic or non-probabilistic.
3. Variables can be independent, dependent or moderating variable.
4. Parameter is the descriptive characteristics of the population, while statistics is that of the sample drawn from that population.

Self-Assessment Questions (SAQs) for Session 1

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 1.1 (tests learning outcome 1.1)

Population refers to the aggregate of ------------ and ------------ in a given place and specific time, while sample is a selected representative of ---------------------------

SAQ 1.2 (tests learning outcome 1.2)

While _______________is concerned with the numerical properties of population; ______________ is concerned with the numerical properties of sample.

SAQ 1.3 (tests learning outcome 1.3)

$\sum$ is the Greek symbol and means

SAQ 1.4 (tests learning outcome 1.4)

Define Questionnaire

SAQ 1.5 (tests learning outcome 1.5)

Data

SAQ 1.6 (tests learning outcome 1.6)

Mention three may types of Variable

SAQ 1.7 (tests learning outcome 1.7)

Describe Sampling Frame

SAQ 1.8 (tests learning outcome 1.8)

Define Sample space

SAQ 1.9 (tests learning outcome 1.9)

When is an experiment said to be random?

SAQ 1.10 (tests learning outcome 1.10)

Describe the concept of 'Equally likely'?

Notes on the Self-Assessment Question

SAQ 1.1: animate and inanimate beings; population

SAQ 1.2: parameter; statistics

SAQ 1.3: Summation

SAQ 1.4: Questionnaire could be set of stimuli, questions or statements prepared to elicit response from the respondents

SAQ 1.5: Data is a piece of information about a given phenomenon and the feeling of individual about a particular occurrence.

SAQ 1.6: Independent variable; dependent variable; and moderating variable

SAQ 1.7: Sampling frame is a systematic way of assigning number to all the elements in the target population

SAQ 1.8: Sample space refers to all the possible outcome in an experiment

SAQ 1.9: An experiment is said to be random when all possible outcomes (sample space) is known in advance that is prior performing the experiment

SAQ 1.10: Equally likely events refers to the chances of each possible outcome among many being equal

References

Forguson, (1959) GA. *Statistical Analysis in Psychology and Education.* New York: McGraw Hill Book Company, Inc.

Guilford, J.P. and Fruchter, B. (1973) *Fundamental Statistics in Psychology and Education.* Koyakusha: McGraw — Hill.

Study Session 2: Meaning and Purpose of Statistics in Education

Expected duration: 1 week or 2 contact hours

Introduction
Statistics is an important aspect of a successful research or study. Without statistics, inferences and interpretation of data collected may become an effort in futility. Statistics are basic to research activities everywhere. Thus designing of experiments and research require statistics to enable the educator to describe the results and make inferences from data. Therefore, it is very important to understand the purposes of statistics in education, which is our central focus in this session.

- o What is your perception of statistics?

- Statistics is not difficult, but require constant practice and diligence.

Learning Outcomes for Session 2

When you have studied this session, you should be able to:

2.1. Define statistics (SAQ 2.1)
2.2. Mention types of statistics (SAQ 2.2)
2.3. Mention some purposes of statistics in research (SAQ 2.3)

2.1 Meaning of Statistics in Education

Statistics is the theory and method of analyzing quantitative data obtained from samples of observations in order to study and compare sources of variance of the phenomena. It helps to make decisions to accept or reject hypothesized relations between the phenomena, and to aid in making reliable inferences from empirical observations. Statistics is a science that deals with collection, representation, organization, presentation, analysis and interpretation of numerical data in order to make valid decision(s). Statistics therefore is the study of methods of handling such quantitative information, including techniques for organizing and summarizing as well as for making generalizations and inferences from data. In other words Statistics is the study of how to collect, organizes, analyze, and interpret numerical information from data.

Activity 2.1

Take a moment to reflect on what you have read so far. How would you define statistics?

Activity 2.1 Feedback:

Statistics is the scientific study of how to collect, organizes, analyze, and interpret numerical information from data.

2.2: Types of statistics

(a) Descriptive statistics (e.g. percentage, bar chart, mean, etc.)
(b) Inferential statistics (t-test, ANOVA, etc.)

Descriptive statistics involves methods of organizing, picturing and summarizing information from data. Inferential statistics involves methods of using information from a sample to draw conclusions about the population.

Activity 2.2

Take a moment to reflect on what you have read so far. How many types of statistics do we have?

Activity 2.2 Feedback: Two types. They are descriptive and inferential statistics.

2.3. The Purpose of Statistics in Education

Knowledge of statistics is important, both to be able to conduct and analyze data from educational experiments and to read, understand, and interpret research results published in textbooks and journals. Statistics is required primarily because almost all data in education contains variability.

Organize Data: When dealing with an enormous amount of information, it is all too easy to become overwhelmed. Statistics allow psychologists to present data in ways that are easier to comprehend. Visual displays such as graphs, pie charts, frequency distributions, and scatterplots make it possible for researchers to get a better overview of the data and to look for patterns that they might otherwise miss.

Describe Data: Think about what happens when researchers collect a great deal of information about a group of people. The U.S. census is a great example. Using statistics, we can accurately describe the information that has been gathered in a way that is easy to understand. Descriptive statistics provide a way to summarize what already exists in a given population, such as how many men and women there are, how many children there are, or how many people are currently employed.

Make Inferences Based Upon Data: By using what's known as inferential statistics, researchers can infer things about a given sample or population. Psychologists use the data they have collected to test a hypothesis, or a guess about what they predict will happen. Using this type of statistical analysis, researchers can determine the likelihood that a hypothesis should be either accepted or rejected.

- Abiodun is confused on purpose of statistics, how would you summarise it for him?

- Give your answer in line with collection, organization, analysis and interpretation of numerical information from data.

2.4 Why Statistics are Important in Research

You should know that statistical thinking is required because of the following advantages

i. Statistics permit the most exact kind of description.
ii. They force us to be definite and exact in our procedures and in our thinking.
iii. They enable us to summarize our results in a meaningful and convenient form. Statistics provide an unrivaled device for bringing order out of chaos, for seeing the general picture in one's results.
iv. They enable us to draw general conclusions, and the process of extracting conclusions is carried out according to accepted rules. Furthermore, by means of statistical steps, we can say about how much faith should be placed in any conclusion and about how far we may extend our generalization.
v. Statistics enable us to predict "how much" of a thing will happen under conditions we know and have measured. For example, we can predict the probable mark you will earn in this course if we know your score in a general academic-aptitude test, your score in a special statistics- aptitude test, your average mark in high school mathematics, and perhaps the number of hours per week that you devote to studying statistics.
vi. They enable us to analyze some of the causal factors underlying complex and otherwise bewildering events. For example, the reasons why a man fails in his examinations or in his profession are many and varied.
vii. It makes use of descriptive statistics for collection of data and inferential statistics for drawing inferences and meaning of the set of data collected from the respondents.
viii. Statistics is very important in research because that is the backbone of the research.
ix. Statistics provides a platform for research as to; How to go about your research, either to consider a sample or the whole population, the Techniques to use in data collection and observation, how to go about the data description (using measure of central tendency).
x. Statistics help in making reliable decision and formulate programmes.

Box 1.1: Research Design, Population and Sampling Procedure

It is important to note the following:

- Statistics was defined as the theory and method of data analysis. One of the purposes of' statistics in education is that it is an essential part of the professional training. Statistical thinking permits the most exact kind of description and enables us to draw general conclusions.

Summary of Study Session 2

In session 2, you have learned that:

1. Statistics is a science that deals with collection, representation, organization, presentation, analysis and interpretation of numerical data in order to make valid decision(s).
2. Statistics can be either descriptive or inferential.
3. Statistics is the backbone of research and has many purposes in research.

Self-Assessment Questions (SAQs) for Session 2

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 2.1 (tests learning outcome 2.1)
What is statistics?
SAQ 2.2 (tests learning outcome 2.2)
Mention types of statistics?
SAQ 2.3 (tests learning outcome 2.3)
Mention some purposes of statistics in research?

Notes on the Self-Assessment Question

SAQ **2.1:** Statistics is a science that deals with collection, representation, organization, presentation, analysis and interpretation of numerical data in order to make valid decision(s)**.**

SAQ 2.2: Descriptive statistics and inferential statistics.

SAQ 2.3: Statistics helps to collect, organize, and interpret data necessary for beneficial decisions.

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Ferguson, G.A., (1959). *Statistical Analysis in Psychology and Education.,* New York*:* McGraw—Hill Book Company, Inc., ,

Guilford, J.P. and Fruchter, B., (1973) *Fundamental Statistics in Psychology and Education.* Koyakusha,: McGraw—Hill,

Study Session 3: The Kinds of Statistics

Expected duration: 1 week or 2 contact hours

Introduction
We have discussed the concept of statistics and its purposes. We also mention that there are two basic kind of statistics, viz: (a) descriptive statistics; and (b) inferential statistics. In this session, we shall dwell more on these kinds or types statistics.

Learning Outcomes for Session 3

When you have studied this session, you should be able to:

3.1. Describe and give examples of descriptive statistics. (SAQ 3.1)
3.2. Describe and give examples of inferential statistics. (SAQ 3.2)

3.0 The Kinds of Statistics

The' two main types of statistics are (a) descriptive statistics — used for describing the properties of samples, or of populations where complete population data are available; and (b) inferential statistics — used for making inferences from observational data. This lecture will thus introduce you to these two kinds of statistics.

- o Mention at least two characteristics that need descriptive statistics among student?

- Age, gender, socio-economic status, etc.

3.1 Descriptive Statistics

Descriptive statistics is the discipline of quantitatively describing the main features or characteristics of a collection of information, or the quantitative description itself. Descriptive statistics are distinguished from inferential statistics (or inductive statistics), in that descriptive statistics aim to summarize a sample, rather than use the data to learn about the population that the sample of data is thought to represent. This generally means that descriptive statistics, unlike inferential statistics, are not developed on the basis of probability theory.
Some measures that are commonly used to describe a data set are measures of central tendency and measures of variability or dispersion. Measures of central tendency include the mean, median and mode, while measures of variability include the standard deviation (or variance), the minimum and maximum values of the variables, kurtosis and skewness.

3.1.1 Univariate analysis

Univariate analysis involves describing the distribution of a single variable, including its central tendency (including the mean, median, and mode) and dispersion (including the range and quartiles of the data-set, and measures of spread such as the variance and standard deviation). The shape of the distribution may also be described via indices such as skewness and kurtosis. Characteristics of a variable's distribution may also be depicted in graphical or tabular format, including histograms and stem-and-leaf display.

3.1.2 Bivariate analysis

When a sample consists of more than one variable, descriptive statistics may be used to describe the association between pairs of variables. In this case, descriptive statistics include:

- Cross-tabulations and contingency tables
- Graphical representation via scatterplots
- Quantitative measures of dependence
- Descriptions of conditional distributions

Finally, descriptive statistics (e.g. mean, standard, etc.) are needed in calculating inferential statistics.

Activity 3.1

Take a moment to reflect on univariate and bivariate analysis, what difference can you note between them?

Activity 3.1 Feedback:

Univariate analysis consist distribution of a single variable, while bivariate consists of more than one variable

3.2. Inferential Statistical

Inferential Statistical is the process of deducing properties of an underlying distribution by analysis of data. Inferential statistical analysis infers properties about a population: this includes testing hypotheses and deriving estimates. The population is assumed to be larger than the observed data set; in other words, the observed data is assumed to be sampled from a larger population. Statistical inference makes propositions about a population, using data drawn from the population with some form of sampling. Given a hypothesis about a population, for which we wish to draw inferences, statistical inference consists of (firstly) selecting a statistical model of the process that generates the data and (secondly) deducing propositions from the model.
The conclusion of a **statistical inference** is a statistical proposition. Some common forms of statistical proposition are the following:

- a point estimate, i.e. a particular value that best approximates some parameter of interest;
- an interval estimate, e.g. a confidence interval (or set estimate), i.e. an interval constructed using a dataset drawn from a population so that, under;
- rejection of a hypothesis and so on.

3.2.1 Statistical inferences have two characteristics

i. The inferences are usually made from samples to populations. When we say that variables A and B are related because the statistical evidence is $r = 0.67$, or near 0.67, in the population from which the sample was drawn.

ii. The second type of inference is used when investigators are not interested in the populations, or only interested secondarily in them.

- One of your classmates is confused on characteristics of inferential statistics. What would you tell him or her?

- Compare your response to the characteristics mentioned above

Box 3.1: Kinds of Statistics

It is important to note the following:

- Statistics is the study of methods of handling data.
- All areas of education rely on one or both of the two basic types of statistics:
- (1) Descriptive statistics organizes, summarizes, and describes data, and (2) Inferential statistics is used for making inferences from small samples to larger populations.

Summary of Study Session 3

In session 1, you have learned that:

1. There two kinds of statistics. One is descriptive, while the other one is inferential. Descriptive statistics focuses on describing the characteristics of the population
2. Inferential focuses on making interpretation based on the data analysis result. Descriptive statistics (e.g. mean, standard deviation, etc.) are very useful in calculating and give further explanation on the data analysis.

Self-Assessment Questions (SAQs) for Session 3

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 3.1 (tests learning outcome 3.1)
Describe and give examples of descriptive statistics. (SAQ 3.1)
SAQ 3.2 (tests learning outcome 3.2)
Describe and give examples of inferential statistics. (SAQ 3.2)

Notes on the Self-Assessment Question

SAQ 3.1: Descriptive statistics organizes, summarizes, and describes data. Examples are frequency, percentage, graph, mean, median, standard deviation, variance etc.

SAQ 3.2: Inferential statistics is meant for making inferences or judgment from small samples to larger populations. Examples are t-test, chi-square, ANOVA, correlation analysis, etc.

References

Ferguson, G.A., (1959) *Statistical Analysis in Psychology and Education.* New York: McGraw—Hill Book Company, Inc.

Guilford, J.P. and Fruchter, B. (1973).*Fundamental Statistics in Psychology and Education,* Koyakusha: McGraw—Hill.

Study Session 4: Frequency Distribution and Graphing

Expected duration: 1 week or 2 contact hours

Introduction

The classification and description of numbers are required to assist data interpretation. There are many advantages attached to the classification of data in the form of frequency distributions. This type of classification assists the comprehension of important properties of the data and may reduce the arithmetical labour in calculating certain statistics. In this session, you shall therefore be reading about frequency distributions, how frequency distributions are constructed from grouped data, the graphs of frequency distributions, the differences in frequency distributions and their uses.

Learning Outcomes for Session 4

When you have studied this session, you should be able to:

4.1. Mention how data distribution can be represented. (SAQ 4.1)
4.2. Prepare a frequency table of any distribution of their choice (SAQ4.2)
4.3. Mention some factors that may influence graphical representation (SAQ 4.3)

4.1 Frequency Distributions and Graphing

4.1.1 Frequency Distributions

A frequency distribution is an arrangement of data which shows the frequency of occurrence of the different values of the variable or defined groupings of the values of the variable.

- Suppose a short statistics quiz was given to a class of 10 students, and the verbal distribution goes thus: two students scores five, one score three, three of them score four, while the rest score six. You are to translate this statement into a table.

Number of students	Frequency of score
2	5
1	3
3	4
4	6

Example of Table of frequency distribution

Example I: Table 4.2 Frequency Distribution of Scores on a Statistics quiz

X	Tally	F
10	1	1
9	11	2
8	111	3
7	111	3
6	1	2
	N	**= 10**

Example II: A second example of a frequency distribution might involve an opinion researcher who gives a questionnaire that asks a sample of people to what extent they approve of the way the Minister of Education is carrying out his duties. The researcher provides five possible responses and arbitrarily attaches a number for each level of response.

1. disapprove greatly
2. disapprove
3. ambivalent
4. approve
5. approve greatly.

Suppose 80 people were questioned and asked to indicate one of these five opinions. The frequency of their responses is presented in table 4.3.

Table 4.3 Frequency distribution of opinions on Ministerial Policy

	X	f	Rel. f
Approve greatly	5	9	1125
Approve	4	30	.375
Ambivalent	3	10	.125
Disapprove	2	25	.313
Disapprove greatly	1	6	.075
		N = 80	**1.000**

It is clear from this frequency distribution that people seem to be somewhat split on their opinion about the Minister's actions. A sizeable group generally disapproves while another sizeable group approves of his actions. Few people seem either ambivalent (X = 3) or adamantly positive or negative. Such descriptive conclusions would be dificu1t to arrive at if all 80 scores were written down without the assistance of the frequency distribution. Hence, not only does a frequency distribution save time in displaying data, it also organizes the numbers in a way in which the data may be summarized more easily than a complete listing of each score would allow.

Example III: Use this data to prepare a frequency table:

Table 4. 4

79	51	67	50	78	71	77	75	55	65
62	89	83	73	80	67	74	63	32	88
88	48	60	71	79	79	47	55	70	34
89	63	55	93	71	81	72	68	75	93.
41	81	46	50	61	72	86	66	54	58-
59	50	90	75	61	82	73	57	87	41
75	98	53	79	80	64	67	51	36	52
70	37	42	72	74	78	91	69	95	76
67	73	79	67	85	74	70	62	76	. 69
91	73	77	36	77	45	39	59	63	57
53	67	85	74	77	78	73	61	47	43
71	43	42	96	83	83	84	67	81	75
70	92	59	86	53	71	49	68	42	46
32	67	67	71	71	59	80	66	39	49
82	68	30	72	57	92	50	38	73	56

Class Interval	Class Boundaries	Class Mark (X)	f	Rel. f (f/N)	Cum. F.	Cum. Re. f (Cum. f/N)
30—39	29.5—39.5	34.5	10	.07	10	.07
40—49	39.5—49.5	44.5	15	.10.	25	17
50—59	49.5 —59.5	54.5	23	.15	48	.32
60—69	59.5 — 69.5	64.5	27	.18	75	.50
70—79	69.5 —79.5	74.5	44	.29	119	.79
80—89	79.5—89.5	84.5	21	.14	140	.93
90—99	89.5 – 99.5	94.5	10 .	07	150	1.00
			N = 150	**1.00**		

Activity 4.1

Take a moment to reflect on what you have read so far. Based on your learning experience, A look at the interval 30—39 shows that it has ________________ as its class interval

Activity 4.1 Feedback:

Its class interval is 10

4.2. Constructing Frequency Distributions for Grouped Data

Number of class intervals

In the construction of the frequency distribution for the above case, an interval such as 30 — 39 was selected.

The Size of the Class Interval

To determine the size of a class interval from a distribution already constructed, subtract the lower real limit from the upper real limit of any interval. For the interval 30 — 3 (30 and 39 are called the stated limits), the lower real limit is 29.5 and the upper real limit is 39.5 which yields an interval size of 10:

39.5 — 29.5 =10

Midpoint of an Interval

The midpoint of a class interval is the precise centre of that interval or halfway between ‘the endpoints, it can be determined by adding one-.half of the size of the interval to its lower real limit. Thus for the intended 30-39 whose size is ten (half of which is 5) and whose lower real limit is 29.5, the midpoint for class mark) is 29.5 + 5 = 34.5

Activity 4.2

Take a moment to reflect on what you have read so far. Based on your learning experience, The midpoint of any interval is given by

Activity 4.2 Feedback:

Adding upper limit score to lower limit score and divide by 2.

4.3 Graphs of Frequency Distributions

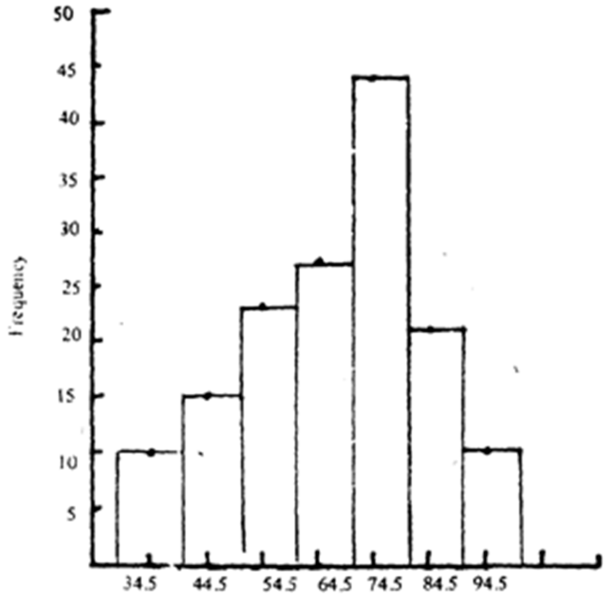
a. Graphs

Graphic representation is often of great help in enabling us to comprehend the essential features of frequency distributions and in comparing one frequency distribution with another. A graph is the geometrical image of a set of data. It is a mathematical picture.

b. *Histograms*

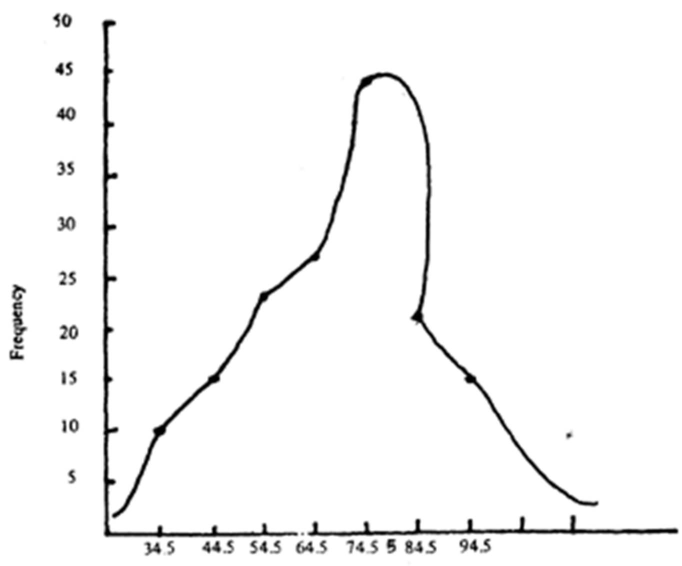
A histogram is a graph in which the frequencies are represented by areas in the form of bars. Using the previous case of table 4.5., Figure 4.1 shows the frequencies plotted in the form of a histogram.

Figures 4.1 A Frequency histogram for the statistics ability scores of 150 students as presented in Table 4.5



c. *Frequency Polygons*

In a histogram we assume that all the cases within a class interval are uniformly distributed over the range of the interval. In a frequency polygon we assume that all cases in each interval are concentrated at the mid-point of the interval. In this fact resides the essential difference between a histogram and a frequency polygon.



Figures 4.3 A Frequency polygon for the statistics ability scores of 150 students as presented in Table 4.5

d. *Cumulative Frequency Polygon*

The drawing of a cumulative frequency polygon differs from that of a frequency polygon in two respects. (i) instead of plotting points corresponding to frequencies, we plot points corresponding to cumulative frequencies. (ii) instead of plotting points above the mid-point of each interval, we plot our points above the top of the exact limits of the interval. This is done because we wish our graph to visually represent the number of cases falling above or below particular values. Figure 4.3 shows the cumulative frequency distribution for the data appearing in the sixth column on table 4.5

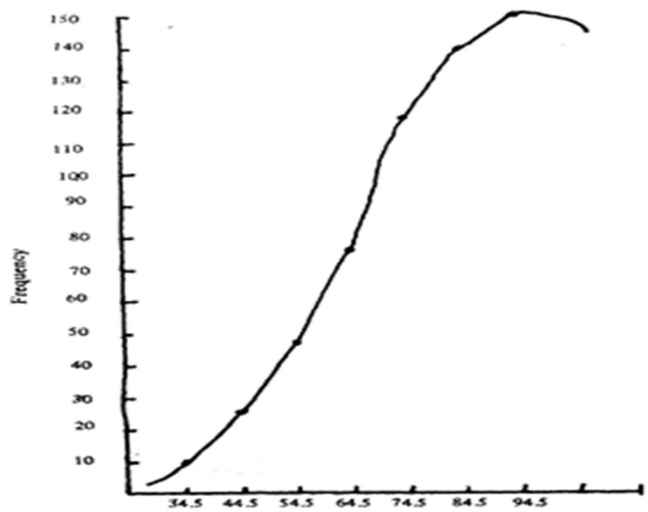


Figure 4.3 A Cumulative Frequency Polygon for the statistics ability scores of 150 students as presented in Table 4.5

4.4 Some Conventions for the Construction of Graphs

1. In the graphing of frequency distributions it is customary to let the horizontal axis represent scores and the vertical axis frequencies.

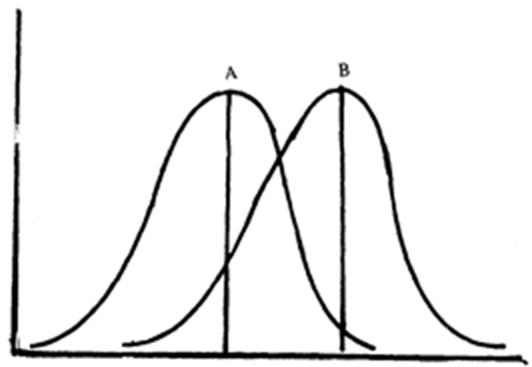
2. The arrangement of the graph should proceed from left to right.

3. Both the horizontal and vertical axes should be appropriately labelled.

4. Every graph should be assigned a descriptive title which states precisely what it is all about.

4.5 Variability

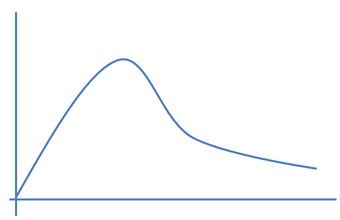
Variability is the degree to which scores deviate from their central tendency.



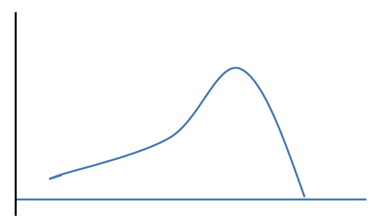
4.6: Skewness

Skewness refers to the symmetry or asymmetry of the frequency distribution. If a distribution is asymmetrical and the larger frequencies tend to be concentrated toward the low end of the variable and the smaller frequencies toward the high end, it is said to be positively skewed. If the opposite holds, the larger frequencies being concentrated toward the high end of the variable and the smaller frequencies toward the low end, the distribution is said to be negatively skewed.

Feedback:



+ skewness



- skewness

4.7: Kurtosis

Kurtosis refers to the flatness or peakedness of one distribution in relation to another. If one distribution is more peaked than another, it may be spoken of as more leptokurtic. When it is less peaked, it is said to be platier kurtic.

Activity 4.3

Take a moment to reflect on what you have read so far. Based on your learning experience, what factors could influence the graph of distribution?

Activity 4.3 Feedback:

Variability, skewness and kurtosis

Box 4.1: Frequency Distribution and Graphing

It is important to note the following:

- Frequency distribution helps to present a large data in a concise and short form.
- There is table for group and ungroup data, one must therefore follow some considerations.
- Data can also be presented in graph or chart.
- Variability, skewness and kurtosis all affect distribution of data on graph.

Summary of Study Session 4

In Session 4, you have learned that:

1. A frequency distribution is a tallying of the number of times each score value occurs in a group of scores.
2. The relative frequency distribution is a distribution that indicates the proportion of the total number of cases which were observed at each score value.
3. A graph is the geometrical image of a set of data. A histogram is a graph in which the frequencies are represented by areas in the form of bars.
4. Distributions differ from one another with respect to (a) central tendency (b) variability (c) skewness, and (d) kurtosis.

Self-Assessment Questions (SAQs) for Session 4

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 4.1 (tests learning outcome 4.1)
Mention how data distribution can be represented.
SAQ 4.2 (tests learning outcome 4.2)
Below is a set of scores on a mathematics examination. Construct a frequency distribution appropriate for these data. Present the distribution in the manner of Table 4.5 with 1ess interval, class boundaries, class mark, and frequency indicated (use an interval size of (9). Then compose a relative frequency distribution, cumulative frequency distribution, and cumulative relative frequency distribution for these data. Also, construct polygons and histograms for the frequency and relative frequency distributions

54	81	18	44	24
63	67	60	34	39
91	47	75	72	36
87	49	86	57	74

26	41	90	59	14
13	31	68	13	29
29	70	22	63	35
50	42	27	95	77
42	31	69	73	11
3.1	45	51	56	40

SAQ 4.3 (tests learning outcome 4.3)
Mention some factors that may make graphical representation differ?

Notes on the Self-Assessment Question

SAQ 4.1: frequency table and graph or chart.

SAQ 4.2: Compare your result with the examples given to you in this session.
SAQ 4.3: (a) central tendency (b) variability (c) skewness, and (d) kurtosis.

References

Ferguson, G.A., (1959) *Statistical Analysis in Psychology and Education,* New York: McGraw — Hill Book Company. Inc.

Guilford, J .P. and Fruchter, B., (1973) *Fundamental Statistics in Psychology and Education,* Kyakusha :McGraw—Hill,.

Study Session 5: Measures of Central Tendency

Expected duration: 1 week or 2 contact hours

Introduction

Measures of central tendency used in the description of frequency distributions are called averages. In common usage, the word average is often employed to refer to a value obtained by adding a set of measurements and then dividing by the number of measurements in the set. This type of average is called the arithmetic mean. Averages in common use are the *arithmetic mean, median, mode, geometric mean, a harmonic mean.* However, the most important and widely used of these is the arithmetic mean.

Activity 5.1

Take a moment to reflect on what you have read so far. Based on your learning experience, mention three measure of central tendency

Activity 2.2 Feedback:

Mean, median and Mode

Learning Outcomes for Session 5

When you have studied this session, you should be able to:

5.1. Describe and use formula for mean in a distribution (SAQ 5.1)
5.2. Describe and use formula for median in a distribution (SAQ 5.2)
5.3. Describe and use formula for mode in a distribution (SAQ 5.3)
5.4. Indicate which of the three measures of central tendency is most accurate and reliable (SAQ 5.4)

5.1 Measures of Central Tendency

5.1.1 The Arithmetic Mean

The mean is commonly known as the average. Consider the following set of data: 18, 12, 13, 8, 18, 16, 12, 17, and 12. The mean is 14.

By definition, the arithmetic mean is the sum of a set of measurements divided by the number of measurements in the set. The mean, symbolized by X (read “X bar”), is given by the formula:

$$\bar{X} = \frac{\Sigma X}{N}$$

Activity 5.2

Take a moment to reflect on what you have read so far. Try to recall the formula for mean.

Activity 2.2 Feedback:

$$\bar{X} = \frac{\Sigma X}{N}$$

a. Mean from an Ungrouped Data

The mean is calculated in the following manner if values of X are 8, 3, 4, 10, 7, 5 and 6.

X
8
3
4
10
7
5
6

$\Sigma X = 43$

$N = 7$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{43}{7} = 6.14$$

i. *Deviations about the Mean*

Symbolically,

$$\sum_{i=1}^{N} (x_i - \bar{X}) = 0$$

Consider the following numerical example

X_i	$\bar{X}$	$(X_i - \bar{X})$
3	5	3 – 5 = –2
6	5	6 – 5 = 1
5	5	5 – 5 = 0
1	5	1 – 5 = –4
10	5	10 – 5 = 5

$\Sigma X_i = 25$

$N = 5$

$\bar{X} = 5$

$$\sum_{i=1}^{N} (X_i - \bar{X}) = 0$$

b. C*alculating the Means from a Grouped Data*

Using the figures or scores from Tables 4.4 and 4.5, with only Table 4.5 reproduced below. However, the formula for the mean from a grouped data will have to change. It is usually written this way.

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$ f = number of cases within the interval;

fx = frequency times the x value for the interval. All such products are found first and then added together.

Table 5.1

Class Interval	Frequency	x	Fx
30 – 39	10	34.5	345.0
40 – 49	15	44.5	667.5
50 – 59	23	54.5	1253.5
60 – 69	27	64.5	1741.5
70 – 79	44	74.5	3278.0
80 – 89	21	10	1774.5
90 – 99	10	94.5	945.0
	∑f 150		**∑fX = 10005**

From table 5.1 therefore,

$$\bar{X} = \frac{\Sigma fx}{\Sigma f}$$

$$\vec{X} = \frac{10005}{150} = \underline{\underline{66.7}}$$

The arithmetic mean is to be preferred whenever possible because of several desirable properties.
(1). It is generally the most reliable or accurate of the three measures of central values. This means that from sample to sample from the same population; the mean ordinarily fluctuates less widely than either the mode or the median.

(2). The mean is better suited for further arithmetical computations.

Activity 5.3

Take a moment to reflect on what you have read so far. Based on your learning experience, why do you think mean is most useful among measures of central tendency?

Activity 2.2 Feedback:

It is better suited for further arithmetical computation

5.2. The Median

Another commonly used measure of central location is the median. The median is a point on a scale such that half the observations fall above it and half below it.

(a). *Calculating the Median from Ungroup Frequency Distribution*

The observations 2, 7, 16, 19, 20, 25, and 27 are arranged in order of magnitude. Here N is an odd number and the median is 19; three observations fall above it and three below it. If another observation, say 31, is included, the median is then taken as the arithmetic mean of the two middle values 19 and 20; that is, the median is (19 + 20)/2, or 19.5.

(b). *Calculating the Median from Group Frequency Distributions*

In calculating the median from data grouped in the form of a frequency distribution the problem is to determine a value of the variable such that one- half of the observations fall above this value and the other half below. The method will be illustrated with reference to the data in Table 4.5 which is reproduced below again.

Table 5.2: Frequency Distribution of Statistical Test Scores

(1) Class Interval	(2) Frequency	(3) Cumulative Frequency
30 – 39	10	10
40 – 49	15	25
50 – 59	23	48
60 -69	27	75
70 – 79	44	119
80 – 89	21	140
90 – 99	10	150

The formula for the median is given a is

$$\text{Median} = L + \left(\frac{N/2 - F}{fm}\right)h$$

Where f = exact lower limit of interval on containing the median

F = sum of all frequencies below L

fm = frequency of interval containing median

N = number of cases

h = class interval

Thus L = 59.5, F = 48, fm = 27, N = 150, and h = 9.

We then have

$$\text{Median} = 59.5 + \frac{\left(\frac{150}{2} - 48\right)}{27} \times 9$$

= 59.5 + (1)9

= 68.5

5.3 The Mode

Another measure of central tendency is the mode. The mode, symbolized Mo, is the most frequently occurring score.

(a) Ungroup Data Mode

Consider the observations 11, 11, 12, 12, 12, 13, 13, 13, 13, 13, 14, 14, 14, *15, 15, 15,* 16, 16, 17, 17, 18. Here the value 13 occurs 5 times, more frequently than any other value; hence the mode is 13.

Consider this series and indicate its mode: 11, 11, 12, 12, 12, 13, 13, 13, 13, 14, 14. 14, 14, 15, 15, 16, 16, 17, 18. Here the values 13 and 14 both occur with a frequency of 4, which is greater than the frequency of occurrence of the remaining values. The distribution is called bimodal. The mode may then be taken as (13 + 14)/2, or 13.5.

(b) Group Data Mode

With data grouped in the form of a frequency distribution the mode is taken as the mid-point of the class interval with the largest frequency.

- A friend of yours requests you to tell him the most reliable measure of central tendency, what will be your response?
- Mean is the most reliable and accurate measure of central tendency.

Activity 2.2

Take a moment to reflect on what you have read so far. Based on your learning experience, what is mode?

Activity 2.2 Feedback:

Mode is the number that has the highest frequency in a distribution.

Box 5.1: Measures of Central Tendency

In this session 5, it is important to note the following:

- An average is a central reference value which is usually fairly close to the point of greatest concentration of the measurements.
- By definition the arithmetic mean is the sum of a set measurements divided by the number of measurements in the set.
- The arithmetic mean is the only value in a distribution from which the deviations always sum exactly to zero. It is also the most reliable or accurate of the three measures of central values.
- The median is a point on a scale such that half of the observations fall above it and half below it.
- The mode is the most frequently occurring score in a distribution.

Summary of Study Session 5

In Session 5, you have learned that:

There are three main measure of central tendency. They are mean, median and mode. Of the three, mean is the most useful and reliable. They can be calculated either using group or ungroup data.

Self-Assessment Questions (SAQs) for Session 5

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module. Use this distribution to answer the following questions.

SAQ 5.1 (tests learning outcome 5.1)

1. The ____________ is the sum of a set of measurements divided by the number of measurements in the set.
   (a) mode (b) median (c) arithmetic mean (d) geometric mean (e) both (c) and (d).
2. The mean is usually symbolized by _____________
   (a) $\overline{X}$ (b) $\overline{Y}$ (c) $\overline{A}$ (d) $\overline{N}$ (e) $\overline{X}$ — X
3. The deviations about the mean are usually adding up to
   (a) one (b) zero (c) three (d) two (e) four
4. The formula for calculating for the mean from a grouped data is given by ____________
   (a) $\overline{X} = \frac{\Sigma fx}{\Sigma f}$ (b) $\overline{X} = \frac{\Sigma x}{N}$ (c) $\overline{X} = \frac{\Sigma N}{\Sigma fx}$
   (d) $\overline{X} = \frac{\Sigma f}{\Sigma fx}$ (e) $\overline{X} = \frac{\Sigma N}{X}$
5. This observations (8, 8, 8, 4, 5, 6, 10) has a mean of………
   (a) 6 (b) 7 (c) 5 (d) 4 (e) 9

SAQ 5.2 (tests learning outcome 5.2)

1. The median is obtained by the formula below from a grouped data.

(a) $Md = L + \frac{N/2 - F}{fm} h$

(b) $Md = H + N/2 - F\ h$

(c) $Md = M + \frac{2/N - F}{fm} h$

(d) $Md = M + \frac{2/N - F}{fm} h$

(e) $Md = L - \frac{N/2 - F}{fm} h$

2. The median of question 5 above is __________ (a) 5.4 (b) 4.5 (c) 6.875 (d) 7.875 (e) 4.35

SAQ 5.3 (tests learning outcome 5.3)

1. ______________ is the most frequently occurring score (a) mean (b) parameter (c) mode (d) median (e) variance:

2. The mode of question 5 above is ____________ (a) 5.4 (b) 4.5 (c) 8 (d) 7.675 (e) 5.675

SAQ 5.4 (tests learning outcome 5.4)
Arithmetic mean is the most reliable of the three measures of central values. True or False.

Notes on the Self-Assessment Question

SAQ 5.1:

1. (C) arithmetic mean
2. (A) $\overline{X}$
3. (B) Zero
4. (A) $\overline{X} = \frac{\Sigma fx}{\Sigma f}$ f
5. $\overline{X} = 7$

SAQ 5.2:

1. (A) L+ $\frac{N/2 - F}{fm}$ h
2. Median= 8 (4, 5, 6, 8, 8, 8, 10)

SAQ 5.3:

1. (C) mode
2. Mo = 8

References

Edwards, A.L. (1954) *Statistical Methods for the behavioural sciences*, New York: Rhinehart and Company, Inc.,

Wallis, W.A. and Roberts, H.V. (1963) Statistics*: A New Approach*, New York: Free Press.

Study Session 6: Measures of Variability

Expected duration: 1 week or 2 contact hours

Introduction

Variability refers to the measures of dispersion, spread or extent to which the scores dispersed from each other. The variation in the events of nature is of great concern to the educationist. Thus, the populations which are the objects of statistical study always display variation in one or more respects. In this session, our focus shall be on the idea of measures of variability, what variations are, how to calculate the indices of variation — the standard deviation, variance, and range. However, more prominence shall be given to the standard deviation, how it is calculated from both ungrouped and grouped data.

- o Suggest some variables we measure in education that could vary along different individual?
- Your answer may be: intelligence, interests, ability, aptitudes, academic achievement motivation, personality, etc.

Learning Outcomes for Session 6

When you have studied this session, you should be able to:

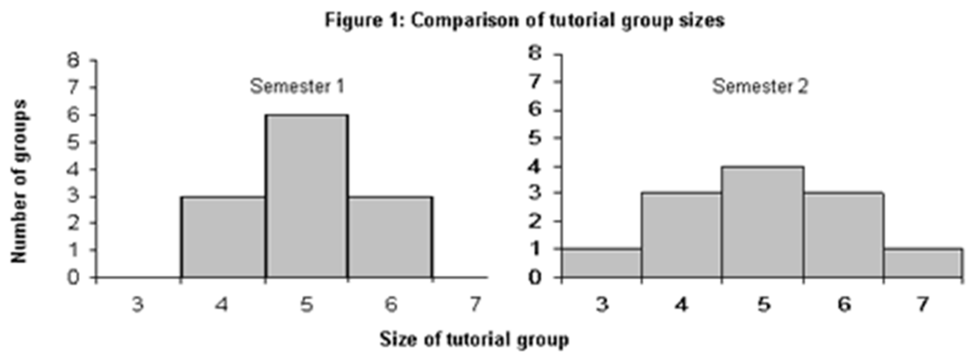
6.1. Define variability and give examples. (SAQ 6.1)
6.2 Define range and give good example (SAQ 6.2)
6.3 Establish relationship between standard deviation and variance (SAQ 6.3)
6.4: Compute the means deviation, variance and standard deviations (SAQ 6.4)

6.1 Concept of Variability

Variability refers to the extent to which the scores differ from each other. The extent to which the median and mean are good representatives of the values in the original dataset depends upon the variability or dispersion in the original data. Datasets are said to have high dispersion when they contain values considerably higher and lower than the mean value. Dispersion within a dataset can be measured or described in several ways including the range, inter-quartile range and standard deviation.

Example of Variability or Dispersion

In figure 1 the number of different sized tutorial groups in semester 1 and semester 2 are presented. In both semesters the mean and median tutorial group size is 5 students, however the groups in semester 2 show more dispersion (or variability in size) than those in semester 1.



6.2.The Range

The range is the most obvious measure of dispersion and is the difference between the lowest and highest values in a dataset. In figure 1, the size of the largest semester 1 tutorial group is 6 students and the size of the smallest group is 4 students, resulting in a range of 2 (6-4). In semester 2, the largest tutorial group size is 7 students and the smallest tutorial group contains 3 students, therefore the range is 4 (7-3).

- The range is simple to compute and is useful when you wish to evaluate the whole of a dataset.
- The range is useful for showing the spread within a dataset and for comparing the spread between similar datasets.

Activity 2.1

Take a moment to write sentences on the advantages of range.

Activity 2.1 Feedback:

Range is simple to compute. It helps to show and compare spread sheet data

- Explain to your friend how to calculate range using this spread sheet:

Table 1: Comparison of coursework and examination marks for 14 students

Student	A	B	C	D	E	F	G	H	I	J	K	L	M	N
Coursework mark	27	44	39	23	41	48	37	34	40	43	30	43	29	27
Examination mark	12	47	26	25	38	45	35	35	41	39	32	25	18	30

- Coursework: 48-27 =21; while Examination mark = 45-12 = 33. It means there was wider variation in the students' performance in the examination than in the coursework for this module.

Note that range is not a good measure of variability, as it only takes care of highest and lowest data.

6.2.The Inter-quartile Range

The inter-quartile range is a measure that indicates the extent to which the central 50% of values within the dataset are dispersed. It is based upon, and related to, the median.

In the same way that the median divides a dataset into two halves, it can be further divided into quarters by identifying the upper and lower quartiles. The lower quartile is found one quarter of the way along a dataset when the values have been arranged in order of magnitude; the upper quartile is found three quarters along the dataset. Therefore, the upper quartile lies half way between the median and the highest value in the dataset whilst the lower quartile lies halfway between the median and the lowest value in the dataset. The inter-quartile range is found by subtracting the lower quartile from the upper quartile.

For example, the examination marks for 20 students following a particular module are arranged in order of magnitude.

Lower quartile — Median — Upper quartile

Student	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
Mark	43	48	50	50	52	53	56	58	59	60	62	65	66	68	70	71	74	76	78	80

The median lies at the mid-point between the two central values (10th and 11th)

= half-way between 60 and 62 = 61

The lower quartile lies at the mid-point between the 5th and 6th values

= half-way between 52 and 53 = 52.5

The upper quartile lies at the mid-point between the 15th and 16th values

= half-way between 70 and 71 = 70.5

The inter-quartile range for this dataset is therefore **70.5 - 52.5 = 18** whereas the range is: **80 - 43 = 37.**

The inter-quartile range provides a clearer picture of the overall dataset by removing/ignoring the outlying values.

Like the range however, the inter-quartile range is a measure of dispersion that is based upon only two values from the dataset. Statistically, the standard deviation is a more powerful measure of dispersion because it takes into account every value in the dataset. The standard deviation is explored in the next section of this guide.

6.3. Variance is the average squared distance from the mean.

Population variance is defined as:

$$\sigma^2=\frac{\sum_{i=1}^{N}(y_i-\mu)^2}{N}$$

In this formula μ is the population mean and the summation is over all possible values of the population. *N* is the population size.

The sample variance that is computed from the sample and used to estimate σ^2 is:

$$s^2=\frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$$

Why do we divide by *n* - 1 instead of by *n*? Since μ is unknown and estimated by $\bar{y}$, the y_i's tend to be closer to $\bar{y}$ than to μ. To compensate, we divide by a smaller number, *n* - 1. The sample variance (and therefore sample standard deviation) is the common default calculations used by software. When asked to calculate the variance or standard deviation of a set of data, assume - unless otherwise instructed - this is sample data and therefore calculating the sample variance and sample standard deviation.

For example, let's find S^2 for the data set from vending machine A: 1, 2, 3, 3, 4, 5

$\bar{y}=(1+2+3+3+4+5)/6=3$

$S^2=(y_1-\bar{y})^2+\cdots+(y_n-\bar{y})^2/n-1=[(1-3)^2+(2-3)^2+(3-3)^2+(3-3)^2+(4-3)^2+(5-3)^2]/6-1=2$

6.4. The Standard Deviation

The standard deviation is a measure that summarizes the amount by which every value within a dataset varies from the mean. Effectively it indicates how tightly the values in the dataset are bunched around the mean value. It is the most robust and widely used measure of dispersion since, unlike the range and inter-quartile range, it takes into account every variable in the dataset. When the values in a dataset are pretty tightly bunched together the standard deviation is small. When the values are spread apart the standard deviation will be relatively large. The standard deviation is usually presented in conjunction with the mean and is measured in the same units.

Activity 6.2

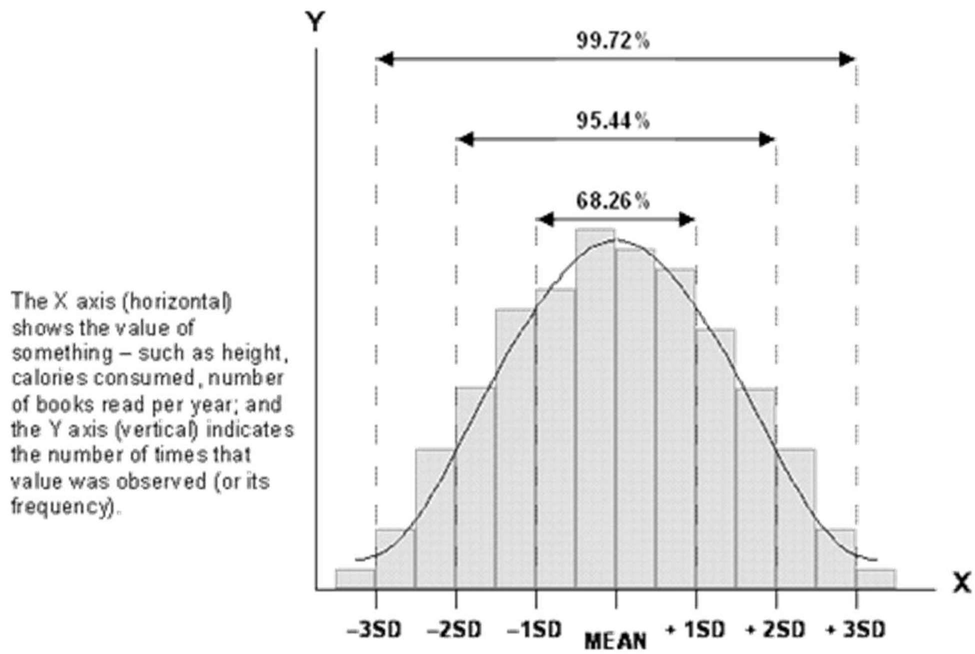
How would you describe standard deviation?

Activity 6.2 Feedback: standard deviation is summarizes the amount by which every value within a dataset varies from the mean.

Range is simple to compute. It helps to show and compare spread sheet data

In many datasets the values deviate from the mean value due to chance and such datasets are said to display a normal distribution. In a dataset with a normal distribution most of the values are clustered around the mean while relatively few values tend to be extremely high or extremely low. Many natural phenomena display a normal distribution.

For datasets that have a normal distribution the standard deviation can be used to determine the proportion of values that lie within a particular range of the mean value. For such distributions it is always the case that 68% of values are less than one standard deviation (1SD) away from the mean value, that 95% of values are less than two standard deviations (2SD) away from the mean and that 99% of values are less than three standard deviations (3SD) away from the mean. Figure 3 shows this concept in diagrammatical form.

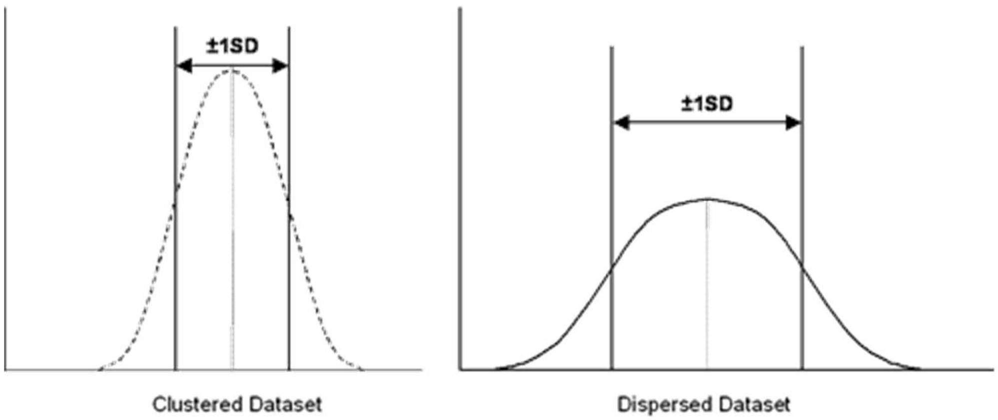


If the mean of a dataset is 25 and its standard deviation is 1.6, then

- 68% of the values in the dataset will lie between **MEAN-1SD** (25-1.6=23.4) and **MEAN+1SD** (25+1.6=26.6)
- 99% of the values will lie between **MEAN-3SD** (25-4.8=20.2) and **MEAN+3SD** (25+4.8=29.8).

If the dataset had the same mean of 25 but a larger standard deviation (for example, 2.3) it would indicate that the values were more dispersed. The frequency distribution for a dispersed dataset would still show a normal distribution but when plotted on a graph the shape of the curve will be flatter as in figure 4.

Example 4: Typical frequency distributions for clustered and dispersed datasets



6.6. Population and sample standard deviations

There are two different calculations for the Standard Deviation. Which formula you use depends upon whether the values in your dataset represent an entire population or whether they form a sample of a larger population. For example, if all student users of the library were asked how many books they had borrowed in the past month then the entire population has been studied since all the students have been asked. In such cases the population standard deviation should be used. Sometimes it is not possible to find information about an entire population and it might be more realistic to ask a sample of 150 students about their library borrowing and use these results to estimate library borrowing habits for the entire population of students. In such cases the sample standard deviation should be used.

6.7.Formulae for the standard deviation

Whilst it is not necessary to learn the formula for calculating the standard deviation, there may be times when you wish to include it in a report or dissertation.

The standard deviation of an entire population is known as **σ** (sigma) and is calculated using:

$$\sigma = \sqrt{\frac{\sum (x-\mu)^2}{N}}$$

Where **x** represents each value in the population, **μ** is the mean value of the population, **Σ** is the summation (or total), and **N** is the number of values in the population.

The standard deviation of a sample is known as **S** and is calculated using:

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Where **x** represents each value in the population, **x** is the mean value of the sample, **Σ** is the summation (or total), and **n-1** is the number of values in the sample minus 1.

Activity 6.3

Take a moment to recall the formula for calculating standard deviation for sample.

Activity 6.3 Feedback: $s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$

The sample standard deviation will always be greater than the population standard deviation when they are calculated for the same dataset. This is because the formula for the sample standard deviation has to take into account the possibility of there being more variation in the true population than has been measured in the sample. If all of the scores are grouped around the average, then your standard deviation will be lower. If your scores are all over the map and not grouped together at all, then your standard deviation will be huge. The steps for calculating it are:

- Calculate the average
- Calculate the **deviations**, which are the scores minus the average
- Square the deviations
- Sum up the squared deviations
- Divide this by the number of scores in your data set (or multiply by 1/N, same thing)
- Take the square root

The formula takes advantage of statistical language and is not as complicated as it seems. The part in the parentheses is the first two steps, subtracting the average (the *x* with the line over it) and the score (represented by *xi*). Then you square each result. The big, funny ∑ (called sigma) means that you add up all the squared deviations. Then you multiply the sum by one divided by the number of scores in your sample. The last step is square rooting to get your standard deviation, which is represented on the left side of the equation by the *Sn (√n)*.

If you have a group of scores and they're all clustered around the mean, then our second step of calculating the squared deviations would result in a smaller number. This would make all the maths later much smaller, and thus our standard deviation smaller.

When all our scores are clustered around the middle, it would look like this, with all the scores making a huge bump in the middle. If the scores are all spread out or clumped in weird places, then the standard deviation will be really high.

Standard deviation is important to understanding samples and populations because it lets you know how varied the scores are. First off, if you're looking at a study involving weight with the average being 200 and the standard deviation being 50 pounds, then that means about 68% of the data is between 150 and 250 pounds. That's not bad, depending on how big of a weight difference you want. Many statistical tests could be compromised because the data set is too widely spread.

6.8.Obtaining the Standard Deviation from a grouped data

You will now be required to go back to the figures presented in table 4.4 of 1eture 4.1. You will notice that the derivation of the standard deviation from a grouped data is quite simple if you follow the steps from column (1) down to the last column (9). This has been presented in table 6.1 below.

(1) Class interval	**(2) Class boundaries**	**(3) Class mark**	**(4) Frequency**	**(5) Cumulative**	**(6) fx**	**(7) $(X - \bar{X})$**	**(8) $(X - \bar{X})^2$**	**(9) $f(X - \bar{X})^2$**
30 – 39	29.5 – 39.5	34.5	10	10	345.0	-32.2	1036.84	10369.4
40 – 49	39.5 -49.5	44.5	15	25	667.5	-22.2	492.84	7392.6
50 – 59	49.5 – 59.4	54.5	23	48	1253.5	-12.2	148.84	3423.32
60 - 69	59.5 -69.5	64.5	27	75	1741.5	-2.2	4.84	130.68
70 – 79	69.5 - 79.5	74.5	44	119	3278.0	7.8	60.84	2676.96
80 – 89	89.5 -89.5	84.5	21	140	1774.5	17.8	316.84	6653.64
90 – 99	80.5 – 99.5	94.5	10	150	945	27.8	772.84	7728.2
			$\sum f = 150$		**$\sum fx = 10005$**	**$\sum(X - X)^2$**	**$\sum f(x - \bar{X})^2$**	
						2833.88 =	**38374**	

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{10005}{150} = 66.7$$

$$S^2 = \frac{\Sigma f(X-X)^2}{N-1} = \frac{38374}{149} = 257.54$$

$$S = \sqrt{\frac{\Sigma f(X-X)^2}{N-1}} = \sqrt{\frac{38374}{149}} = 16.05$$

It provides a more stable estimate of the standard deviation in the population than the sample mean deviation, for example, those of the mean deviation in the population. This is one of the reasons why it has come to be' accepted as the basic measure of variation. The standard deviation is more amenable to mathematical manipulation than other measures.

6.9. Advantages of the Standard Deviation

The standard deviation and variance have many advantages over other measures of variation. Many branches of statistical methods involve their use. The sample standard deviation is a non-stable or accurate estimate of the population parameter than other measure. It provides a more stable estimate of the standard deviation in the population than the sample mean deviation, for example, those of the mean deviation in the population. This is one of the reasons why it has come to be accepted as the basic measure of variation. The standard deviation is more amenable to mathematical manipulation than other measures.

- Take your time to tell one of your classmates some advantages of standard deviation.
- Your answer may be: It is a non- stable or accurate estimate of the population parameter than other measure.

Box 6.1: Measure of Variability

It is important to note the following:

- Measures that indicate the amount of variability within a dataset.
- The range is the simplest measure of variability to calculate but can be misleading if the dataset contains extreme values.
- The inter-quartile range reduces this problem by considering the variability within the middle 50% of the dataset.
- The variance and standard deviation is the most robust measure of variability since it takes into account a measure of how every value in the dataset varies from the mean.
- However, care must be taken when calculating the standard deviation to consider whether the entire population or a sample is being examined and to use the appropriate formula.

Summary of Study Session 6

In Session 6, you have learned that:

Self-Assessment Questions (SAQs) for Session 6

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 6.1 (tests learning outcome 6.1)
What is variability and give two examples of it?
SAQ 6.2 (tests learning outcome 6.1)
Define range and give good example to support your definition?

SAQ 6.3 (tests learning outcome 6.1)
__________ is the square root of the variance?
SAQ 6.4 (tests learning outcome 6.1)

Compute the means deviation, variance and standard deviations of the following figures. (a) 6, 7, 7, 8, (b) 4, 5, 9, 10 (c) 6, 4, 4, 5, 9

Notes on the Self-Assessment Question

SAQ 6.1: Variability refers to the extent to which the scores differ from each other.

SAQ 6.2: Range is the difference between the highest score or event and the lowest score or events. For example in a series of 20, 15, 13, 17, 20, 6; the range is 20 – 6 = 14.
SAQ 6.3: Standard deviation
SAQ 6.4: Compare your result with what you have in session 6.7.

References

Ferguson. G.A., (1959) *Statistical Analysis in Psychology and Education.* New York: McGraw-Hill Book Company, Inc.

McCall, B. Robert, (1975) *Fundamental Statistics for Psychology* New York**:** Harcourt Brace Jovanovich, Inc, 2nd Edition.

Study Session 7: Correlation

Expected duration: 1 week or 2 contact hours

Introduction

Psychologist and educators are constantly confronted with situations in which they are required to determine whether two or more variables are related in any way. Correlation helps in achieving this task. Correlation was introduced by Pearson Karl. Correlational studies usually compare members of a single group in which the variables under investigation are present in varying degrees.

Learning Outcomes for Session 7

When you have studied this session, you should be able to:

7.1. Define correlation. (SAQ 7.1)
7.2. State the range of co-efficient of correlation (SAQ 7.2)
7.3. Draw a chart to differentiate between positive and negative relationship (SAQ 7.3)
7.4. State some misuse of correlation. (SAQ 7.4)

7.1 Correlation

7.1.2 The Meaning of Correlation

The coefficient of correlation is one of those summarizing numbers, like a mean or a standard deviation, which, though they are single numbers, tell a story. In different situations it can vary from a value of +1 .00, which means perfect positive correlation through zero, which means complete independence, or no correlation whatever on down to -—1.00, which means perfect negative correlation.

Therefore, correlation can assume values between $-1 \leq r \leq 1$ when r – assume values between $-1 \leq r \leq -0.5$, this can be interpreted as strong but negative correlation, $-0.49 \leq r < 0$, this can be interpreted as negative and weak correlation. Also when $r = 0$, this indicates that there is no relationship between the two variables at all, while the values between $0 < r \leq 0.49$ indicates that the correlation is weak but positive and when $0.5 \leq r \leq 1$indicates that the correlation is positive and strong. From the above information one can say that there are two type of correlation positive and negative correlation.

Activity 7.1

Take a moment to reflect on what you have read so far. What word can be said to be synonymous with correlation?

Activity 7.1 Feedback: Relationship.

The most commonly used techniques for investigating the relationship between two quantitative variables are correlation and linear regression. Correlation quantifies the strength of the linear relationship between a pair of variables, whereas regression expresses the relationship in the form of an equation. For example, in patients attending an accident and emergency unit (A&E), we could use correlation and regression to determine whether there is a relationship between age and urea level, and whether the level of urea can be predicted for a given age.

- o Abiodun is confused on range of co-efficient of correlation, what would be your response?

- Co-efficient is between $-1 \leq r \leq 1$

7.3 Scatter diagram

When investigating a relationship between two variables, the first step is to show the data values graphically on a scatter diagram. Consider the data given in Table 1.1. These are the ages (years) and the logarithmically transformed admission serum urea (natural logarithm [ln] urea) for 20 patients attending an A&E. The reason for transforming the urea levels was to obtain a more Normal distribution. The scatter diagram for ln urea and age (Fig. 1.1) suggests there is a positive linear relationship between these variables.

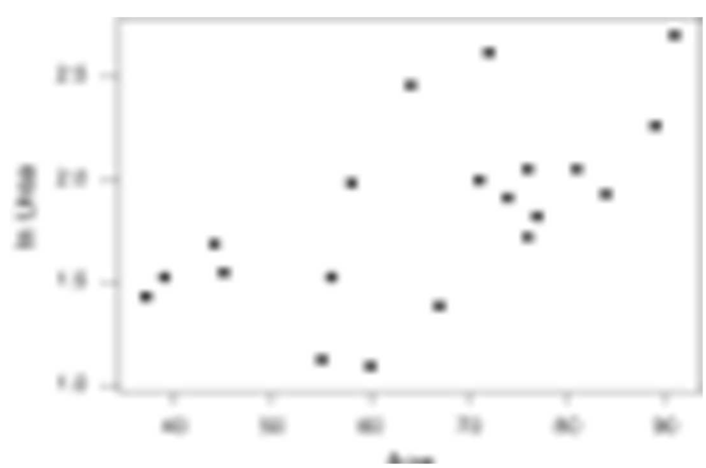


Figure 1 (http://www.ncbi.nlm.nih.gov/pmc/articles/PMC374386/figure/F1/
Scatter diagram for ln urea and age

Table 1
Age and ln urea for 20 patients attending an accident and emergency unit

Subject	Age (years)	ln urea
1	60	1.099
2	76	1.723
3	81	2.054
4	89	2.262
5	44	1.686
6	58	1.988
7	55	1.131
8	74	1.917
9	45	1.548
10	67	1.386
11	72	2.617
12	91	2.701
[illegible]	76	2.054

Table 1
Age and ln urea for 20 patients attending an accident and emergency unit

7.4 Correlation

On a scatter diagram, the closer the points lie to a straight line, the stronger the linear relationship between two variables. To quantify the strength of the relationship, we can calculate the correlation coefficient. In algebraic notation, if we have two variables x and y, and the data take the form of n pairs (i.e. $[x_1, y_1]$, $[x_2, y_2]$, $[x_3, y_3]$ ... $[x_n, y_n]$), then the correlation coefficient is given by the following equation:

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

where $\bar{x}$ is the mean of the x values, and $\bar{y}$ is the mean of the y values.

This is the product moment correlation coefficient (or Pearson correlation coefficient). The value of r always lies between -1 and +1. A value of the correlation coefficient close to +1 indicates a strong positive linear relationship (i.e. one variable increases with the other; Fig. 2). A value close to -1 indicates a strong negative linear relationship (i.e. one variable decreases as the other increases; Fig. 3). A value close to 0 indicates no linear relationship (Fig.4); however, there could be a nonlinear relationship between the variables (Fig. 5).

Figure 2
Correlation coefficient (r) = +0.9. Positive linear relationship.

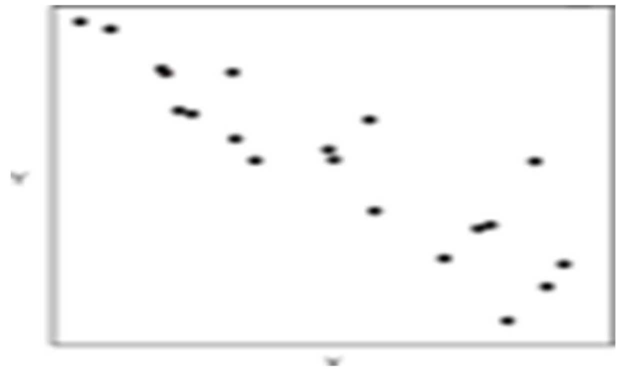


Figure 3
Correlation coefficient (r) = -0.9. Negative linear relationship.

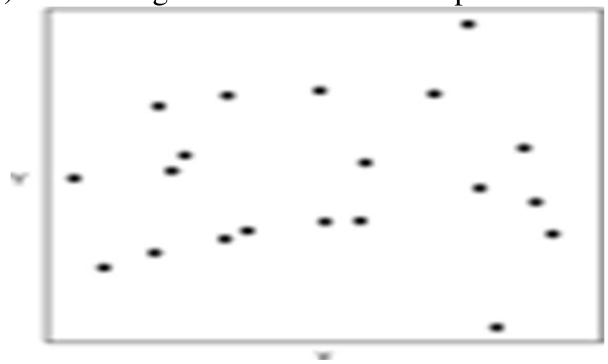


Figure 4
Correlation coefficient (r) = 0.04. No relationship.

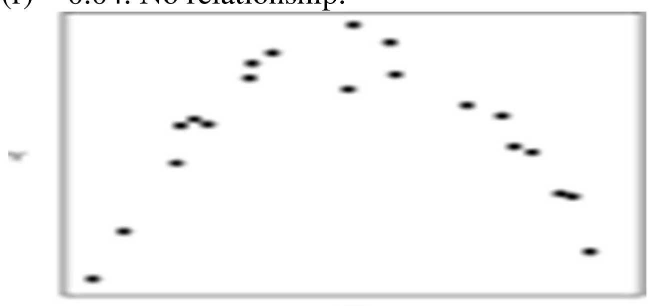


Figure 5
Correlation coefficient (r) = -0.03. Nonlinear relationship.

For the A&E data, the correlation coefficient is 0.62, indicating a moderate positive linear relationship between the two variables.

Activity 7.2

Take a moment to recall the main types of correlation?

Activity 7.2 Feedback: positive correlation and negative correlation.

7.6 Hypothesis test of correlation

We can use the correlation coefficient to test whether there is a linear relationship between the variables in the population as a whole. The null hypothesis is that the population correlation coefficient equals 0. The value of r can be compared with those given in Table Table2, or alternatively exact *P* values can be obtained from most statistical packages. For the A&E data, r = 0.62 with a sample size of 20 is greater than the value highlighted bold in Table Table2 for *P* = 0.01, indicating a *P* value of less than 0.01. Therefore, there is sufficient evidence to suggest that the true population correlation coefficient is not 0 and that there is a linear relationship between ln urea and age.

Table 2

5% and 1% points for the distribution of the correlation coefficient under the null hypothesis that the population correlation is 0 in a two-tailed test

	r values for two-tailed probabilities (*P*)			Two-tailed probabilities (*P*)	
Sample size	0.05	0.01	Sample size	0.05	0.01
3	1.00	1.00	23	0.41	0.53
4	0.95	0.99	24	0.40	0.52
5	0.88	0.96	25	0.40	0.51
6	0.81	0.92	26	0.39	0.50
7	0.75	0.87	27	0.38	0.49
8	0.71	0.83	28	0.37	0.48
9	0.67	0.80	29	0.37	0.47
10	0.63	0.76	30	0.36	0.46
[illegible]	0.60	0.73	40	0.31	0.40

Table 2

5% and 1% points for the distribution of the correlation coefficient under the null hypothesis that the population correlation is 0 in a two-tailed test

7.7 Confidence interval for the population correlation coefficient

Although the hypothesis test indicates whether there is a linear relationship, it gives no indication of the strength of that relationship. This additional information can be obtained from a confidence interval for the population correlation coefficient.

To calculate a confidence interval, r must be transformed to give a Normal distribution making use of Fisher's z transformation [2]:

$$z_r = \frac{1}{2}\log_e\left(\frac{1+r}{1-r}\right)$$

The standard error [3] of z_r is approximately:

$$\frac{1}{\sqrt{n-3}}$$

and hence a 95% confidence interval for the true population value for the transformed correlation coefficient z_r is given by z_r - (1.96 × standard error) to z_r + (1.96 × standard error). Because z_r is Normally distributed, 1.96 deviations from the statistic will give a 95% confidence interval.

For the A&E data the transformed correlation coefficient z_r between ln urea and age is:

$$\frac{1}{2}\log_e\left(\frac{1+0.62}{1-0.62}\right) = 0.725$$

The standard error of z_r is:

$$\frac{1}{\sqrt{20-3}} = 0.242$$

The 95% confidence interval for z_r is therefore 0.725 - (1.96 × 0.242) to 0.725 + (1.96 × 0.242), giving 0.251 to 1.199.

We must use the inverse of Fisher's transformation on the lower and upper limits of this confidence interval to obtain the 95% confidence interval for the correlation coefficient. The lower limit is:

$$\frac{e^{2\times0.251}-1}{e^{2\times0.251}+1}$$

giving 0.25 and the upper limit is:

$$\frac{e^{2\times1.199}-1}{e^{2\times1.199}+1}$$

giving 0.83. Therefore, we are 95% confident that the population correlation coefficient is between 0.25 and 0.83.

The width of the confidence interval clearly depends on the sample size, and therefore it is possible to calculate the sample size required for a given level of accuracy.

7.8 Misuse of correlation

There are a number of common situations in which the correlation coefficient can be misinterpreted.

One of the most common errors in interpreting the correlation coefficient is failure to consider that there may be a third variable related to both of the variables being investigated, which is responsible for the apparent correlation. Correlation does not imply causation. To strengthen the case for causality, consideration must be given to other possible underlying variables and to whether the relationship holds in other populations.

A nonlinear relationship may exist between two variables that would be inadequately described, or possibly even undetected, by the correlation coefficient.

A data set may sometimes comprise distinct subgroups, for example males and females. This could result in clusters of points leading to an inflated correlation coefficient (Fig.6). A single outlier may produce the same sort of effect.

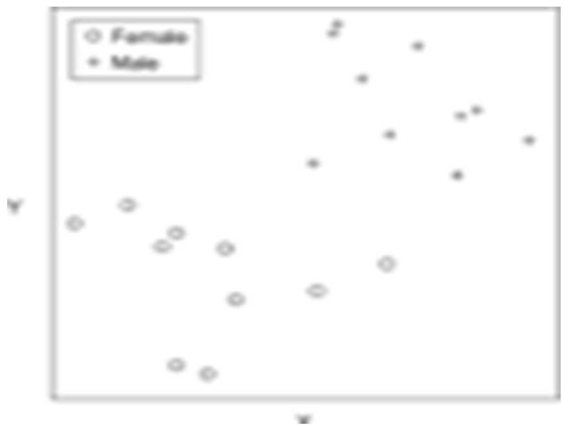


Figure 6
Subgroups in the data resulting in a misleading correlation. All data: r = 0.57; males: r = -0.41; females: r = -0.26.

It is important that the values of one variable are not determined in advance or restricted to a certain range. This may lead to an invalid estimate of the true correlation coefficient because the subjects are not a random sample.

Another situation in which a correlation coefficient is sometimes misinterpreted is when comparing two methods of measurement. A high correlation can be incorrectly taken to mean that there is agreement between the two methods. An analysis that investigates the differences between pairs of observations, such as that formulated by Bland and Altman [5], is more appropriate.

Activity 7.3

Take a moment to reflect on misuse of correlation and form a note

Activity 7.3 Feedback

Compare your points with the one in section 7.8

7.8 The Application of Correlation

Correlational studies can and have been used to investigate almost the entire range of problems of psychological and educational interest. It is used to answer such questions in developmental psychology. Is the child's performance in school related to the social economic status of his or her family?

Is there a relationship between late toilet training and compulsiveness in adulthood? Is the frequency of dating behaviour during adolescence related to later marital success and happiness? Another area of interest that has been studied using the correlational method is the, relationship between Down's syndrome (Mongolism) and the age of the mother giving birth to a retarded child.

You should note however that correlation does not in any way mean causality. If two variables are correlated with each other, there is a tendency to conclude that one variable causes the other. On the strength of a correlation between the amount of TV violence and real-life violence, some people concluded that TV violence causes people to be more violent in their everyday behaviour, but we cannot draw such a causal conclusion from correlations. For there may be a third factor, for example, the increased stress of modern living, causes both TV and real-life violence.

Thus correlational analysis finds application in:

(a) Dealing with causality problems when experimental methods are inappropriate.

(b) A descriptive and predictive capacity, for example, personality traits and sociological characteristics associated with suicide cannot be studied with the experimental approach. After a person commits suicide, the researcher can only look at the historical records of the person's life.

7.9 Regression

In the A&E example we are interested in the effect of age (the predictor or x variable) on ln urea (the response or y variable). We want to estimate the underlying linear relationship so that we can predict ln urea (and hence urea) for a given age. Regression can be used to find the equation of this line. This line is usually referred to as the regression line.

Note that in a scatter diagram the response variable is always plotted on the vertical (y) axis.

7.10 Equation of a straight line

The equation of a straight line is given by $y = a + bx$, where the coefficients a and b are the intercept of the line on the y axis and the gradient, respectively. The equation of the regression line for the A&E data (Fig. 7) is as follows: ln urea = 0.72 + (0.017 × age) (calculated using the method of least squares, which is described below). The gradient of this line is 0.017, which indicates that for an increase of 1 year in age the expected increase in ln urea is 0.017 units (and hence the expected increase in urea is 1.02 mmol/l). The predicted ln urea of a patient aged 60 years, for example, is 0.72 + (0.017 × 60) = 1.74 units. This transforms to a urea level of $e^{1.74}$ = 5.70 mmol/l. The y intercept is 0.72, meaning that if the line were projected back to age = 0, then the ln urea value would be 0.72. However, this is not a meaningful value because age = 0 is a long way outside the range of the data and therefore there is no reason to believe that the straight line would still be appropriate.

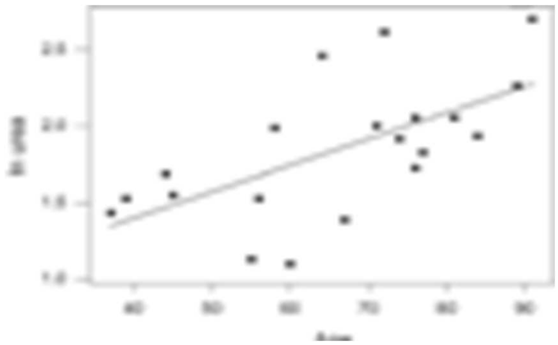


Figure 7
Regression line for ln urea and age: ln urea = 0.72 + (0.017 × age).

Activity 7.4

Take a moment and recall the formula for simple linear regression.

Activity 7.4 Feedback: y = a + bx.

7.11 Method of least squares

The regression line is obtained using the method of least squares. Any line y = a + bx that we draw through the points gives a predicted or fitted value of y for each value of x in the data set. For a particular value of x the vertical difference between the observed and fitted value of y is known as the deviation, or residual (Fig.8). The method of least squares finds the values of a and b that minimise the sum of the squares of all the deviations. This gives the following formulae for calculating a and b:

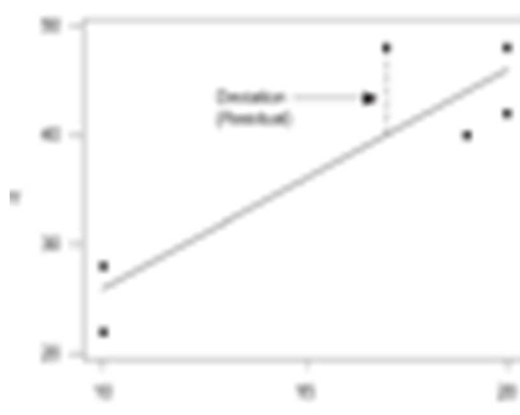


Figure 8
Regression line obtained by minimizing the sums of squares of all of the deviations.

$$a = \bar{y} - b\bar{x} \qquad b = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Usually, these values would be calculated using a statistical package or the statistical functions on a calculator.

7.12 Hypothesis tests and confidence intervals

We can test the null hypotheses that the population intercept and gradient are each equal to 0 using test statistics given by the estimate of the coefficient divided by its standard error.

$$\text{The standard error of the intercept} = s\sqrt{\left[\frac{1}{n} + \frac{\bar{x}^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right]}$$

$$\text{and for the gradient} = \frac{s}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

$$\text{where } s = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2 - b\sum_{i=1}^{n}(x_i - \bar{x})^2}{(n-2)}}$$

The test statistics are compared with the t distribution on n - 2 (sample size - number of regression coefficients) degrees of freedom.

The 95% confidence interval for each of the population coefficients are calculated as follows: coefficient ± (t_{n-2} × the standard error), where t_{n-2} is the 5% point for a t distribution with n - 2 degrees of freedom.

For the A&E data, the output (Table 3) was obtained from a statistical package. The *P* value for the coefficient of ln urea (0.004) gives strong evidence against the null hypothesis, indicating that the population coefficient is not 0 and that there is a linear relationship between ln urea and age. The coefficient of ln urea is the gradient of the regression line and its hypothesis test is equivalent to the test of the population correlation coefficient discussed above. The *P* value for the constant of 0.054 provides insufficient evidence to indicate that the population coefficient is different from 0. Although the intercept is not significant, it is still appropriate to keep it in the equation. There are some situations in which a straight line passing through the origin is known to be appropriate for the data, and in this case a special regression analysis can be carried out that omits the constant.

Table 3

Regression parameter estimates, P values and confidence intervals for the accident and emergency unit data

	Coefficient	Standard error of coefficient	t	P	Confidence interval
Constant, or intercept	0.72	0.346	2.07	0.054	-0.01 to +1.45
ln area	0.017	0.005	3.35	0.004	0.006 to 0.028

Table 3

Regression parameter estimates, *P* values and confidence intervals for the accident and emergency unit data.

Box 7.1: Correlation

It is important to note the following:

- A coefficient of correlation is a single number that tells us to what extent two things are related.
- The correlation coefficient can vary from a value of 1.00, which means perfect positive correlation, through zero, which means complete independence, or no correlation whatever, down to -1.00.
- While correlation is for relationship, regression is for prediction.
- Researcher should avoid misuse of correlation analysis.

Summary of Study Session 7

In Session 7, you have learned that:

Correlation is for determining the co-efficient of relationship between or among two or more variables. It could range from -1 to +1.

Self-Assessment Questions (SAQs) for Session 7

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 7.1 (tests learning outcome 7.1)
Define correlation?
SAQ 7.2 (tests learning outcome 7.2)
What is the range of co-efficient of correlation?
SAQ 7.3 (tests learning outcome 7.3)
Draw a chart to differentiate between positive and negative relationship.

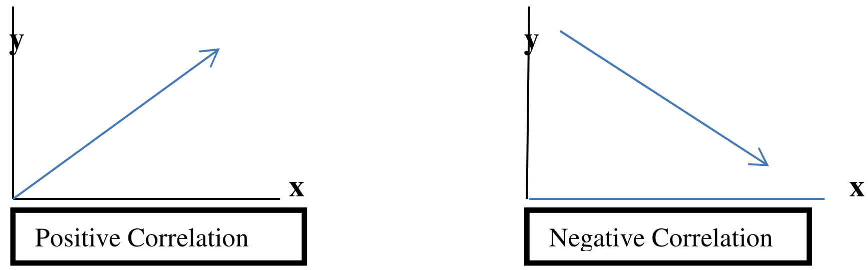
SAQ 7.4 (tests learning outcome 7.4)
State some misuse of correlation

Notes on the Self-Assessment Question

SAQ 7.1: Correlation is the meant for calculating the co-efficient of relationship between or among two or more variables. It could be positive or negative.

SAQ 7.2: The co-efficient is (r) = -1 to +1

SAQ 7.3:



SAQ 7.4: Correlation can be misused when: (1) failure to consider that there may be a third variable related to both of the variables being investigated; (2) wrongful assumption of linear relationship, when the actual relationship is non-linear, and so on.

References

Ferguson, G.A. (1959) *Statistical Analysis in Psychology and Education*. New York: McGraw—Hill Book Company, Inc.

McCall B. Robert. (1959) *Fundamental Statistics for Psychology*, New York: Harcourt Brace Jovanovich, Inc., 2nd Edition.

Study Session 8: The Principles of Probability

Expected duration: 1 week or 2 contact hours

Introduction

The term probability may be used subjectively to refer to an attitude of doubt with respect to some future event as to possibility of occurring or not occurring. For example, the assertions may be made that, "It will probably rain tomorrow", or "The probability is small that you shall live to be 90 years old", or "There is high probability that a particular football club will win in the league season", The other usage defines the probability of an event as the ratio of the number of favourable cases to the total number of equally likely cases. This usage stems from a consideration of games of chance involving cards, dice, and coins. For example, on examining the structure of a die the assertion may be made that no basis exists for choosing one of the six alternatives in preference to another; consequently all six alternatives may be considered equally likely. The probability of throwing a particular result, say, a 3, in a single toss is then 1/6, there being one favourable case among six equally likely alternatives. More of the usages of the term probability will be brought to you in the subsequent sub-units of this lecture.
In rolling a die, what is the probability of obtaining either a 5 or a 6?

- What is the probability that you are going to pass this course?

- Your answer should be: 1/2.

Learning Outcomes for Session 8

When you have studied this session, you should be able to:

8.1. Discuss the term probability. (SAQ 8.1)
8.2. Recall the mathematical notation for probability (SAQ 8.2)
8.3. State the theorems that are in probability and the word that indicate each of them (SAQ 8.3)
8.4. Calculate probability based on addictive theorem (SAQ8.4)
8.5. Calculate probability based on multiplication theorem (SAQ8.5)
8.6. Calculate probability based on both additive and multiplication theorems (SAQ8.6)

8.0 The Principles of Probability

8.1. What Is Probability?

The notion of "the probability of something" is one of those ideas, like "point" and "time," that we can't define exactly, but that are useful nonetheless. The following should give a good working understanding of the concept.

Activity 8.1

Take a moment to reflect on what you have read. What word can be synonymous to 'probability'?

Activity 8.1 Feedback: The word is "Chance"

8.2. Events

First, some related terminology: The "some things" that we consider the probabilities of are usually called *events*. For example, we may talk about the event that the number showing on a die we have rolled is 5; or the event that it will rain tomorrow; or the event that someone in a certain group will contract a certain disease within the next five years.

8.3. Four Perspectives on Probability

Four perspectives on probability are commonly used: *Classical, Empirical, Subjective, and Axiomatic*.

1. Classical (sometimes called "A priori" or "Theoretical")

This is the perspective on probability that most people first encounter in formal education (although they may encounter the subjective perspective in informal education). For example, suppose we consider tossing a fair die. There are six possible numbers that could come up ("outcomes"), and, since the die is fair, each one is equally likely to occur. So we say each of these outcomes has probability 1/6. Since the event "an odd number comes up" consists of exactly three of these basic outcomes, we say the probability of "odd" is 3/6, i.e.1/2.

More generally, if we have a situation (a "random process") in which there are n equally likely outcomes, and the event A consists of exactly m of these outcomes, we say that the probability of A is m/n. We may write this as "P(A) = m/n" for short.
This perspective has the advantage that it is conceptually simple for many situations. However, it is limited, since many situations do not have finitely many equally likely outcomes. Tossing a weighted die is an example where we have finitely many outcomes, but they are not equally likely. Studying people's incomes over time would be a situation where we need to consider infinitely many possible outcomes, since there is no way to say what a maximum possible income would be, especially if we are interested in the future.

2. Empirical (sometimes called "A posteriori" or "Frequentist")

This perspective defines probability via a thought experiment. To get the idea, suppose that we have a die which we are told is weighted, but we don't know how it is weighted. We could get a rough idea of the probability of each outcome by tossing the die a large number of times and using the proportion of times that the die gives that outcome to estimate the probability of that outcome.

This idea is formalized to define the probability of the event A as P(A) = the limit as n approaches infinity of m/n, where n is the number of times the process (e.g., tossing the die) is performed, and m is the number of times the outcome A happens.
(*Notice that m and n stand for different things in this definition from what they meant in Perspective1.*)

In other words, imagine tossing the die 100 times, 1000 times, 10,000 times, ... Each time we expect to get a better and better approximation to the true probability of the event A. The mathematical way of describing this is that the true probability is the limit of the approximations, as the number of tosses "approaches infinity" (that just means that the number of tosses gets bigger and bigger indefinitely).

This view of probability generalizes the first view: If we indeed have a fair die, we expect that the number we will get from this definition is the same as we will get from the first definition (e.g., P(getting 1) = 1/6; P(getting an odd number) = 1/2). In addition, this second definition also works for cases when outcomes are not equally likely, such as the weighted die. It also works in cases where it doesn't make sense to talk about the probability of an individual outcome. For example, we may consider randomly picking a positive integer (1, 2, 3, ...) and ask, "What is the probability that the number we pick is odd?" Intuitively, the answer should be 1/2, since every other integer (when counted in order) is odd. To apply this definition, we consider randomly picking 100 integers, then 1000 integers, then 10,000 integers, ... Each time we calculate what fraction of these chosen integers are odd. The resulting sequence of fractions should give better and better approximations to 1/2.

However, the empirical perspective does have some disadvantages. First, it involves a thought experiment. In some cases, the experiment could never in practice be carried out more than once. Consider, for example the probability that the Dow Jones average will go up tomorrow. There is only one today and one tomorrow. Going from today to tomorrow is not at all like rolling a die. We can only imagine all possibilities of going from today to a tomorrow (whatever that means). We can't actually get an approximation.

A second disadvantage of the empirical perspective is that it leaves open the question of how large n has to be before we get a good approximation. The example linked above shows that, as n increases, we may have some wobbling away from the true value, followed by some wobbling back toward it, so it's not even a steady process.

The empirical view of probability is the one that is used in most statistical inference procedures. These are called frequentist statistics. The frequentist view is what gives credibility to standard estimates based on sampling. For example, if we choose a large enough random sample from a population (for example, if we randomly choose a *sample* of 1000 students from the *population* of all 50,000 students enrolled in the university), then the average of some measurement (for example, college expenses) for the *sample* is a reasonable estimate of the average for the *population.*

3. Subjective

Subjective probability is an individual person's measure of belief that an event will occur. With this view of probability, it makes perfectly good sense intuitively to talk about the probability that the Dow Jones average will go up tomorrow. You can quite rationally take your subjective view to agree with the classical or empirical views when they apply, so the subjective perspective can be taken as an expansion of these other views.

However, subjective probability also has its downsides. First, since it is subjective, one person's probability (e.g., that the Dow Jones will go up tomorrow) may differ from another's. This is disturbing to many people. Still, it models the reality that often people do differ in their judgments of probability. The second downside is that subjective probabilities must obey certain "coherence" (consistency) conditions in order to be workable. For example, if you believe that the probability that the John will go up tomorrow is 60%, then to be consistent you cannot believe that the probability that the John will go *down* tomorrow is also 60%. It is easy to fall into subjective probabilities that are not coherent.

The subjective perspective of probability fits well with Bayesian statistics, which are an alternative to the more common frequentist statistical methods. (This website will mainly focus on frequentist statistics.)

4. Axiomatic

This is a unifying perspective. The coherence conditions needed for subjective probability can be proved to hold for the classical and empirical definitions. The axiomatic perspective codifies these coherence conditions, so can be used with any of the above three perspectives.

The axiomatic perspective says that probability is any function (we'll call it P) from events to numbers satisfying the three conditions (axioms) below. (Just what constitutes events will depend on the situation where probability is being used.)

The three axioms of probability:

I. $0 \leq P(E) \leq 1$ for every allowable event E. (In other words, 0 is the smallest allowable probability and 1 is the largest allowable probability).
II. The certain event has probability 1. (The *certain event* is the event "some outcome occurs." For example, in rolling a die, the certain event is "One of 1, 2, 3, 4, 5, 6 comes up." In considering the stock market, the certain event is "The Dow Jones either goes up or goes down or stays the same.")

III. The probability of the union of mutually exclusive events is the sum of the probabilities of the individual events. (Two events are called *mutually exclusive* if they cannot both occur simultaneously. For example, the events "the die comes up 1" and "the die comes up 4" are mutually exclusive, assuming we are talking about the same toss of the same die. The *union* of events is the event that at least one of the events occurs. For example, if E is the event "a 1 comes up on the die" and F is the event "an even number comes up on the die," then the union of E and F is the event "the number that comes up on the die is either 1 or even."

If we have a fair die, the axioms of probability *require* that each number comes up with probability 1/6: Since the die is fair, each number comes up with the same probability. Since the outcomes "1 comes up," "2 comes up," ..."6 come up" are mutually exclusive and their union is the certain event, Axiom III says that P(1 comes up) + P(2 comes up) + ... + P(6 comes up) = P(the certain event), which is 1 (by Axiom 2). Since all six probabilities on the left are equal, that common probability must be 1/6.

Activity 8.3

Take a moment to write about 4 sentences on each of the perspective discussed above..

Activity 8.3 Feedback:

The four perspectives are: Classical, Empirical, Subjective, and Axiomatic. (Form two sentences on each)

8.4. Some Mathematical Theorems about Probability

Mathematically, a probability is symbolized by "**p**", which may range from zero, when there is no chance whatever of the favoured event, to 1.0, when there is absolute certainty; nothing else could happen. It is very well known that if you should toss a coin a finite number of times, the ratio of the number of heads to the number of tosses would probably not come out exactly ½. For a definition of probability that is more fully in accord with the outcomes of events, therefore, you need the following modification, although it is not always explicitly stated

$$p = \frac{nf}{n}$$

$$n \rightarrow 00$$

This expression states that as the number of times (n) the event occurs becomes indefinitely large, the ratio of the number of favoured outcomes (nf) to n approaches the probability p as a limit. It is also necessary to stress two requirements: that the ways of occurrence of the event must be equally likely and that they must be mutually independent. "Mutual independence" means that one event has no effect whatsoever on any other event. Granting unbiased coins, dice, and cards, the occurrences of heads and tails are equally likely; and in drawing a card from a well-shuffled deck, any card has as much chance of being drawn as any other.

- o Biola asked you whether it is possible to have negative probability, what would be your response?

- Your answer should be: No. P ranges between 0 and 1.

8.5. The Addition Theorem

Other kinds of questions may be asked regarding events in games of chance. In tossing a die, what is the probability of either a 1 or 2 coming up? The probability of a 1 is 1/6, and the probability of a 2 is 1/6. There are two ways in which the favoured event can occur, out of a total of 6 ways; therefore, by definition, the probability is 2/6. Note that this probability is the sum of two separate probabilities. Thus, probabilities are additive.

You may ask what the probability is of obtaining four or more spots.

The specification "four or more" includes the outcomes 4, 5 and 6. Adding the three probabilities, we have 3/6 or Y2 as the probability of the alternatives outcomes. In tossing a coin, what is the probability of getting a head or a tail? The addition 1/2 + 1/2 gives you 1.00, which means that you are certain to obtain either a head or a tail. The same principle you can apply to drawing cards. The two separate probabilities are 1/4 and 6/4 which, summed give 1/2. What is the probability of drawing a queen or a king? The answer is 1/13 + 1/13. In general, the probability of alternative outcomes is the sum of the probabilities of the outcomes taken separately.

Activity 8.3

Take a moment to reflect on what you have read. What is the probability that a bird will be dead or alive?

Activity 8.3 Feedback: alive = ½; dead = ½; dead or alive = ½ + ½ = 1.

- o What word connotes additive theorem?

- It is "OR".

8.6. The Multiplication Theorem

You have read and treated either kinds of case. Here you will be concerned with "this and that" kind of case the probability of combined outcomes. In two independent throws of a coin, what is the probability that, the outcome will be two heads (a head and a head)? It will help you if you can go back to the basic definition of probability, the relative frequency of the outcome, the ratio of the number of favoured ways to the total number of ways. In tossing two coins (either two different coins simultaneously or one coin in succession two times), 'what is the total number of ways in which to get the outcome? If all these are written out, you will have the following: HH HT TH and TT. That is, when the first is a head, the second can be either a head or a tail, and when the first is a tail, the second can be either a head or a tail. There are four total ways and only one favoured way (HH), and so the probability is 1/4. Here the probability that the first event will come out H is 1/2, and the probability that the second event will come out H is 1/2.

And the probability of the combination of the two outcomes is 1/2 + 1/2, or 1/4. The probability of each of the other outcomes is also 1/4, and the four add up to 1 .0, as they should. Thus you are certain to have one of the four.

Another question can be stated: What is the probability of getting one (and only one) head? This specification describes the HT and TH cases, of which these are two. The addition theorem applies. The probability for this is 2/4, or 1/2. What is the probability of obtaining at least one head? This statement also includes the case HH, and so the probability is 3/4.

Activity 8.4

Take a moment to reflect on what you have read. What is the probability that a boy will pick letter B and N in the word IBADAN

Activity 8.4 Feedback: B = 1/6; N = 1/6; B and N = 1/6 * 1/6 = 1/36.

- o Suggest a word that connotes additive theorem?

- It is “AND”.

Box 8.1: The Principles of Probability

It is important to note the following:

- Probability has to do with chance that an event will occur or not.
- Probability (P) is calculated by possible event (e.g. Head of a coin) divide by total possible outcome (Two sides of a coin).
- There are four perspectives on probability. These are: Classical, Empirical, Subjective, and Axiomatic.
- We have additive (OR) and multiplication (AND) theorems.

Summary of Study Session 8

In Session 8, you have learned that:

- Probability may be considered as the relative frequency or number of chances an event will occur or not in the population, and thus it is a population parameter.
- Four perspectives on probability are: Classical, Empirical, Subjective, and Axiomatic
- The two theorems about probability are, the addition theorem, and the multiplication theorem.
- You cannot have probability below 0 or above 1.

Self-Assessment Questions (SAQs) for Session 8

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 8.1 (tests learning outcome 8.1)
Define probability?
SAQ 8.2 (tests learning outcome 8.2)
What is the mathematical notation for probability?
SAQ 8.3 (tests learning outcome 8.3)
State the theorems that are in probability and the word that indicate each of them?
SAQ 8.4 (tests learning outcome 8.4)
In rolling a die, what is the probability of obtaining either a 5 or a 6?
(a) 2/3 (b) 1/3 (c) 1/6 (d) 2/4 (e) 5/6
SAQ 8.5 (tests learning outcome 8.5)
In tossing four coins, what is the probability of obtaining the sequence H T H T?
(a) 4/24 (b) 3/16 (c) 1/16 (d) 2/16 (e) 8/24
SAQ 8.6 (tests learning outcome 8.6)
In tossing four coins, what is the probability of obtaining four heads or three tails?
(a) 4/16 (b) 5/16 (c) 3/16 (d) 10/16 (e) 5/36

Notes on the Self-Assessment Question

SAQ 8.1: Probability is the chance that an event will occur or not occur out of total possible outcome.

SAQ 8.2: $p = \frac{nf}{n}$ (where *nf* = number an event occur or not; *n* = total possible outcomes)

SAQ 8.3: (1) Addictive theorem; signifies by the word OR
(2) Multiplication theorem; signifies by the word AND.

SAQ 8.4: p (5) = 1/6; p (6) = 1/6); p (5 or 6) = 1/6 +1/6 = 2/6 = 1/3.

SAQ 8.5: p (H) = ½; p (T) = ½; p (HTHT) = 1/2*1/2*1/2*1/2 = 1/16.

SAQ 8.6: p (H) = ½; p (T) = ½; p (HHHH) = 1/2*1/2*1/2*1/2 = 1/16;
p (TTT) = 1/2*1/2*1/2 = 1/8; p (HHHH or TTT) = 1/16 + 1/8 = 3/16

References

Ferguson, G.A., (1959) *Statistical Analysis in Psychology and Education,* New York: McGraw-Hill Book Company, Inc..

McCall B. Robert, (1975) *Fundamental Statistics for psychology Harcourt* New York: Brace Jovanovich, Inc., 2nd Edition.

Study Session 9: Set Theory

Expected duration: 1 week or 2 contact hours

Introduction

Session 1 and 7 of this course have been concerned with descriptive statistics that is, the procedures that describe and summarize groups of measurements. In study session and in this present session, however, attention is now turned to inferential statistics which includes techniques for making inferential decisions when someone has only partial information. The crux of inferential statistics is the probability and set theory; probability has been dealt with already in study session 8.

Learning Outcomes for Session 9

When you have studied this session, you should be able to:

9.1. Define set. (SAQ 9.1)
9.2. Mention types of set (SAQ 9.2)
9.3. Use Combination method effectively (SAQ 9.3)

9.1 Set Theory

9.1 .1 Sets Theory and Relations among Sets

A Set is a collection of distinct and defined elements. Sets are represented by using French braces { } with commas to separate the elements in a Set.

There are various types of sets as follows:

- Finite Set
- Infinite Set
- Empty or Null Set
- Equal Sets
- Disjoint Sets
- Intersecting Sets

Activity 9.1

Take a moment to reflect on what you have read. Then what is a set and mention its types?

Activity 9.1 Feedback: A set is a collection of distinct and defined elements. Types are: Finite Set, Infinite Set, Empty or Null Set, Equal Sets, Disjoint Sets and Intersecting Sets.

Examples of Sets

- The set of all points on a particular line.
- The set of all lines in a particular plane.
- A set can also contain elements which are themselves sets. For example, a set may contain 4, 5, {6, 7}.

  - Tell your friend the set of a coin?

- Your answer should be: {1,2,3,4,5,6}.

Set Operations

There are three basic Set Operations.

- Intersection of Sets
- Union of Sets
- Complement of Sets

Properties on Operations of Sets

$A \subset B, B \subset C \Rightarrow A \subset C$ (Property of Transitivity)

$A \subset B, B \subset A \Rightarrow A = B$

$A \cup A = A$

$A \cap A = \emptyset$

$A \cup \emptyset = A$

$A \cap \emptyset = \emptyset$

$A \cup B = B \cup A$ (Commutative law for addition)

$A \cap B = B \cap A$ (Commutative law for multiplication)

$(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law for addition)

$(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law for multiplication)

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law for addition)

$A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$ (Distributive law for multiplication)

Set Basic Concept

If S is any set, every object in S is called an element of the set. For example, let S be the set of all natural numbers less than 100. Then, S = {1, 2, 3, 4, ..., 10, 11, ...97, 98, 99}

10 is an element of the set. Also, 36 is an element of the set. The fact that 10 is an element of the set expressed in symbols as $10 \in S$ which is read as "10 belongs to S" or "10 is an element of S".

150 is not an element of the set S. We represent it as $150 \notin S$ which is read as "150 does not belong to S" or "150 is not an element of S".

The elements of a set are generally denoted by small letters a, b, c... x ,y, z. The sets are denoted by capital letters A, B, C ...., X, Y, Z.

In general,

- If an element x is in set A, then we say x belongs to A and we write $x \in A$.
- If an element x is not in set A, then we say x does not belongs to A and we write $x \notin A$.

- Tell your friend which of this is true: 12 ∉ numerals or 12 ∈ numerals?

- 12 ∈ numerals is true because 12 belongs to numerals.

9.2 Types of Sets

There are various types of sets:

- Finite Set
- Infinite Set
- Null Set
- Singleton Set
- Equivalent Sets
- Equal Sets

9.2.1 Finite Set

A set is said to be finite if it contains only finite number of elements.

Let A={1,2,3,4,5} and B={a,b,c,d,e,f}
Here, A contains 5 elements and B contains 6 elements. So, A and B are finite sets

Examples of Finite Set

The set {2, 4, 6} Is a finite set, as it contains only 3 elements.

- If A is the set of all days in a week, then A is a finite set containing 7 elements.

9.2.3 Infinite Set

A set is said to be infinite if it contains an infinite number of elements.

Let C = {number of men living in different parts of the world}
It is difficult to find the number of elements in C. But, it is a definite number, may be quite a big number. And so, C is an infinite set.

Examples of Infinite Set

- The set of all even numbers is an infinite set.
- The set of points on a particular straight line is an infinite set.
- The sets N (Natural Numbers), Z (Integers), Q (Rational Numbers), R (Real Numbers) and C(Complex Numbers) are all infinite sets, where:
    1. N = { 1, 2, 3, 4, ...}
    2. Z = {............., -3, -2, -1, 0, 1, 2, 3, ...}
    3. C = { x + iy : x, y in R, i = −1−−−√}

- What kind of set is star?

- Your answer should be: infinite set.

9.2.4 Null Set

A set is said to be a null set if it does not contain any element. A null set is also called as an empty set or void set. A null set is denoted by Φ.

Therefore, Φ = { }
The set { 0 } is not a null set, because this set contains one element "0".

Examples of Null Set

- Let A = { x : x ∈ N, 2 < x <3}. A does not contain any element , because there is no natural number between 2 and 3.
- Let B = { x : x ∈ Q, 2 < x <3}. B is not a null set, because rational numbers like 52

  - Suggest a null set to your friend?

- Your answer may be: (1) men that breast feed etc.

9.2.5 Singleton Set

A set is said to be a singleton set, if it contains only one element.

Examples of the Singleton Set

- The set {7} , {-15} are singleton sets.
- {x : x + 4 = 0, x ∈ Z} is a singleton set, because this set contains only one integer namely, -4.

9.2.6 Equivalent Sets

Two sets A and B are said to be equivalent sets if the elements of A can be paired with the elements of B, so that each element of A corresponds exactly to one element of B, and each element of B there corresponds exactly to one element of A.

Examples of Equivalent Sets

- The sets {a, b, c} and {4, 7, 10} are equivalent.
- The sets {w, x, y, z} and {1, 2, 3, 4} are equivalent.

9.2.7 Equal Sets

Two sets are said to be equal sets if every element of one set is in the other set and vice-versa. So, two sets are equal, if x in A ⇒ x in B and x ∈ B ⇒ x ∈ A. If sets A and B are not equal, then we write A ≠ B

Examples of Equal Sets

- Let A = {x : x ∈ N, 2 ≤ x ≤ 6 } and B = { 2, 3, 4, 5, 6} , then A = B
- Let A = {x : x ∈ N, 10< x<11} and B = {10.5} , then A ≠ B since 10.5 ∉ A

Activity 9.3

Are these sets equal: A = {x : x ∈ N, 10< x<11} and B = {10.5}.

Activity 2.1 Feedback: NO. A ≠ B since 10.5 ∉ A.

9.3. Intersection of Sets

The Intersection of two or more sets is a set containing only the common elements among all the sets under consideration. The intersection operation is denoted by the symbol ∩. The Intersection of the sets A and B is expressed as A ∩ B.

Examples on Intersection of Sets

Given below are some examples that explain the intersection of sets.

Example 1:

If A = {Cat, Dog, Mouse, Lion, Tiger} and B = {Cat, Lion, Elephant, Tiger}, Find A ∩ B.

Solution:

A = {Cat, Dog, Mouse, Lion, Tiger}
B = {Cat, Lion, Elephant, Tiger}

A ∩ B = {Cat, Lion, Tiger}

Example 2:

If A = {1, 2, 3, 4, 5, 6, 7, 8} and B = {1, 3, 5, 7, 9}, Find A ∩ B.

Solution:

A = {1, 2, 3, 4, 5, 6, 7, 8}
B = {1, 3, 5, 7, 9}

A ∩ B = {1, 3, 5, 7}

Union of Sets
The Union of two or more sets is a set that contains all the elements of all the sets under consideration. The union of sets is denoted by the symbol ∪. The Union of sets C and D is expressed by C ∪ D.

Examples on the Union of Sets

Given below are some of the examples that explain the union of sets.

Example 1:

If A = {Cat, Dog, Mouse, Lion, Tiger} and B = {Cat, Lion, Elephant, Tiger}, Find A ∪ B.

Solution:

A = {Cat, Dog, Mouse, Lion, Tiger}
B = {Cat, Lion, Elephant, Tiger}

A ∪ B = {Cat, Dog, Mouse, Lion, Tiger, Elephant}

Example 2:

If A = {1, 2, 3, 4, 5, 6, 7, 8} and B = {1, 3, 5, 7, 9}, Find A ∪ B.

Solution:

A ={1, 2, 3, 4, 5, 6, 7, 8}
B = {1, 3, 5, 7, 9}

A ∩ B = {1, 2, 3, 4, 5, 6, 7, 8, 9}
Complement of Sets
The Complement of a set is a set that contains only the elements which are not in the given set but are contained in the universal set. The Complement of a set is denoted by A'.

- The Complement of an Universal Set is a Null Set. That is, U' = ∅
- The Complement of a Null Set is an Universal Set. That is, ∅' = U

Examples on the Complement of Sets

Given below are some examples that explain the complement of sets.

Example 1:

If U = {Red, Blue, Green, Yellow, Violet} and A = {Blue, Yellow}, Find complement of A.

Solution:

The Complement of A will have those elements which are not in A but are in U.

So, A' = {Red, Green, Violet}

Example 2:

If U = {1, 2, 3, 4, 5, 6, 7, 8} and B = {1, 3, 5, 7}, Find A'.

Solution:

U = {1, 2, 3, 4, 5, 6, 7, 8}
A = {1, 3, 5, 7}

A' = {2, 4, 6, 8}
Cardinal Number of the Set
Cardinality is a measure of a size of a set or simply the total number of elements in a set.

- o Johnson does not know the complement of a nuclear family, with father husband and wife given. Suggest possible answer.

- Your answer should be: child or children.

9.4. Examples of Cardinal Number of a Set

- Cardinality of a set of vowels is 5 as it has 5 elements (a, e ,i , o, u).
- Cardinality of a Set of the number of months in a year is 12 as it has 12 elements (January, February, ..., December).

Notation: Cardinality of a set is denoted by |A| or #A. The Cardinality of a set of natural numbers is denoted by N_0 and that of the Real numbers is C.

Properties of Cardinal number or Cardinality of Sets

- Two Sets have the same Cardinal number if the function from A to B is Bijective. That is, injective (one-one) as well as surjective (onto). That is |A| = |B|.

Example: If A = {0, 1, 2, 3,........} and B = {0, 5, 10,.........}, then the function can be defined as, f(a) = b (where, a and b represents the elements of Sets A and B). So, the function is Bijective. Hence, |A| = |B|.

- Cardinal number of a set A is less than or equal to that of set B, if the function from A to B is injective but not surjective. That is, |A| ≤ |B|.

Example: If A = {1, 2, 3, 4, 5, ...} and B = {0, 1, 2, 3, 4, ...} and if a function is defined from A to B such that f(a) = a, then the element 0 of Set B does not have a pre-image in A. So, the function is Injective but not Surjective. Hence, |A| ≤ |B|.

Cardinality helps in determining the types of Sets.

- Any set is a finite set, if its cardinality is less than that of natural number. That is, |A| ≤ |N|
- Any set is a countably infinite set, if its cardinality is equal to that of the natural number. That is, |A| = |N|
- Any set is an uncountable set, if its cardinality is greater than that of the natural number. That is, |A| ≥ |N|

Set Theory Examples
Given below are some solved examples in the set theory.

Example 1:

If U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10},
A = {2, 4, 6, 8, 10}, B = (1, 3, 6, 7, 8} and
C = {3, 7},
find A ∩ B, A ∪ C, B ∩ A′, B ∩ C′

Solution:

U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A ={2, 4, 6, 8, 10}
B =(1, 3, 6, 7, 8}
C ={3,A7}

A ∩ B = {6, 8}
A ∪ C = {2, 3, 4, 6, 7, 8, 10}
B ∩ A′ = {1, 3, 7}
B ∩ C′ = {1, 6, 8}

Example 2:

If U = {Pencil, Pen, Eraser, Notebook}, P = {Pencil, Notebook} and Q = {Pen, Eraser}, find $P \cup Q$, $P \cap Q$, $P \cup Q'$, $P \cap Q'$

Solution:

U = {Pencil, Pen, Eraser, Notebook}
P = {Pencil, Notebook}
Q = {Pen, Eraser}

$P \cup Q$ = {Pencil, Notebook, Pen, Eraser} = U
$P \cap Q = \emptyset$
$P \cup Q'$ = {Pen, Eraser, Pencil, Notebook}
$P \cap Q' = \emptyset$

Example 3:
At a breakfast buffet, 93 people preferred coffee as a beverage, 47 people preferred juice, 25 preferred both coffee and juice. If each person prefers atleast one of the beverages, then how many people visited the buffet?

Solution:

Let A be the set of people who prefer coffee and B be the set of people who prefer juice.

$n(A) = 93$, $n(B) = 47$, $n(A \cap B) = 25$

$n(AUB) = n(A) + n(B) - n(A \cap B)$

Plugging-in all the values,

$n(AUB) = 93 + 47 - 25$
$n(AUB) = 115$

Hence, the number of people who visited the buffet is 115.

Example 4:

In a class of 50 students, 30 speak Spanish, 15 speak both Spanish and English. How many students speak English?

Solution:

Let A be the Set of students who speak Spanish and B be the Set of students who speak English.

n(AUB) = 50, n(A) = 30, n(A ∩ B) = 25
n(AUB) = n(A) + n(B) - n(A ∩ B)

Plugging-in all the values,
50 = 30 + n(B) - 15
n(B) = 35

Hence, the number of students who speak English is 35.

Laws of Set Theory

Given below are some laws of set theory:

1	¬(¬A) = A	Law of Double Complement
2	¬(A ∪ B) = ¬A ∩ ¬B ¬(A ∩ B) = ¬A ∪ ¬B	DeMorgan's Laws
3	A ∪ B = B ∪ A A ∩ B = B ∩ A	Commutative Laws
4	A ∪ (B ∪ C) = (A ∪ B) ∪ C A ∩ (B ∩ C) = (A ∩ B) ∩ C	Associative Laws
5	A ∪ (B ∩ C) = (A ∪ B) ∩ (A ∪ C) A ∩ (B ∪ C) = (A ∩ B) ∪ (A ∩ C)	Distributive Laws
6	A ∪ A = A A ∩ A = A	Idempotent Laws
7	A ∪ ∅ = A A ∩ U = A	Identity Laws
8	A ∪ ¬A = U A ∩ ¬A = ∅	Inverse Laws
9	A ∪ U = U A ∩ ∅ = ∅	Domination Laws
10	A ∪ (A ∩ B) = A A ∩ (A ∪ B) = A	Absorption Laws
11	A - B = A ∩ ¬B	Definition of Set Difference
12	A ∆ B = (A ∪ B) - (A ∩ B)	Definition of Symmetric Difference

Subsets

A Set is called as a subset of another set if all it's elements are contained in another set. The symbol used for the subset is ⊂. The set A is a subset of the set B and is expressed as A ⊂ B. Each non empty set has at least two subsets - Empty Set (or Null Set) and the set itself. The number of subsets of a set is calculated by the formula 2n, where n is the number of elements of the Set.

Empty Set or Null Set is represented by either { } or Φ (read as phi)

- Set A contains all the letters in alphabet, and set B has {a, d, &, u, i}; is set B a subset of set A?

- Your answer should be: NO. Set A does not contain element "&"

Examples on Subsets

Given below are some examples on subsets.

Example 1:

S = {a, b, c}
Here, the number of elements is 3.
So, the total number of Subsets is $2^3 = 8$.
The Subsets are: Φ , {a}, {b}, {c}, {a,b}, {b,c}, {a,c}, {a,b,c}.

Example 2:
E = Φ (an empty set)
Here, the number of elements is 0.
So, the total number of Subsets is $2^0 = 1$.
The only Subset is: Φ

Proper Subset

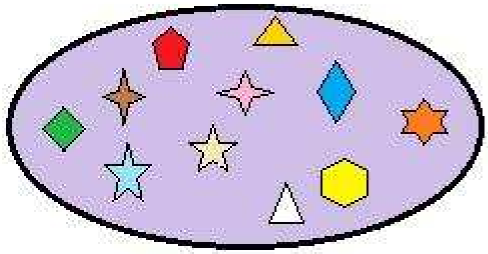
A set A is said to be a proper subset of a set B, if A is a subset of B and A is not equal to B. If A is a proper subset of B, then we write $A \subset B$. If A is a proper subset of B, then B must have at least one element which is not in A.

Examples of Proper Subset

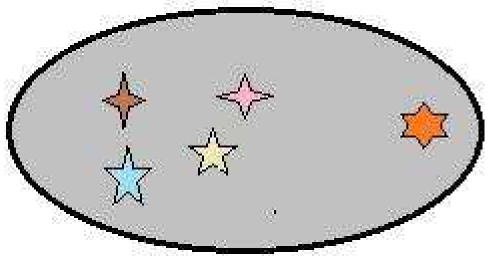
Given below are some examples of proper subset.

Example 1:

Let B be the set of closed objects as shown below:



Let us consider the set A as follows:



Then, clearly every object in A is an object in B. Hence, A ⊂ B

Example 2:

The set of natural numbers, N ⊂ Z, the set of integers, because -2 ∈ Z and -2 ∉ N. If A = {1, 2, 3} , then proper subsets are Φ, {1}, {2}, {3}, {1,2}, {2,3}, {1,3}

If A ⊆ B, then every element of x in A is in B and there is a chance that A may be equal to B, that is, every element of B is in A. But, in case A ⊂ B, then every element of A is in B and there is no chance that A may be equal to B, that is, there will exist at least one element in B which is not in A.

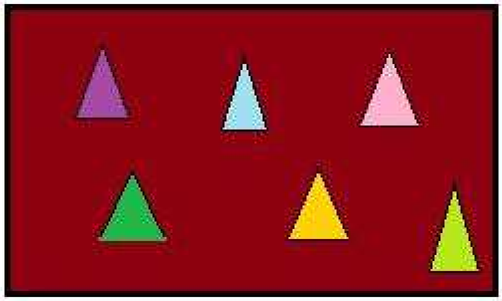
Properties of Proper Subset

- If a set has only one element, then it has two subsets.
- If B sub A and if A has one element more than B, then A has twice as many subsets as B.
- A set with two elements has 2^2 subsets, A set with three elements has 2^3 subsets, and so on. Hence a set with n elements has 2^n subsets.

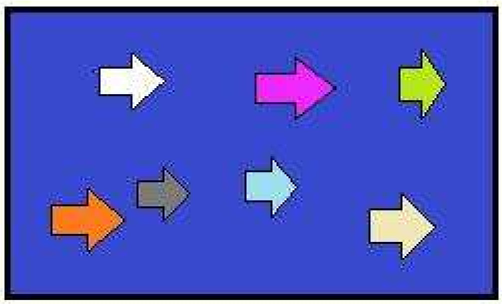
9.5. Disjoint Sets

Two sets A and B are called disjoint sets if there is no element which is both in A and B. Disjoint sets can be also defined as those sets which do not overlap or are not duplicated. Let us consider two sets as follows:

Let A be the set of triangles as shown



Let B be the set of arrows as shown



Here, the set A and B have no elements in common. So, A ∩ B = Φ. Hence, such sets are called as disjoint sets.

Examples of Disjoint Sets:

Given below are some examples of disjoint sets.

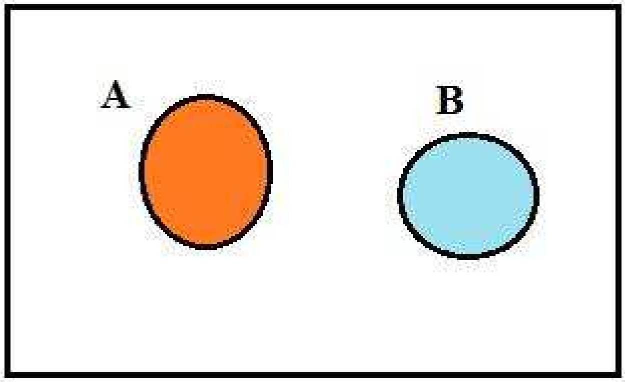
Example 1:

Let A ={ 4, 6, 10} and B = {7, 11, 15}
Since there is no number common between A and B, we get A ∩ B = Φ
So, A and B are disjoint.

Example 2:

The set of intervals { [1, 3], [2,5], (7,9)} is not disjoint, since [1, 3] is overlapping [2,5]. Disjoint sets are also said to be mutually exclusive or independent.

Disjoint sets can be represented by using venn-diagram as:



Activity 9.5

A disjoint set is said to be what?

Activity 9.5 Feedback: Two sets that has no element in common.

Mutually Disjoint Sets

Set A_1, A_2, A_3, ..., A_n are mutually disjoint if $A_i \cap A_j = \Phi$ for $i \neq j$. A partition of set A is a collection of subsets A_i of A such that $A_i \neq \Phi$, A_1, A_2, A_3, ..., A_n are mutually disjoint and $\cup A_i = A$

For example, let the universal set be U = {1, 2, 3, 4, 5}
and let its subsets be A = {1, 2, 3} and B = {4, 5}.
Then, $A \cap B = \Phi$ and $A \cup B = U$. So, A and B are partitions of U

9. 6. Permutations

A permutation of a set of objects or events is an ordered sequence. The number of ordered sequences of r objects which can be selected from a total of n objects is symbolized by

$^{n}p_{r}$

which is read "the number of permutations of n things taken r at a time".

If one has-four objects, for example, A, B, C, D, then ABCD, ACBD, ADBC, ADCB are some of the 24 possible permutations of the four objects taken four at a time. If however, these four objects were taken two at a time, then AB, BA, AC, AD, DA, BC, CB, etc., are some of the 12 permutations of four objects taken two at a time. You should note that the definition states "ordered sequences". That means that AB and BA are two different permutations, that is, order makes a difference.

The number of permutations on n things taken r at a time, $^{n}p_{r}$, equals

$$^{n}p_{r} = \frac{n!}{(n - r)!}$$

This expression involves the symbol n!, reed "n factorial", which is defined as

$n! = n(n-1)(n-2)(n-3)...(1)$

Thus, $5! = (5)(4)(3)(2)(1)\ 120$. Remember that $0! = 1$.

If r = n in the expression for the number of permutations of n things taken n at a time) equals

$$^{n}p_{r} = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n$$

More generally, there are n! permutations of n things (taken n at a time),

$^{n}p_{r} = n!$

Now you are to consider the number of permutations of five things taken only three at a time, that is, $^{n}p_{r}$ = 5P3. There are five ways to fill the first position, four ways to fill the second position, and three ways to fill the third position and that is all. Thus one multiplies

$^{n}p_{r} = n(n-1)(n-2)...(n-r+1)\ (5)(4)(3) = 60$

However, mathematically the same result is arrived at by dividing n! by (n—r)!:

$$^{n}p_{r} = \frac{n!}{(n-n)!} = \frac{(5)(4)(3)\cancel{(2)}\cancel{(1)}}{\cancel{(2)}\ \cancel{(1)}} = (5)(4)(3) = 60$$

Thus, the expression for the number of permutations of n things taken r at a time is

$$^{n}p_{r} = n(n-1)(n-2)...(n-R+1) = \frac{n!}{(n-n)!}$$

Worked Examples: What is the probability of taking first, second, and third in a race involving 8 men? There is only one way to take the three men and assign them to the proper-place (event A) but there are 8P3 ways of ordering three of 8 men. The required probability is

$$P(A) = \frac{(A)}{(S)} = \frac{1}{8P3} = \frac{1}{\frac{8!}{(8-3)!}}$$

$$= \frac{1}{\frac{(8)(7)(6)\cancel{(5)}\cancel{(4)}\cancel{(3)}\cancel{(2)}\cancel{(1)}}{\cancel{(5)}\cancel{(4)}\cancel{(3)}\cancel{(2)}\cancel{(1)}}} = \frac{1}{(8)(7)(6)} = \frac{1}{336}$$

$\therefore P(A) = 0.003$

9.6. Combinations

A combination is any set of objects or events regardless of their internal order. The number of groups of r objects that can be selected from n objects is symbolized by n Cr and given by

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

The number of different ways of selecting objects from a set, ignoring the order in which they are arranged, is the number of combinations. Given the objects A, B, C, and D, the number of permutations of two from this set is $4 \times 3 = 12$. The arrangements are AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, and DC. Note that each arrangement occurs in two different orders. If you ignore the order in which each pair of objects is arranged, you have the number of combinations. In this example each pair occurs in two different orders. The number of combinations is then $4 \times \frac{3}{2} = 6$. In general, the number of different combinations of N things taken r at a time is given above and given here also for quick understanding as

$$^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

The number of combinations of 10 things taken 3 at a time is

$$10\ C/_3 = \frac{10!}{(10-3)!3!}$$

$$= \frac{(10)(9)(8)\cancel{(7)}\cancel{(6)}\cancel{(5)}\cancel{(4)}\cancel{(3)}\cancel{(2)}\cancel{(1)}}{\cancel{(7)}\cancel{(6)}\cancel{(5)}\cancel{(4)}\cancel{(3)}\cancel{(2)}\cancel{(1)}(3)(2)(1)}$$

$$= \frac{(20)(9)(8)}{(3)(2)(1)} = \frac{720}{6} = 120$$

Activity 9.6

What is the $6p_4$ and $6C_4$?

Activity 9.6 Feedback: $6p_4 = \frac{n!}{(n-n)!} = \frac{(6)(5)\cancel{(4)}\cancel{(3)}\cancel{(2)}\cancel{(1)}}{\cancel{(4)}\cancel{(3)}\cancel{(2)}\cancel{(1)}} = (6)(5) = 30$

$$6C_4 = \frac{n!}{(n-r)!r!} = \frac{10!}{(10-3)!3!} = \frac{\cancel{(6)}\cancel{(5)}(4)(3)(2)(1)}{\cancel{(2)}\cancel{(1)}(4)(3)(2)(1)}$$

$$= \frac{(6)(5)}{(2)(1)} = \frac{30}{2} = 15$$

Box 9.1: Set Theory

It is important to note the following:

- A set, defined as a well-defined collection of things;
- An element of a set, which is one of its members;
- The universal set - which includes all objects to be considered in any one discussion;
- The empty or null set, which contains no elements
- An operation which is the union of two sets;
- Permutations, which means the permutations of a set of objects or events in an ordered sequence;
- Combinations — a combination is any set of objects or events regardless of their internal order.

Summary of Study Session 9

In Session 9, you have learned that:

- A set is a well-defined collection of things
- There are types of set, such as universal set, sub set, null set, complementary set, etc.
- Permutation is permutation of a set of objects in order of sequence.
- Combination is a set of objects or events regardless of their internal order.

Self-Assessment Questions (SAQs) for Session 9

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 9.1 (tests learning outcome 9.1)

What is a set?

SAQ 9.2 (tests learning outcome 9.2)

Mention types of set?

SAQ 9.3 (tests learning outcome 9.3)

How would you arrange A, B, C, and D, using a combination method?

Notes on the Self-Assessment Question

SAQ 9.1: A set is defined as a well-defined and definite collection of things

SAQ 9.2**:** Types of set include: universal set, null set, complementary set, finite set, infinite set, equal set, subset, etc.

SAQ 9.3: The arrangements are AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, and DC.

References

Guilford, J. P. and Fruchter, B. (1973) *Fundamental Statistics in Psychology and Education*, Kogakusha: McGraw-Hill.

McCall B. Robert (1975) *Fundamental Statistics for Psychology* New York: Harcourt Brace Jovanovich, Inc., 2nd Edition.

Study Session 10: Binomial Probability and Normal Curve

Expected duration: 1 week or 2 contact hours

Introduction

In this lecture you shall be learning or considering mathematical models that are based on the principles of probability, the first of which is the binomial probability. When a coin is thrown, generally there are situations in which the coin is biased, so that heads and tails have different probabilities. In the present section, we consider probability distributions for which there are just two possible outcomes with fixed probabilities summing to one. These distributions are called binomial distribution which is readily explained by reference to coin tossing.

Activity 10.1

What are the two theorems of probability?

Activity 10.1 Feedback: Additive theorem and Multiplication theorem.

Learning Outcomes for Session 10

When you have studied this session, you should be able to:

10.1. To be able to define the concept of binomial probability. (SAQ 10.1)
10.2. The binomial expansion. (SAQ 10.2)
10.3. The normal curve (SAQ 10.3)
10.4. The areas under the Normal curve (SAQ 10.4)
10.5. The applications of the concept of binomial expansion (SAQ 10.5)

10.0 Binomial Probability and Normal Curve

10.1 Binomial Probability

The binomial distribution model is an important probability model that is used when there are two possible outcomes (hence "binomial"). In a situation in which there were more than two distinct outcomes, a multinomial probability model might be appropriate, but here we focus on the situation in which the outcome is dichotomous.

- When you flip a coin, there are two possible outcomes. Name them and their probabilities?

- Your answer should be: Head (1/2); Tail (1/2).

For example, adults with allergies might report relief with medication or not, children with a bacterial infection might respond to antibiotic therapy or not, adults who suffer a myocardial infarction might survive the heart attack or not, a medical device such as a coronary stent might be successfully implanted or not. These are just a few examples of applications or processes in which the outcome of interest has two possible values (i.e., it is dichotomous). The two outcomes are often labelled "success" and "failure" with success indicating the presence of the outcome of interest. Note, however, that for many medical and public health questions the outcome or event of interest is the occurrence of disease, which is obviously not really a success. Nevertheless, this terminology is typically used when discussing the binomial distribution model. As a result, whenever using the binomial distribution, we must clearly specify which outcome is the "success" and which is the "failure".

The binomial distribution model allows us to compute the probability of observing a specified number of "successes" when the process is repeated a specific number of times (e.g., in a set of patients) and the outcome for a given patient is either a success or a failure. We must first introduce some notation which is necessary for the binomial distribution model.

First, we let "n" denote the number of observations or the number of times the process is repeated, and "x" denotes the number of "successes" or events of interest occurring during "n" observations. The probability of "success" or occurrence of the outcome of interest is indicated by "p".

The binomial equation also uses **factorials**. In mathematics, the factorial of a non-negative integer k is denoted by k!, which is the product of all positive integers less than or equal to k. For example,

- 4! = 4 x 3 x 2 x 1 = 24,
- 2! = 2 x 1 = 2,
- 1!=1.
- There is one special case, 0! = 1.

With this notation in mind, the binomial distribution model is defined as:

$P(X)= {}^{n}CxP^{x}q^{n-x}$

Use of the binomial distribution requires three assumptions:

1. Each replication of the process results in one of two possible outcomes (success or failure),
2. The probability of success is the same for each replication, and
3. The replications are independent, meaning here that a success in one patient does not influence the probability of success in another.

Activity 10.2

What sign denotes factorial? And how would you write 4 as a factorial?.

Activity 10.2 Feedback: Sign of factorial is “!”. 4! = 4 x 3 x 2 x 1 = 24

Examples of Use of the Binomial Model

1. Relief of Allergies

Suppose that 80% of adults with allergies report symptomatic relief with a specific medication. If the medication is given to 10 new patients with allergies, what is the probability that it is effective in exactly seven?

First, do we satisfy the three assumptions of the binomial distribution model?

1. The outcome is relief from symptoms (yes or no), and here we will call a reported relief from symptoms a 'success.'
2. The probability of success for each person is 0.8.
3. The final assumption is that the replications are independent, and it is reasonable to assume that this is true.

We know that:

- Number of observation is n=10
- Number of successes or events of interest is x=7
- p=0.80

The probability of 7 successes is:

$P(X=7) = 10!/7!(10-7)!\ [0.8^7\ (1-0.8)^{10-7}]$

$= 10!/7!(3!)[0.8^7(0.2)^3]$

$= 0.2013$

But many of the terms in the numerator and denominator cancel each other out, so this can be simplified to:

The Probability of Dying after a Heart Attack

The likelihood that a patient with a heart attack dies of the attack is 0.04 (i.e., 4 of 100 die of the attack). Suppose we have 5 patients who suffer a heart attack, what is the probability that all will survive? For this example, we will call a success a fatal attack (p = 0.04). We have n=5 patients and want to know the probability that all survive or, in other words, that none are fatal (0 successes).

We again need to assess the assumptions. Each attack is fatal or non-fatal, the probability of a fatal attack is 4% for all patients and the outcome of individual patients are independent. It should be noted that the assumption that the probability of success applies to all patients must be evaluated carefully. The probability that a patient dies from a heart attack depends on many factors including age, the severity of the attack, and other comorbid conditions. To apply the 4% probability we must be convinced that all patients are at the same risk of a fatal attack. The assumption of independence of events must also be evaluated carefully. As long as the patients are unrelated, the assumption is usually appropriate. Prognosis of disease could be related or correlated in members of the same family or in individuals who are co-habitating. In this example, suppose that the 5 patients being analyzed are unrelated, of similar age and free of comorbid conditions.

$P(0) = {}^5C_0(0.04)^0(1-0.04)^5$

$= 0.8154$

There is an 81.54% probability that all patients will survive the attack when the probability that any one dies is 4%. In this example, the possible outcomes are 0, 1, 2, 3, 4 or 5 successes (fatalities). Because the probability of fatality is so low, the most likely response is 0 (all patients survive). The binomial formula generates the probability of observing exactly x successes out of n.

A binomial experiment is an experiment which satisfies these four conditions

- A fixed number of trials
- Each trial is independent of the others
- There are only two outcomes
- The probability of each outcome remains constant from trial to trial.

These can be summarized as: An experiment with a fixed number of independent trials, each of which can only have two possible outcomes.

The fact that each trial is independent actually means that the probabilities remain constant.

Scenario 10.2

- Suggest some examples of **binomial experiments**?
- Your answer may be: (1) Tossing a coin 20 times to see how many tails occur; (2) Asking 200 people if they watch ABC news; (3) Rolling a die to see if a 5 appears, etc.

- What are some of the examples of experiments **that are not binomial**?

- Your answer may be: (1) Rolling a die until a 6 appears (not a fixed number of trials); (2) Asking 20 people how old they are (not two outcomes); (3) Drawing 5 cards from a deck for a poker hand (done without replacement, so not independent); etc.

Binomial Probability Function

Example: What is the probability of rolling exactly two sixes in 6 rolls of a die?

There are five things you need to do to work a binomial story problem.

1. Define Success first. Success must be for a single trial. Success = "Rolling a 6 on a single die"
2. Define the probability of success (p): p = 1/6
3. Find the probability of failure: q = 5/6
4. Define the number of trials: n = 6
5. Define the number of successes out of those trials: x = 2

Anytime a six appears, it is a success (denoted S) and anytime something else appears, it is a failure (denoted F). The ways you can get exactly 2 successes in 6 trials are given below. The probability of each is written to the right of the way it could occur. Because the trials are independent, the probability of the event (all six dice) is the product of each probability of each outcome (die)

1 FFFFSS 5/6 * 5/6 * 5/6 * 5/6 * 1/6 * 1/6 = (1/6)^2 * (5/6)^4
2 FFFSFS 5/6 * 5/6 * 5/6 * 1/6 * 5/6 * 1/6 = (1/6)^2 * (5/6)^4
3 FFFSSF 5/6 * 5/6 * 5/6 * 1/6 * 1/6 * 5/6 = (1/6)^2 * (5/6)^4
4 FFSFFS 5/6 * 5/6 * 1/6 * 5/6 * 5/6 * 1/6 = (1/6)^2 * (5/6)^4
5 FFSFSF 5/6 * 5/6 * 1/6 * 5/6 * 1/6 * 5/6 = (1/6)^2 * (5/6)^4
6 FFSSFF 5/6 * 5/6 * 1/6 * 1/6 * 5/6 * 5/6 = (1/6)^2 * (5/6)^4
7 FSFFFS 5/6 * 1/6 * 5/6 * 5/6 * 5/6 * 1/6 = (1/6)^2 * (5/6)^4
8 FSFFSF 5/6 * 1/6 * 5/6 * 5/6 * 1/6 * 5/6 = (1/6)^2 * (5/6)^4
9 FSFSFF 5/6 * 1/6 * 5/6 * 1/6 * 5/6 * 5/6 = (1/6)^2 * (5/6)^4
10 FSSFFF 5/6 * 1/6 * 1/6 * 5/6 * 5/6 * 5/6 = (1/6)^2 * (5/6)^4
11 SFFFFS 1/6 * 5/6 * 5/6 * 5/6 * 5/6 * 1/6 = (1/6)^2 * (5/6)^4
12 SFFFSF 1/6 * 5/6 * 5/6 * 5/6 * 1/6 * 5/6 = (1/6)^2 * (5/6)^4
13 SFFSFF 1/6 * 5/6 * 5/6 * 1/6 * 5/6 * 5/6 = (1/6)^2 * (5/6)^4
14 SFSFFF 1/6 * 5/6 * 1/6 * 5/6 * 5/6 * 5/6 = (1/6)^2 * (5/6)^4
15 SSFFFF 1/6 * 1/6 * 5/6 * 5/6 * 5/6 * 5/6 = (1/6)^2 * (5/6)^4

Notice that each of the 15 probabilities are exactly the same: (1/6)^2 * (5/6)^4.

Also, note that the 1/6 is the probability of success and you needed 2 successes. The 5/6 is the probability of failure, and if 2 of the 6 trials were success, then 4 of the 6 must be failures. Note that 2 is the value of x and 4 is the value of n-x.

Further note that there are fifteen ways this can occur. This is the number of ways 2 successes can be occur in 6 trials without repetition and order not being important, or a combination of 6 things, 2 at a time.

The probability of getting exactly x success in n trials, with the probability of success on a single trial being p is:

P(X=x) = nCx * p^x * q^(n-x)

Example:

A coin is tossed 10 times. What is the probability that exactly 6 heads will occur.

1. Success = "A head is flipped on a single coin"
2. p = 0.5
3. q = 0.5
4. n = 10
5. x = 6

P(x=6) = ${}_{10}C_6$ * 0.5^6 * 0.5^4 = 210 * 0.015625 * 0.0625 = 0.205078125

Mean, Variance, and Standard Deviation

The mean, variance, and standard deviation of a binomial distribution are extremely easy to find.

U = np

Σ^2 = npq

Σ = Sqrt npq

Another way to remember the variance is mu-q (since the np is mu).

Example:

Find the mean, variance, and standard deviation for the number of sixes that appear when rolling 30 dice.

Success = "a six is rolled on a single die". p = 1/6, q = 5/6.

The mean is 30 * (1/6) = 5. The variance is 30 * (1/6) * (5/6) = 25/6. The standard deviation is the square root of the variance = 2.041241452 (approx)

Activity 10.3

Find the mean, variance, and standard deviation for the number of blues that appear when rolling 21 rainbow colour balls.

Activity 10.3 Feedback: Success = "a blue is rolled on a ball". p = 1/7, q = 6/7.

The mean is 21 * (1/7) = 3. The variance is 21 * (1/7) * (6/7) = 18/7. The standard deviation is the square root of the variance (√18/7 = 1.5714 (approx)

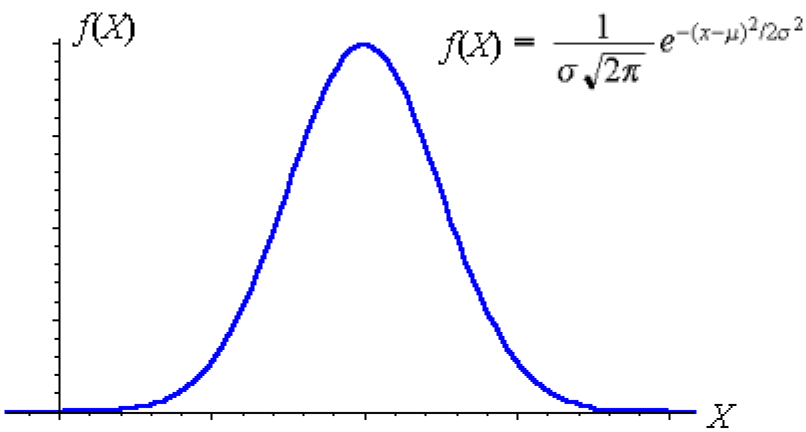
10.3 Normal Probability Distributions

The Normal Probability Distribution is very common in the field of statistics.

Whenever you measure things like people's height, weight, salary, opinions or votes, the graph of the results is very often a normal curve.

10.3.1. The Normal Distribution

A random variable *X* whose distribution has the shape of a **normal curve** is called a **normal random variable**.



Normal Curve

This random variable *X* is said to be normally distributed with mean μ and standard deviation σ if its probability distribution is given by

`f(X)=1/(sigmasqrt(2pi))e^(-(x-mu)^2 "/"2\ sigma^2`

Continues below ⇩

Properties of a Normal Distribution

1. The normal curve is symmetrical about the mean μ;
2. The mean is at the middle and divides the area into halves;
3. The total area under the curve is equal to 1;
4. It is completely determined by its mean and standard deviation σ (or variance σ^2)

Note: In a normal distribution, only **2** parameters are needed, namely μ and σ^2.

Activity 10.4

Take a moment to reflect on characteristics of normal curve?

Activity 10.4 Feedback: It is symmetrical about the mean μ; and its area equals 1; etc.

10.3.2 Area Under the Normal Curve using Integration

The probability of a continuous normal variable *X* found in a particular interval [*a*, *b*] is the area under the curve bounded by $x = a$ and $x = b$ and is given by

$$P(a<X<b)=\int_a^b f(X)\,dx$$

and the area depends upon the values of μ and σ.

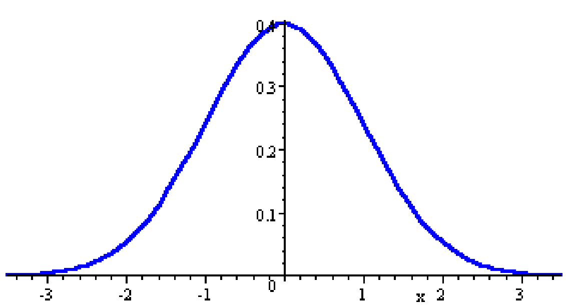
[See Area under a Curve for more information on using integration to find areas under curves. Don't worry - we don't have to perform this integration - we'll use the computer to do it for us.]

10.3.3 **The Standard Normal Distribution**

It makes life a lot easier for us if we **standardize** our normal curve, with a mean of zero and a standard deviation of 1 unit.

If we have the **standardized situation** of $\mu = 0$ and $\sigma = 1$, then we have:

$$f(X)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$$



Standard Normal Curve $\mu = 0$, $\sigma = 1$

We can transform all the observations of any normal random variable X with mean μ and variance σ to a new set of observations of another normal random variable Z with mean `0` and variance `1` using the following transformation:

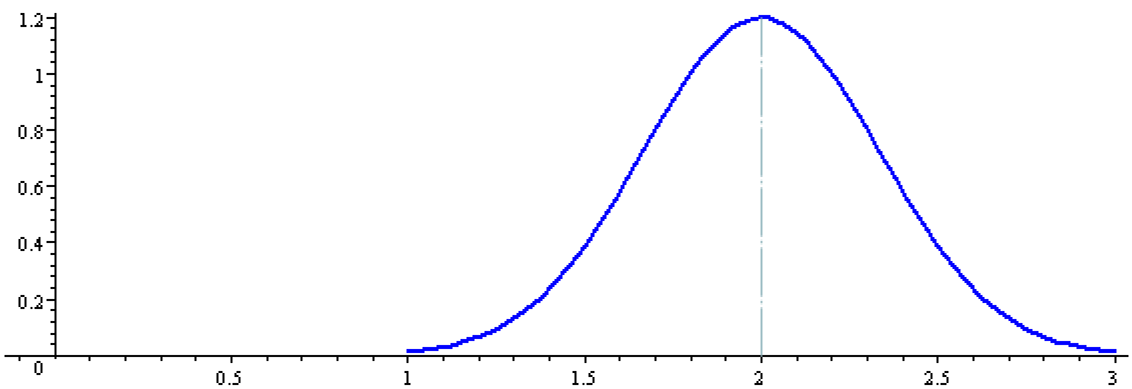
`Z=(X-mu)/sigma`

We can see this in the following example.

Example 1

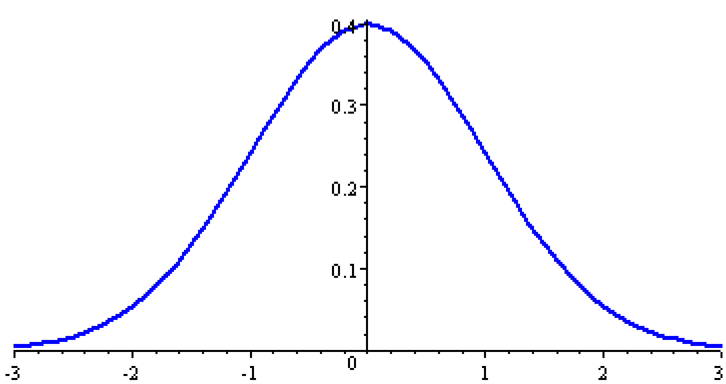
Say `μ = 2` and `sigma = 1/3` in a normal distribution.

The graph of the normal distribution is as follows:



`μ = 2`, `σ = 1/3`

The following graph represents the same information, but it has been **standardized** so that $\mu = 0$ and $\sigma = 1$:



`μ = 0`, ` σ = 1`

The two graphs have different μ and σ, but have the same shape (if we tweak the axes).

The new distribution of the normal random variable Z with mean `0` and variance `1` (or standard deviation `1`) is called a **standard normal distribution**. Standardizing the distribution like this makes it much easier to calculate probabilities.

Formula for the Standardized Normal Distribution

If we have mean μ and standard deviation σ, then `Z=(X-mu)/sigma`

Since all the values of X falling between x_1 and x_2 have corresponding Z values between z_1 and z_2, it means:

The area under the X curve between $X = x_1$ and $X = x_2$ equals the area under the Z curve between $Z = z_1$ and $Z = z_2$.

Hence, we have the following equivalent probabilities:

$P(x_1 < X < x_2) = P(z_1 < Z < z_2)$

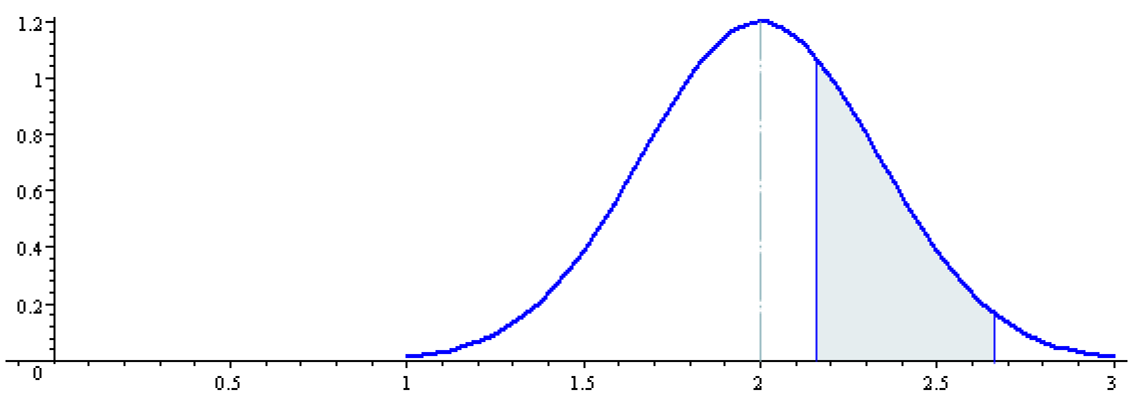
Example 2

Considering our example above where `μ = 2`, `σ = 1/3`, then

One-half standard deviation = `σ/2 = 1/6`, and

Two standard deviations = `2σ = 2/3`

So `1/2` s.d. (standard deviation) to 2 s.d. to the right of `μ = 2` will be represented by the area from `x_1=13/6 = 2 1/6` to `x_2=8/3 = 2 2/3`. This area is graphed as follows:

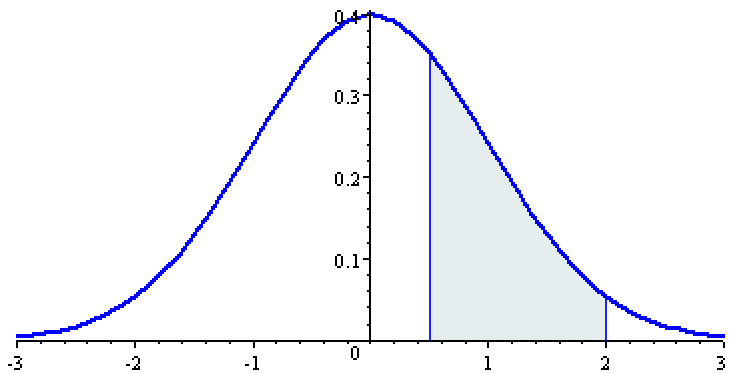


$\mu = 2$, $\sigma = 1/3$

The area above is exactly the same as the area

$z_1 = 0.5$ to $z_2 = 2$

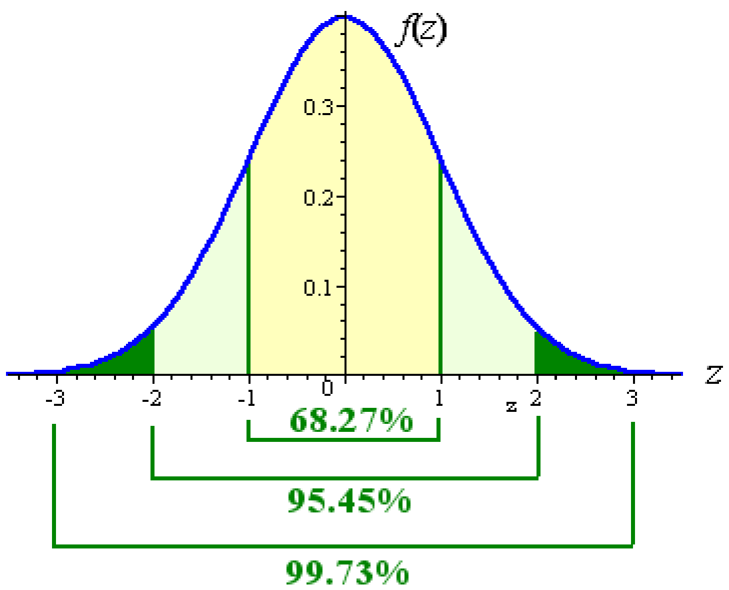
in the standard normal curve:



$\mu = 0, \sigma = 1$

Percentages of the Area Under the Standard Normal Curve

A graph of this standardized (mean `0` and variance `1`) normal curve is shown.



In this graph, we have indicated the areas between the regions as follows:

$-1 \leq Z \leq 1$ 68.27%

$-2 \leq Z \leq 2$ 95.45%

$-3 \le Z \le 3$ 99.73%

This means that 68.27% of the scores lie within 1 standard deviation of the mean.

This comes from: $\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.68269$

Also, 95.45% of the scores lie within 2 standard deviations of the mean.

This comes from: $\int_{-2}^{2} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.95450$

Finally, 99.73% of the scores lie within 3 standard deviations of the mean.

This comes from: $\int_{-3}^{3} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.9973$

The total area from $-\infty < z < \infty$ is 1.

Activity 10.5

What is the total area of a normal curve?

Activity 10.5 Feedback: The total area from $-\infty < z < \infty$ is 1.

10.4 The *z*-Table

The areas under the curve bounded by the ordinates $z = 0$ and any positive value of z are found in the ***z*-Table**. From this table the area under the standard normal curve between any two ordinates can be found by using the symmetry of the curve about $z = 0$. Go here for the actual *z*-Table.

Example 3

Find the area under the standard normal curve for the following, using the *z*-table. Sketch each one.

(a) between $z = 0$ and $z = 0.78$

(b) between $z = -0.56$ and $z = 0$

(c) between $z = -0.43$ and $z = 0.78$

(d) between $z = 0.44$ and $z = 1.50$

(e) to the right of $z = -1.33$.

Example 4

Find the following probabilities:

(a) $P(Z > 1.06)$

(b) $P(Z < -2.15)$

(c) $P(1.06 < Z < 4.00)$

(d) $P(-1.06 < Z < 4.00)$

Example 5

It was found that the mean length of `100` parts produced by a lathe was `20.05\ "mm"` with a standard deviation of `0.02\ "mm"`. Find the probability that a part selected at random would have a length

(a) between `20.03\ "mm"` and `20.08\ "mm"`

(b) between `20.06\ "mm"` and `20.07\ "mm"`

(c) less than `20.01\ "mm"`

(d) greater than `20.09\ "mm"`.

Example 6

A company pays its employees an average wage of `$3.25` an hour with a standard deviation of `60` cents. If the wages are approximately normally distributed, determine

a. the proportion of the workers getting wages between `$2.75` and `$3.69` an hour;
b. the minimum wage of the highest `5%`.

Example 7

The average life of a certain type of motor is `10` years, with a standard deviation of `2` years. If the manufacturer is willing to replace only `3%` of the motors that fail, how long a guarantee should he offer? Assume that the lives of the motors follow a normal distribution.

10.5 The Normal Curve

In tossing N coins the frequency distribution of heads or tails is approximated more closely by the normal distribution as N increases in size. The normal curve is the limiting form of the symmetrical binomial. The equation for the normal curve is:

$$Y = \frac{N}{\sqrt[0]{2^{\pi}}} \; \frac{e - x^{2}}{2} 2$$

where Y = frequency

N = number of observations

= standard deviation of the distribution.

π = 3.1416 (approximately)

e = 2.7 18 (approximately), the base of the Napierian system of logarithms.

x = deviation of a measurement from the mean ($X — \overline{X}$).

The normal curve is usually written in standard score form. Standard scores have a mean of zero and a standard deviation of 1. Thus u = 0 and σ = 1. The area under the curve is taken as unity that is, N 1. With these substitutions the equation may be written as

$$\underset{\pi}{e} - 2^{2}/_{2} Y = \frac{1}{\sqrt{2}}$$

Here Z is a standard score on X and is equal to (X - U)/σ. The score Z is a deviation in standard deviation units measured along the baseline of the curve from a mean of zero, deviations to the right of the mean being positive and those to the left negative. The curve has unit area and unit standard deviation. By substituting different values of Z in the above formula, different values of y may be calculated when Z = 0, y = 1/√2π = .398 9. This follows from the fact that e^{0} 1. Any term raised to the zero power is equal to 1. This is the height of the ordinate at the mean of the normal curve in standard-score form is given by the number .3989. For z = +1, y = .2420, and for Z = +2, y .0540. Similarly the height of the curve may be calculated for any value of z. In practice you are required to substitute different values of z in the normal curve formula and solve for y to obtain the height of the required ordinate.

The general shape of the normal curve can be observed by inspection of figure 10.1. The curve is symmetrical. It is asymptotic at the extremities, that is, it approaches but never reaches the horizontal axis. It can be said to extend from minus infinity to plus infinity. The area under the curve is finite.

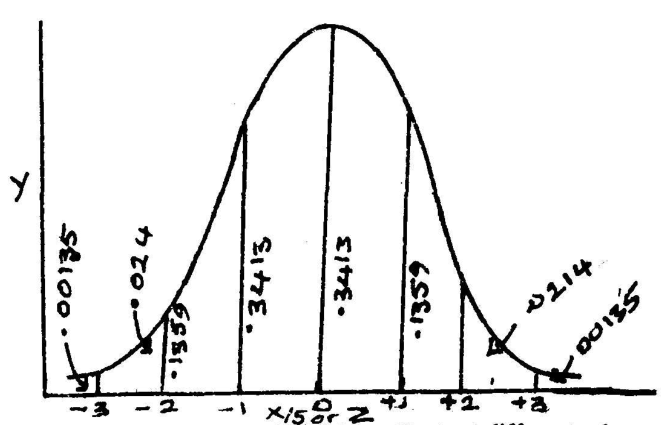


Figure 10.1: Normal curve showing at different values of x/σ, or z.

10.6 Areas under the Normal Curve

For many purposes it is necessary to ascertain the proportion of the area under the normal curve between ordinates at different points on the base line. You may for example, want to know (I) the proportion of the area under the curve between an ordinate at the mean and an ordinate at any specified point either above or below the mean, (2) the proportion of the total area above or below an ordinate at any point on the base line, and (3) the proportion of the area falling between ordinates at any two-points on the base line. In tables of normal curve, you will find that the proportion of the area between the mean of the unit normal curve and ordinates extends from z = 0 to z = 3. Now suppose, you want to find the area under the curve between the ordinates at z = 0 and z +1. From the Table of normal curve, this area is .3413 of the total. Thus approximately 34% of the total area falls between the mean and one standard deviation unit above the mean. The proportion of the area of the curve between z 0 and z = 2 is .4772. Thus about 47.7% of the area of the curve falls between the mean and two standard deviation units above the mean. The proportion of the area between z = 0 and z = 3 is .49865, or a little less than 49.9%. The proportion of the area between z = 0 and z = 3 is .49865, or a little less than 49.9%.

The proportion of the area falling between z = 0 and z = + 1 is .3413. Since the curve is symmetrical the proportion of the area falling between z = — 1 is also .3413. The proportion of the area falling between the limits z = ± 1 is therefore .3413 ÷ .3413 6826, or roughly 68%. The proportion of the area falling between z = ± 2 is .4772 + .4772 .9544, or about 95%. The proportion between z = ± 3 is .49865 + .49865 .99730 or 99.73%. You should note that the determination of any area under the curve can be found in the Table of Normal Curves.

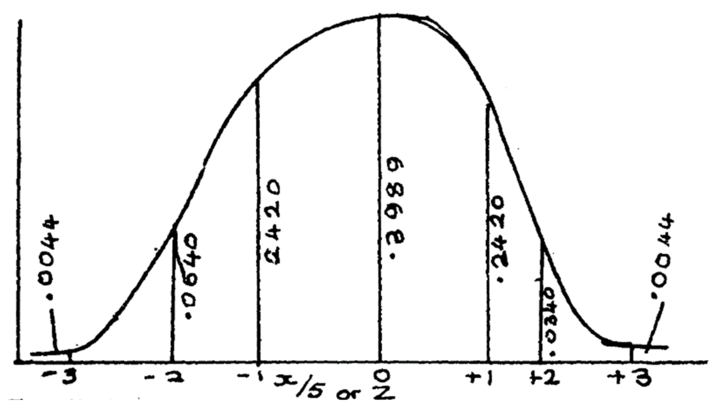


Figure 10:2

- ○ How would you explain figure 10.2 to your classmates?

- Your answer should reflect the one above.

Box 10.1: Binomial Probability and Normal Curve

It is important to note the following:

- The binomial distribution model is an important probability model that is used when there are two possible outcomes (hence "binomial").
- The binomial equation also uses **factorials**.
- the binomial distribution requires three assumptions, which include: Each replication of the process results in one of two possible outcomes (success or failure); The probability of success is the same for each replication; and The replications are independent
- A random variable *X* whose distribution has the shape of a **normal curve** is called a **normal random variable**.
- The normal curve is symmetrical about the mean μ; divides the area into halves; and total area under the curve is equal to 1.
- The binomial expansion is used to answer the question above the normal curve, and areas under the normal curve.

Summary of Study Session 10

In Session 10, you have learned that:

- The binomial distribution model is an important probability model that is used when there are independent two possible outcomes.
- The curve is bell-shaped; its line divide the curve into two equal halves, while it has area coverage equals 1.
- The binomial expansion helps to address issues that are beyond normal curve.

Self-Assessment Questions (SAQs) for Session 10

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 10.1 (tests learning outcome 10.1)
Define the concept of binomial probability?

SAQ 10.2 (tests learning outcome 10.2)

What is the usefulness of the binomial expansion?

SAQ 10.3 (tests learning outcome 10.3)

Draw a normal curve?

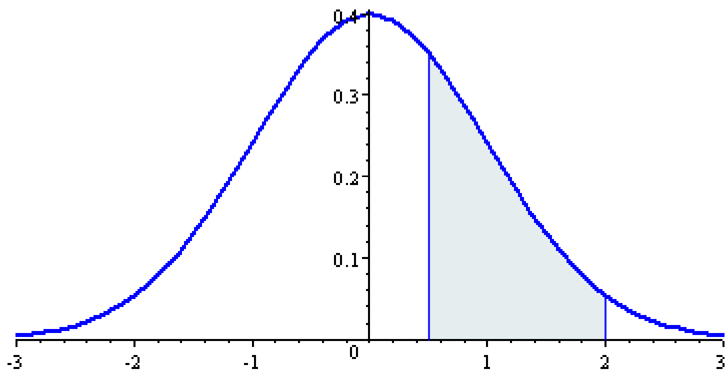
SAQ 10.4 (tests learning outcome 10.4)

What does the areas under the normal curve equals to?

Notes on the Self-Assessment Question

SAQ 10.1: The binomial distribution model is an important probability model that is used when there are independent two possible outcomes.

SAQ 10.2: Binomial expansion helps in addressing issues that are beyond normal curve.
SAQ 10.3.



SAQ 10.4 The area of a normal curve equals to 1

References

Ferguson, G.A., (1959) *Statistical Analysis in Psychology and Education*. New York: McGraw—Hill Book Company, Inc.

Guilford, J.P. and Fruchter, B. (1973) *Fundamental Statistics in Psychology and Education.* Kogakusha: McGraw—Hill.

Study Session 11: Chi Square

Expected duration: 1 week or 2 contact hours

Introduction

Another distribution of considerable theoretical and practical importance is the distribution of chi square, or X^2. In many experimental situations one wishes to compare observed with theoretical frequencies. The observed frequencies are those obtained empirically by direct observation or experiment. The theoretical frequencies are generated on the basis of some hypothesis, or line of theoretical speculation, which is independent of the data at hand. The question arises as to whether the differences between the observed and theoretical frequencies are significant.

Activity 11.1

Take a moment to reflect on Chi-square and state its main usage.

Activity 11.1 Feedback: it is used in finding differences between observed and expected frequencies. It can also be used in finding association between categorical variables.

Learning Outcomes for Session 11

When you have studied this session, you should be able to:

11.1. State the origin of Chi-square. (SAQ 11.1)
11.2. State the assumption guiding the use of Chi square (SAQ 11.2)

11.1 Chi Square

11.1.1 Origin of Chi Square

During the voluntary history of research designs, the chi square test, or X^2, was introduced by Karl Pearson in 1900. Chi square test is used with data in the form of frequencies, or data that can be readily transformed into frequencies. This includes proportions and probabilities. One important feature of chi square is its additive property, which make possible the combination of several statistics or other 'values in the same test. Thus, a hypothesis involving more than one set of data can be tested for significance.

- o Pronounce Chi-square for your friend?

- Your pronunciation should be /kai skua/.

11.2 Assumptions guiding the use of the Chi square

There are four basic assumptions behind the use of the chi square. These assumptions are listed below for your full comprehension.

1. It must be assumed that the two samples are independent of each other. This usually implies that different and unrelated sets of subjects are selected.
2. The subjects within each group must be randomly and independently sampled.
3. Third, each observation must qualify for one and only one category in the classification scheme.
4. The sample size must be fairly large such that no expected frequency is less than 5 for r(row) or c(column) greater than 20 or less than 10 if r = c = 2

> **Box 11.1: Chi Square**
>
> It is important to note the following:
>
> - Chi square test was introduced by Karl Pearson in 1900.
> - The most important use of the chi square test as a statistical tool is additive property which makes possible the combination of several statistics or other values in the same test.

Summary of Study Session 11

In Session 11, you have learned that:

- Chi-square (x^2) was introduced by Karl Pearson in 1900.
- It is used in calculating differences or association between observed and expected frequencies.
- It has basic assumptions that guide its usage.

Self-Assessment Questions (SAQs) for Session 11

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 11.1 (tests learning outcome 11.1)
Who propounded Chi-square?
SAQ 11.2 (tests learning outcome 11.2)
State some assumptions of using Chi-square?

Notes on the Self-Assessment Question

SAQ 11.1: Chi-square (x^2) was introduced by Karl Pearson.

SAQ 11.1: The assumptions include: it must be randomly and independently sampled; the sample should be large not less than 5 for r(row) or c(column) greater than 20 or less than 10 if r = c = 2.

References

Ferguson, G.A., (1959) *Statistical Analysis in Psychology and Education*. New York: McGraw—Hill Book Company.

Guilford, J.P. and Fruchter, B. (1973) *Fundamental Statistics in Psychology and Education*, Koyakusha: McGraw—Hill.

Study Session 12: Computational Procedure for Chi square

Expected duration: 1 week or 2 contact hours

Introduction
In session 11, we studied origin and assumption of Chi-square. In this session 12, we shall study the procedures for the derivation of the chi square (X^2). This session will acquaint you with the procedure and the step-by-step method of arriving at the X^2 for any given data that have been generated from experimental observations or some other research studies.

Learning Outcomes for Session 12

When you have studied this session, you should be able to:

12.1. Recall sigma for Chi-square. (SAQ 12.1)
12.2. Recall formula for calculating expected frequency (SAQ 12.2)
12.3. Recall the formula for calculating Chi-square (SAQ 12.3)

12.1 Computational Procedure for Chi square

12.1.1 Chi-Square Goodness of Fit Test

When an analyst attempts to fit a statistical model to observed data, he or she may wonder how well the model actually reflects the data. How "close" are the observed values to those which would be expected under the fitted model? One statistical test that addresses this issue is the chi-square goodness of fit test. This test is commonly used to test association of variables in two-way tables (see "Two-Way Tables and the Chi-Square Test"), where the assumed model of independence is evaluated against the observed data. In general, the ***chi-square test statistic*** is of the form

$$X^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$

If the computed test statistic is large, then the observed and expected values are not close and the model is a poor fit to the data.

Activity 12.1

Take a moment to reflect on Chi-square formula and make attempt to recall it.

Activity 12.1 Feedback:

$$X^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$

- What is the x^2, if the expected was 8 and the observed was 10?

- Difference = 2. $X^2 = (2)^2 = 4/8 = 1/2 = 0.5$

Review: Uses of the Chi-Squared Statistic

>The chi-squared statistic provides a test of the association between two or more groups, populations, or criteria.
>The chi-square test can be used to test the strength of the association between exposure and disease in a cohort study, an unmatched case-control study, or a cross-sectional study
>McNemar's test can be used to test the strength of the association between exposure and disease in a matched case-control study

Activity 12.2

Take a moment to reflect on uses of Chi-square and recall some of them.

Activity 12.2 Feedback: check your response with the last sub-session.

Example

A new casino game involves rolling 3 dice. The winnings are directly proportional to the total number of sixes rolled. Suppose a gambler plays the game 100 times, with the following observed counts:

Number of Sixes	Number of Rolls
0	48
1	35
2	15
3	3

The casino becomes suspicious of the gambler and wishes to determine whether the dice are fair. What do they conclude?

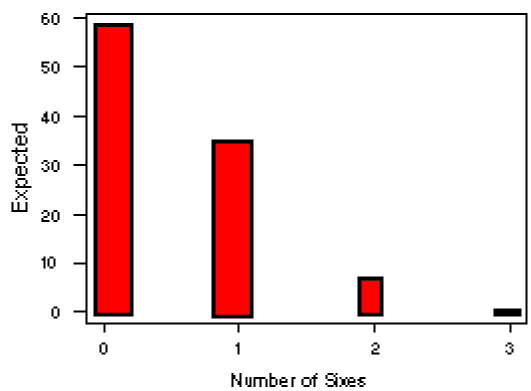
If a die is fair, we would expect the probability of rolling a 6 on any given toss to be 1/6. Assuming the 3 dice are independent (the roll of one die should not affect the roll of the others), we might assume that the number of sixes in three rolls is distributed Binomial(3,1/6). To determine whether the gambler's dice are fair, we may compare his results with the results expected under this distribution. The expected values for 0, 1, 2, and 3 sixes under the Binomial(3,1/6) distribution are the following:

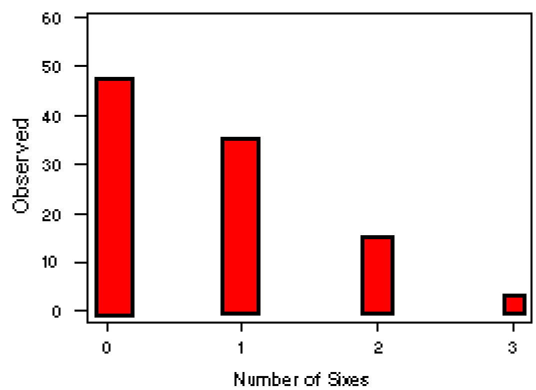
Null Hypothesis:
p_1 = P(roll 0 sixes) = P(X=0) = 0.58
p_2 = P(roll 1 six) = P(X=1) = 0.345
p_3 = P(roll 2 sixes) = P(X=2) = 0.07
p_4 = P(roll 3 sixes) = P(X=3) = 0.005.

Since the gambler plays 100 times, the expected counts are the following:

Number of Sixes	Expected Counts	Observed Counts
0	58	48
1	34.5	35
2	7	15
3	0.5	3

The two plots shown below provide a visual comparison of the expected and observed values:





From these graphs, it is difficult to distinguish differences between the observed and expected counts. A visual representation of the differences is the *chi-gram*, which plots the observed - expected counts divided by the square root of the expected counts, as shown below:

The chi-square statistic is the sum of the squares of the plotted values,
$(48-58)^2/58 + (35-34.5)^2/58 + (15-7)^2/7 + (3-0.5)^2/0.5$
$= 1.72 + 0.007 + 9.14 + 12.5 = 23.367$.

Given this statistic, are the observed values likely under the assumed model?

A random variable χ^2 is said to have a chi-square distribution with *m* degrees of freedom if it is the sum of the squares of *m* independent standard normal random variables (the square of a single standard normal random variable has a chi-square distribution with one degree of freedom). This distribution is denoted $\chi^2(m)$, with associated probability values available in Table G.

The standardized counts (observed - expected)/sqrt(expected) for *k* possibilities are approximately normal, but they are not independent because one of the counts is entirely determined by the sum of the others (since the total of the observed and expected counts must sum to *n*). This results in a loss of one degree of freedom, so it turns out the distribution of the chi-square test statistic based on *k* counts is approximately the chi-square distribution with $m = k-1$ degrees of freedom, denoted $\chi^2(k-1)$.

Hypothesis Testing

We use the chi-square test to test the validity of a distribution assumed for a random phenomenon. The test evaluates the null hypotheses H_0 (that the data are governed by the assumed distribution) against the alternative (that the data are not drawn from the assumed distribution).

Let $p_1, p_2, ..., p_k$ denote the probabilities hypothesized for k possible outcomes. In n independent trials, we let $Y_1, Y_2, ..., Y_k$ denote the observed counts of each outcome which are to be compared to the expected counts $np_1, np_2, ..., np_k$. The chi-square test statistic is q_{k-1} =

$$X^2 = \sum \frac{(\text{observed - expected})^2}{\text{expected}}$$

$$= \frac{(Y_1 - np_1)^2}{np_1} + \frac{(Y_2 - np_2)^2}{np_2} + ... + \frac{(Y_k - np_k)^2}{np_k}$$

Reject H_0 if this value exceeds the upper α critical value of the $\chi^2(k-1)$ distribution, where α is the desired level of significance.

Example

In the gambling example above, the chi-square test statistic was calculated to be 23.367. Since $k = 4$ in this case (the possibilities are 0, 1, 2, or 3 sixes), the test statistic χ^2 is associated with the chi-square distribution with 3 degrees of freedom. If we are interested in a significance level of 0.05 we may reject the null hypothesis (that the dice are fair) if $\chi^2 \geq 7.815$, the value corresponding to the 0.05 significance level for the $\chi^2(3)$ distribution. Since 23.367 is clearly greater than 7.815, we may reject the null hypothesis that the dice are fair at the 0.05 significance level.

Given this information, the casino asked the gambler to take his dice (and his business) elsewhere.

Activity 12.3

When should you reject a hypothesis based on χ^2 result?

Activity 12.3 Feedback: Reject when the calculated is greater than table value.

Example

Consider a binomial random variable Y with mean (expected value) np and variance $\sigma_y^2 = np(1-p)$. From the Central Limit Theorem, we know that $\mathbf{Z = (Y-np)/\sigma_y}$ has an pproximately Normal(0,1) distribution for large values of n. Then Z^2 is approximately $\chi^2(1)$, since the square of a normal random variable has a chi-square distribution.

Suppose the random variable Y_1 has a Bin(n,p_1) distribution, and let $Y_2 = n - Y_1$ and $p_2 = 1 - p_1$.

Then $Z^2 = \frac{(Y_1 - np_1)^2}{np_1(1-p_1)}$

$$= \frac{(Y_1 - np_1)^2(1 - p_1) + (Y_1 - np_1)^2(p_1)}{np_1(1-p_1)}$$

$$= \frac{(Y_1 - np_1)^2}{np_1} + \frac{(Y_1 - np_1)^2}{n(1-p_1)}$$

Since $(Y_1 - np_1)^2 = (n - Y_2 - n + np_2)^2 = (Y_2 - np_2)^2$,
we have $Z^2 = \frac{(Y_1 - np_1)^2}{np_1} + \frac{(Y_2 - np_2)^2}{np_2}$
where Z^2 has a chi-square distribution with 1 degree of freedom. If the observed values Y_1 and Y_2 are close to their expected values np_1 and np_2, then the calculated value Z^2 will be close to zero. If not, Z^2 will be large.

In general, for k random variables Y_i, $i = 1, 2,..., k$, with corresponding expected values np_i, a statistic measuring the "closeness" of the observations to their expectations is the sum

$$\frac{(Y_1 - np_1)^2}{np_1} + \frac{(Y_2 - np_2)^2}{np_2} + ... + \frac{(Y_k - np_k)^2}{np_k}$$

which has a chi-square distribution with $k-1$ degrees of freedom.

Estimating Parameters

Often, the null hypothesis involves fitting a model with parameters estimated from the observed data. In the above gambling example, for instance, we might wish to fit a binomial model to evaluate the probability of rolling a six with the gambler's loaded dice. We know that this probability is not equal to 1/6, so we might estimate this value by calculating the probability from the data. By estimating a parameter, we lose a degree of freedom in the chi-square test statistic. In general, if we estimate d parameters under the null hypothesis with k possible counts the degrees of freedom for the associated chi-square distribution will be $k - 1 - d$.

Example

A two-way table for two categorical variables X and Y with r and c levels, respectively, will have r rows and c columns. The table will have rc cells, with any one cell entirely determined by the sum of the others, so $k-1 = rc - 1$ in this case. A chi-square test of this table tests the null hypothesis of independence against the alternative hypothesis of association between the variables. Under the assumption of independence, we estimate $(r-1) + (c-1)$ parameters to give the marginal probabilities that determine the expected counts, so *d = (r-1) + (c-1)*. The degrees of freedom for the chi-square test statistic are

$(rc - 1) - [(r-1) + (c-1)]$
$= rc - 1 - r + 1 - c + 1$
$= rc - r - c + 1$
$= (r - 1)(c - 1)$.

The chi-square goodness of fit test may also be applied to continuous distributions. In this case, the observed data are grouped into discrete bins so that the chi-square statistic may be calculated. The expected values under the assumed distribution are the probabilities associated with each bin multiplied by the number of observations. In the following example, the chi-square test is used to determine whether or not a normal distribution provides a good fit to observed data.

Activity 12.4

Take a moment to find the parameter of a distribution with row = 5 and column = 4

Activity 12.4 Feedback: *(r - 1)(c - 1)* = (5-1) (4-1) = 4 x 3 = 12

Computationa1 procedure

The 2 × 2 Chi square (test of association) Step 1. Draw up the 2 × 2 contingency table, making sure that each event goes into one of the cells and into not more than one cell.

A hypothetical example Male

	SEX **MALE**	**FEMALE**	
12 – 20 years age	0 = 40 (A) E = 45	0 = 50 (B) E = 45	90
21 – 40 years	(C) 0 = 30 E = 25	(D) 0 = 20 E = 25	50
	70	70	140

Note that 0 stands for the observed frequency. The number in each cell is the observed frequency 0 for that cell.

Step II. Find the row totals, column totals and grand total. In the above example, you will notice that the row totals are 90 and 50: while the column totals are 70 and 70 respectively. The grand total is 140.

a. Row total = 0 frequency for cell A plus
0 frequency for cell B, or 40 + 50 = 90

b. Row total = 0 frequency for cell C plus 0
0 frequency for cell D, or 30 + 20 = 50

c. Column total = 0 frequency for cell A plus 0
Frequency for cell C, or 40 + 30 = 70.

d. Column total = 0 frequency for cell B plus 0
frequency for cell D, or 50 + 20 = 70.

Step III. Work out the expected frequency (E) for each cell separately using this formula.

$$E = \frac{\text{row total} + \text{column total}}{\text{grand total}}$$

For example, using the above figures provided in the table you can obtain the expected frequencies for each cell this way.

$$E_A = \frac{90 \text{ x } 70}{140} = 45$$

$$E_B = \frac{90 \text{ x } 70}{140} = 45$$

$$B_c = \frac{50 \text{ x } 70}{140} = 25$$

$$E_D = \frac{50\ 70}{140} = 25$$

Put the expected frequency for each cell into that cell. Having obtained the expected frequencies you can compute the difference between these and the observed values. X^2 is not based on the simple quantity 0 (0 — E), but on a slightly more complex measure of the difference between 0 and E thus the formula for the chi square is.

$$X^2 = \frac{\sum(0 - E)^2}{E}$$

$\sum$ is summation of the observed frequency 0, minus the expected frequency E all squared, divided by the expected frequency E.

12.2 Worked examples

a. A clinical psychologist studying the symptoms of a random sample of 25 psychotics and 25 neurotics found that only 5 of the psychotics had suicidal feelings, whereas 12 of the neurotics had suicidal feelings. The research question is, this, is there evidence for an association between the two psychiatric groups and presence or absence of suicidal feelings? To answer this question, you need to draw first of all a 2 x 2 contingency table

Step I.

	Psychotics	**Neurotics**
Suicidal	5	12
Non-suicidal	20	13

Step II.

A 5	B 12	17
C 20	D 13	33
25	25	

Step III. For top left cell (cell A) E = $\frac{17 \text{ x } 25}{20} = 8.5$

For top right cell (cell B) E = $\frac{17 \text{ x } 25}{50} = 8.5$

For bottom left cell (cell C) E = $\frac{33 \text{ x } 25}{50} = 1.6.5$

For bottom right cell (cell) D) E = $\frac{33 \text{ x } 25}{50} = 16.5$

Step IV. Work out the difference between 0 and E for each cell, taking the smaller of these from the larger in each case, that is, obtain /0 – E/.

Step V. Square this for each cell $(/0 — E/)^2$

Step VI. Divide by the appropriate E value for that cell

$$\frac{(0 - E)^2}{E}$$

Step VII. Obtain x^2 by dividing all these contributions from the different cells.

$$X^2 = \frac{(0 - E)^2}{E}$$

The degrees of freedom (df) for any statistic Ls the number of components In its calculation that are free to vary.

Thus for this example, the df is obtained this way,

df = (R -1) (C -1)

= (2 - l) (2 - l)

df = 1

Note that R and C stand for both row and column respectively, where R is 2 and C is also 2 for the above case.

Step VIII. If the X^2 obtained exceeds the tabled values at the chosen level of significance,

then there is evidence for an association between the categories.

$$X^2 = \frac{(5 - 8.5)^2}{8.2} + \frac{(12 - 8.5)^2}{8.5} + \frac{(20.16.5)^2}{8.5} + \frac{(13 - 16.5)^2}{16.5}$$

= 1.441 l.44 ÷ 0.7424 ÷ 0.7424

X^2 = 4.3668

X^2 = <u>4.37</u>

This same problem can be approached by using a more simplified formula for the 2 X 2 contingency table. The formula is given as:

$$X^2 = + \frac{N(AD - BC)^2}{(A + B)(A + C)(B + D)(C\ D)}$$

N = 50; A + B = 17; A + C = 25; B + D = 25; and C + D = 33

AD = $5 \times 13 = 65$

BC = $12 \times 20 – 240$

$$X^2 = \frac{50\,(65 - 240)^2}{(17)\,(25)\,(25)\,(33)} = X^2 = \frac{1531250}{350625}$$

b. Numbers of persons in two groups, depressed and not depressed in temperament who responded in each of three categories to the question, “would you rate yourself as an impulsive individual?

GROUP	RESPONSE			
	YES	?	NO	TOTAL
Depressed	72	45	133	350
Not depressed	106	35	109	250
Both	178	242	500	

GROUP	RESPONSE			
	YES	?	NO	TOTAL
Depressed	0 = 72 E = 89	0 = 45 E = 40	0 =133 E = 121	
Not depressed	0 = 106 E = 89	0 = 35 E = 40	0 = 109 E = 121	

To compute the expected frequency for any cell, multiply the total for the row which contains the cell by the total for the column which contains the cells, and divide this product by the total number of cases in the table, for example, the Depressed, yes cell, the expected frequency is given by

$$E = \frac{(178)\,(250)}{500} = 89$$

$$X^2 = \frac{(0 - E)^2}{E}$$

$$= \frac{(72-89)^2}{89} + \frac{(45-40)^2}{40} + \frac{(133-121)^2}{121} + \frac{(106-89)^2}{89} + \frac{(35-40)^2}{40} + \frac{(172-89)^2}{89}$$

$$= \frac{(-17)}{89} + \frac{(5)}{40} + \frac{(12)}{121} + \frac{(17)}{89} + \frac{(-5)}{89} + \frac{(-12)}{121}$$

$$= \frac{289}{89} + \frac{25}{40} + \frac{144}{121} + \frac{289}{89} + \frac{25}{40} + \frac{144}{121}$$

$= 3.24 + 0.63 + 1.19 + 3..24 + 0.63 + 1.19$

$X^2 = 10.12$; df = 2

df = (r -1) (c -1) = (2 -1) (3 -1) 2

Box 12.1: Computational Procedure for Chi square

It is important to note the following:

- The computational procedures for the chi square.
- You must note the following formula:
    - $E \text{ (expected)} = \frac{\text{Row total x column totalq}}{\text{Grand total}}$
    - $X^2 = \frac{\Sigma(0 - E)^2}{E}$

Summary of Study Session 12

In Session 12, you have learned that:

- Chi-square is used in calculating difference between expected and observed frequency.
- There is a procedure to follow in calculating Chi-square.

Self-Assessment Questions (SAQs) for Session 12

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 12.1 (tests learning outcome 12.1)
What is the sigma for Chi-square
SAQ 12.2 (tests learning outcome 12.2)
The formula for obtaining the expected frequency (E) is usually written as:

(a) $E = \frac{Row\,total\ X\ column\,total}{Grand\ total}$

(b) $E = \frac{Grand\ \ total\ X\ column\,total}{Grand\ total}$

(c) $E = \frac{Row\,total - column\,total}{Grand\ total}$

(d) $E = \frac{Row\,total + column\,total}{Grand\ total}$

(e) $E = \frac{Row\,total\ \ column\,total}{Grand\ \ total}$

SAQ 12.3 (tests learning outcome 12.3)

The formula for the chi square is:

(a) $X^2 \quad \frac{(0-E)^2}{0}$ (b) $X^2 = \frac{(0 \; E)^2}{E}$ (c) $X^2 = \frac{(0-E)^2}{E}$

(d) $X^2 = \frac{(0 \quad E)2}{0}$ (e) $X^2 = \frac{(0 \quad 0)^2}{0}$

Notes on the Self-Assessment Question

SAQ 12.1: The sigma is "**X^2**"

SAQ 12.2**: A**

SAQ 12.2: C

References

Ferguson, G.A., (1959) *Statistical Analysis in Psychology and Education,* New York: Hill Book Company, Inc.

Gulford, J.P. and Fruchter, B., (1973) *Fundamental Statistics in Psychology and Education.* Koyakusha :McGraw—Hill.

Study Session 13: Measurement and Statistical Methods

Expected duration: 1 week or 2 contact hours

Introduction

The most important and valuable aspect of educational practice is the measurement of individual differences in various areas. Measurement is thus the systematic assignment of numbers to objects or events, and it forms the very basis of science. Therefore, if an event or attribute, for example, noise level, cannot be measured, it does not find its way into the domain of science. The result of any scientific observation is usually a collection of measurements and these measurements are called data. As you have known what statistics is all about, it can also be regarded as a tool for accurate collection of these measurements or data.

Learning Outcomes for Session 13

When you have studied this session, you should be able to:

13.1. Define the concept of measurement. (SAQ 13.1)
13.2. Mention the categories of statistics (SAQ 13.2)
13.3. Mention the various scales of measurement and the various examples of psychological and educational measurements (SAQ 13.3)
13.4**.** State the reasons for studying statistics (SAQ 13.4)

13.1 Introduction to Measurement and Statistics

13.1.1 What is a Statistic?

Statistics are part of our everyday life. Science fiction author H. G. Wells in 1903 stated, ""Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write." Nowadays, the knowledge and application statistics has been of immense benefits to human endeavour. Therefore, let us examine what is meant by the term statistic; Kuzma (1984) provides a formal definition:

> A body of techniques and procedures dealing with the collection, organization, analysis, interpretation, and presentation of information that can be stated numerically.

Perhaps an example will clarify this definition. Say, for example, we wanted to know the level of job satisfaction nurses experience working on various units within a particular hospital (e.g., psychiatric, cardiac care, obstetrics, etc.). The first thing we would need to do is **collect** some data. We might have all the nurses on a particular day complete a job satisfaction questionnaire. We could ask such questions as "On a scale of 1 (not satisfied) to 10 (highly satisfied), how satisfied are you with your job?". We might examine employee turnover rates for each unit during the past year. We also could examine absentee records for a two month period of time as decreased job satisfaction is correlated with higher absenteeism. Once we have collected the data, we would then **organize** it. In this case, we would organize it by nursing unit.

Absenteeism Data by Unit in Days

	Psychiatric	Cardiac Care	Obstetrics
	3	8	4
	6	9	4
	4	10	3
	7	8	5
	5	10	4
Mean =	5	9	4

Thus far, we have collected our data and we have organized it by hospital unit. You will also notice from the table above that we have performed a simple **analysis**. We found the mean (you probably know it by the name "average") absenteeism rate for each unit. In other words, we added up the responses and divided by the number of them. Next, we would **interpret** our data. In this case, we might conclude that the nurses on the cardiac care unit are less satisfied with their job as indicated by the high absenteeism rate (We would compare this result to the analyses of our other data measures before we reached a conclusion). We could then **present** our results at a conference, in a journal, or to the hospital administration. This might lead to further research concerning job satisfaction on cardiac care units or higher pay for nurses working in this area of specialization.

This example clarifies the process underlying statistical analyses and interpretation. The various techniques and procedures used in the process described above are what make up the content of this chapter. Thus, we will first learn a little bit about the process of data collection/research design. Second, we will examine the use and interpretation of basic statistical analyses used within the context of varying data and design types. And finally, we will examine the process of data presentation.

Activity 13.1

Take a moment to reflect on the definition of statistics. How would you define statistics?

Activity 13.1 Feedback: statistics has to do with the collection, organization, analysis of data, as well as interpretation and presentation of information that can be stated numerically.

Categories of Statistics

To further our understanding of the term statistics, it is important to be aware that statistics can be divided into two general categories: descriptive and inferential statistics. Each of these will be discussed below.

Descriptive statistics are used to organize or summarize a particular set of measurements. In other words, a descriptive statistic will describe that set of measurements. For example, in our study above, the mean described the absenteeism rates of five nurses on each unit. The U.S. census represents another example of descriptive statistics. In this case, the information that is gathered concerning gender, race, income, etc. is compiled to describe the population of the United States at a given point in time. A baseball player's batting average is another example of a descriptive statistic. It describes the baseball player's past ability to hit a baseball at any point in time. What these three examples have in common is that they organize, summarize, and describe a set of measurements.

Inferential statistics use data gathered from a sample to make inferences about the larger population from which the sample was drawn. For example, we could take the information gained from our nursing satisfaction study and make inferences to all hospital nurses. We might infer that cardiac care nurses as a group are less satisfied with their jobs as indicated by absenteeism rates. Opinion polls and television ratings systems represent other uses of inferential statistics. For example, a limited number of people are polled during an election and then this information is used to describe voters as a whole.

Activity 13.2

How many categories of statistics do we have?

Activity 13.2 Feedback: There are two categories of statistics. They are descriptive and inferential statistics.

- What are the instrument we used to measure in education and psychology?

- Psychological test, teacher-made test, and so on.

13.2 What is Measurement?

Normally, when one hears the term measurement, they may think in terms of measuring the length of something (e.g., the length of a piece of wood) or measuring a quantity of something (ie. a cup of flour).This represents a limited use of the term measurement. In statistics, the term measurement is used more broadly and is more appropriately termed **scales of measurement**. Scales of measurement refer to ways in which variables/numbers are defined and categorized. Each scale of measurement has certain properties which in turn determines the appropriateness for use of certain statistical analyses. The four scales of measurement are nominal, ordinal, interval, and ratio.

- Complete this conversation: Kunle asked “What are the scales of measurement”? Jide replied……………..

- Jide replied that there are four types of scale of measurement; and they are nominal, ordinal, interval and ratio..

Nominal: Categorical data and numbers that are simply used as identifiers or names represent a nominal scale of measurement. Numbers on the back of a baseball jersey and your social security number are examples of nominal data. If I conduct a study and I'm including gender as a variable, I may code Female as 1 and Male as 2 or visa versa when I enter my data into the computer. Thus, I am using the numbers 1 and 2 to represent categories of data.

Ordinal: An ordinal scale of measurement represents an ordered series of relationships or rank order. Individuals competing in a contest may be fortunate to achieve first, second, or third place. first, second, and third place represent ordinal data. If Roscoe takes first and Wilbur takes second, we do not know if the competition was close; we only know that Roscoe outperformed Wilbur. Likert-type scales (such as "On a scale of 1 to 10, with one being no pain and ten being high pain, how much pain are you in today?") also represent ordinal data. Fundamentally, these scales do not represent a measurable quantity. An individual may respond 8 to this question and be in less pain than someone else who responded 5. A person may not be in exactly half as much pain if they responded 4 than if they responded 8. All we know from this data is that an individual who responds 6 is in less pain than if they responded 8 and in more pain than if they responded 4. Therefore, Likert-type scales only represent a rank ordering.

Interval: A scale that represents quantity and has equal units but for which zero represents simply an additional point of measurement is an interval scale. The Fahrenheit scale is a clear example of the interval scale of measurement. Thus, 60 degree Fahrenheit or -10 degrees Fahrenheit represent interval data. Measurement of Sea Level is another example of an interval scale. With each of these scales there are direct, measurable quantities with equality of units. In addition, zero does not represent the absolute lowest value. Rather, it is point on the scale with numbers both above and below it (for example, -10degrees Fahrenheit).

Ratio: The ratio scale of measurement is similar to the interval scale in that it also represents quantity and has equality of units. However, this scale also has an absolute zero (no numbers exist below zero). Very often, physical measures will represent ratio data (for example, height and weight). If one is measuring the length of a piece of wood in centimeters, there is quantity, equal units, and that measure cannot go below zero centimeters. A negative length is not possible.

The table below will help clarify the fundamental differences between the four scales of measurement:

	Indications Difference	Indicates Direction of Difference	Indicates Amount of Difference	Absolute Zero
Nominal	X			
Ordinal	X	X		
Interval	X	X	X	
Ratio	X	X	X	X

You will notice in the above table that only the ratio scale meets the criteria for all four properties of scales of measurement.

Interval and Ratio data are sometimes referred to as **parametric** and Nominal and Ordinal data are referred to as **nonparametric**. Parametric means that it meets certain requirements with respect to parameters of the population (for example, the data will be normal--the distribution parallels the normal or bell curve). In addition, it means that numbers can be added, subtracted, multiplied, and divided. Parametric data are analyzed using statistical techniques identified as Parametric Statistics. As a rule, there are more statistical technique options for the analysis of parametric data and parametric statistics are considered more powerful than nonparametric statistics. Nonparametric data are lacking those same parameters and cannot be added, subtracted, multiplied, and divided. For example, it does not make sense to add Social Security numbers to get a third person. Nonparametric data are analyzed by using Nonparametric Statistics.

Activity 13.3

Take a moment to reflect on what you have read. Classify scale of measurement according to either they are parametric or not.

Activity 13.3 Feedback: Interval and Ratio data are sometimes referred to as **parametric** and Nominal and Ordinal data are referred to as **nonparametric**.

As a rule, ordinal data is considered nonparametric and cannot be added, etc.. Again, it does not make sense to add together first and second place in a race--one does not get third place. However, many assessment devices and tests (e.g., intelligence scales) as well as Likert-type scales represent ordinal data but are often treated as if they are interval data. For example, the "average" amount of pain that a person reports on a Likert-type scale over the course of a day would be computed by adding the reported pain levels taken over the course of the day and dividing by the number of times the question was answered. Theoretically, as this represents ordinal data, this computation should not be done.

As stated above, many measures (e.g,. personality, intelligence, psychosocial, etc.) within the psychology and the health sciences represent ordinal data. IQ scores may be computed for a group of individuals. They will represent differences between individuals and the direction of those differences but they lack the property of indicating the amount of the differences. Psychologists have no way of truly measuring and quantifying intelligence. An individual with an IQ of 70 does not have exactly half of the intelligence of an individual with an IQ of 140. Indeed, even if two individuals both score a 120 on an IQ test, they may not really have identical levels of intelligence across all abilities. Therefore, IQ scales should theoretically be treated as ordinal data.

In both of the above illustrations, the statement is made that they should be theoretically treated as ordinal data. In practice, however, they are usually treated as if they represent parametric (interval or ratio) data. This opens up the possibility for use of parametric statistical techniques with these data and the benefits associated with the use of techniques.

13.3 Why Study Statistics?

13.3.1Reason for Studying statistics

There are five primary reasons to study statistics. They are discussed below.

The first reason is to be able to effectively conduct research. Without the use of statistics it would be very difficult to make decisions based on data collected from a research project. For example, in the study cited above, is the difference in recorded absenteeism between psychiatric and obstetrics nurses large enough to conclude that there is meaningful difference in absenteeism between the two units? There are two possibilities: The first possibility is that the difference between the two groups is a result of chance factors. In reality, the two jobs have approximately the same amount of absenteeism. The second possibility is that there is a real difference between the two units with the psychiatric unit demonstrating that these nurses miss more work. Without statistics we have no way of making an educated decision between the two possibilities. Statistics, however, provides us with a tool with which to make an educated decision. We will be able to decide which of the two possibilities is more likely to be true as we base our decision on our knowledge of probability and inferential statistics.

The second reason to study statistics is to be able to read journals. Most technical journals you will read contain some form of statistics. Usually, you will these statistics in something called the results section. Without an understanding of statistics, the information contained in this section will be meaningless. An understanding of basic statistics will provide you with the fundamental skills necessary to read and evaluate most results sections. The ability to extract meaning from journal articles and the ability to critically evaluate research from a statistical perspective are fundamental skills that will enhance your knowledge and understanding in related coursework.

The third reason is to further develop critical and analytic thinking skills. Most students completing high school and introductory undergraduate coursework have at their disposal a variety of critical thinking and analytic skills. The study of statistics will serve to enhance and further develop these skills. To do well in statistics one must develop and use formal logical thinking abilities that are both high level and creative.

The fourth reason to study statistics is to be an informed consumer. Like any other tool, statistics can be used or misused. Yes, it is true that some individuals do actively lie and mislead with statistics. More often, however, well meaning individuals unintentionally report erroneous statistical conclusions. If you know some of the basic statistical concepts, you will be in a better position to evaluate the information you have been given.

The fifth reason to have a working knowledge of statistics is to know when you need to hire a statistician. Most of us know enough about our cars to know when to take it into the shop. Usually, we don't attempt the repair the car ourselves because we do not want to cause any irreparable damage. Also, we try to know enough to be able to carry on an intelligible conversation with the mechanic (or we take someone with us who can) to insure that we do not get a whole new engine (big bucks) when all we need is a new fuel filter (a few bucks). We should be the same way about hiring a statistician. Conducting research is time consuming and expensive. If you are in over your statistical head, it does not make sense to risk an entire project by attempting to compute the data analyses yourself. It is very easy to compute incomplete or inappropriate statistical analysis of one's data. As with the mechanic discussed above, it is also important to have enough statistical savvy to be able to discuss your project and the data analyses you want computed with the statistician you hire. In other words, you want to be able to make sure that your statistician is on the right track.

Activity 13.4

Take a moment to reflect on what you have learnt. Summarize in a sentence importance of studying statistics?

Activity 13.4 Feedback: Knowledge of statistics helps one to be able to effectively conduct research; read journals; develop critical and analytic thinking skills; be an informed consumer; and know when you need to hire a statistician.

13.4 Some Examples of Psychological and Educational measurements

(a) *Psychological measurement*: These are behavioural measurement. The first examples that come to mind are scores of tests on mental ability. These are usually in terms of number of correct responses to test items. A similar kind of measurement is been in scores on a personality questionnaire or a vocational — interest inventory. Other examples are in the sphere of motivation; where you can gauge the strength of derive in terms of the amount of punishment an organism will endure in order to reach his immediate goal. Also, the galvanic skin response, the pupillary response, and the amount of salivation also serve as quantitative indicators of amounts of psychological happenings.

(b) *Educational Measurement*: Many educational problems are also psychological problems. Thus, achievement in many areas of learning, like mental ability, is measurable in terms of test scores. Marks, however obtained, have been the traditional mode of evaluating students in specific units of formal education. Attendance records and data on size of classes, on budgets, on supplies, and on other material aspects of the well regulated school system constitute another list

of measurements in education. Outcomes of educational effort are often expressed quantitatively in terms of promotion, statistics, achievement ratios, and estimates of teaching success.

- o What would be your response if your friend asked you to tell him/her examples of psychological measurement?

- Your answer may be: intelligence test, aptitude test, interest test, etc.

Box 13.1: Measurement and Statistical Methods

It is important to note the following:

- Statistics has to do with collection, organization, analysis, presentation and interpretation of data. It may be descriptive or inferential.
- There are four scale of measurement: nominal, ordinal, interval and ratio.
- The five reasons to study statistics are to be able to effectively conduct research, to be able to read and evaluate journal articles, to further develop critical thinking and analytic skills, to act as an informed consumer, and to know when you need to hire outside statistical help.

Summary of Study Session 13

In Session 13, you have learned that:

- Statistics is very important and it has to do with collection, organization, analysis, presentation and interpretation of data result; which helps in facilitating reliable decision.
- Categories of statistics are: descriptive and inferential.
- There are four scale of measurement: nominal, ordinal, interval and ratio.
- There are five reasons for studying statistics.

Self-Assessment Questions (SAQs) for Session 13

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 13.1 (tests learning outcome 13.1)
What is statistics?
SAQ 13.2 (tests learning outcome 13.2)
Mention the categories of statistics?
SAQ 13.3 (tests learning outcome 13.3)

Mention the scale of measurement we have?

SAQ 13.4 (tests learning outcome 13.4)

What are the reasons for studying statistics?

Notes on the Self-Assessment Question

SAQ 13.1: Statistics has to do with collection, organization, analysis, presentation and interpretation of data.

SAQ 13.2: Descriptive and Inferential statistics

SAQ 13.3: There are four scale of measurement: nominal, ordinal, interval and ratio.

SAQ 13.4: The five reasons to study statistics are to be able to effectively conduct research, to be able to read and evaluate journal articles, to further develop critical thinking and analytic skills, to act as an informed consumer, and to know when you need to hire outside statistical help.

References

McCall, B. Robert, (1915) *Fundamental Statistics for Psychology* New York: Harcourt Brace Jovanovich, Inc., 2nd Edition.

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Study Session 14: The Four Levels of Measurement

Expected duration: 1 week or 2 contact hours

Introduction

When a physical scientist talks about measurement, he usually means the assigning of numbers to observations in such a way that the numbers are amenable to analysis by manipulation or operation according to certain rules. He may, for example, determine how, much a homogeneous mass of material would weigh if cut in half by simply dividing its weight by 2. The educational psychologists has thus, taken some cues from the physical scientist when he attempts to assign scores to certain social variables.

The theory of measurement consists of a set of separate or distinct theories, each concerning a distinct *level* of measurement. The operations allowable on a given set of scores are dependent on the levels of measurement achieved.

Learning Outcomes for Session 14

When you have studied this session, you should be able to:

14.1. Define and distinguish among nominal, ordinal, interval, and ratio scales. (SAQ 14.1)
14.2. State the scale that is mostly used in education, psychology and social sciences (SAQ 14.2)

14.1 The Four Levels of Measurement

14.1.1 Types of Scales

Before we can conduct a statistical analysis, we need to measure our dependent variable. Exactly how the measurement is carried out depends on the type of variable involved in the analysis. Different types are measured differently. To measure the time taken to respond to a stimulus, you might use a stop watch. Stop watches are of no use, of course, when it comes to measuring someone's attitude towards a political candidate. A rating scale is more appropriate in this case (with labels like "very favourable," "somewhat favourable," etc.). For a dependent variable such as "favourite colour," you can simply note the colour-word (like "red") that the subject offers.

Although procedures for measurement differ in many ways, they can be classified using a few fundamental categories. In a given category, all of the procedures share some properties that are important for you to know about. The categories are called "scale types," or just "scales," and are described in this section.

Nominal scales

When measuring using a nominal scale, one simply names or categorizes responses. Gender, handedness, favourite colour, and religion are examples of variables measured on a nominal scale. The essential point about nominal scales is that they do not imply any ordering among the responses. For example, when classifying people according to their favourite colour, there is no sense in which green is placed "ahead of" blue. Responses are merely categorized. Nominal scales embody the lowest level of measurement.

- o Give one example to explain nominal scale for your friend?

- Your answer may be: house numbering, jersey number, raffle number, GSM number, etc.

Activity 14.1

Take a moment to reflect on nominal scale; and state its main purpose

Activity 14.1 Feedback: Nominal scale is only for identification

Ordinal scales

A researcher wishing to measure consumers' satisfaction with their microwave ovens might ask them to specify their feelings as either "very dissatisfied," "somewhat dissatisfied," "somewhat satisfied," or "very satisfied." The items in this scale are ordered, ranging from least to most satisfied. This is what distinguishes ordinal from nominal scales. Unlike nominal scales, ordinal scales allow comparisons of the degree to which two subjects possess the dependent variable. For example, our satisfaction ordering makes it meaningful to assert that one person is more satisfied than another with their microwave ovens. Such an assertion reflects the first person's use of a verbal label that comes later in the list than the label chosen by the second person.

On the other hand, ordinal scales fail to capture important information that will be present in the other scales we examine. In particular, the difference between two levels of an ordinal scale cannot be assumed to be the same as the difference between two other levels. In our satisfaction scale, for example, the difference between the responses "very dissatisfied" and "somewhat dissatisfied" is probably not equivalent to the difference between "somewhat dissatisfied" and "somewhat satisfied." Nothing in our measurement procedure allows us to determine whether the two differences reflect the same difference in psychological satisfaction. Statisticians express this point by saying that the differences between adjacent scale values do not necessarily represent equal intervals on the underlying scale giving rise to the measurements. (In our case, the underlying scale is the true feeling of satisfaction, which we are trying to measure.)

What if the researcher had measured satisfaction by asking consumers to indicate their level of satisfaction by choosing a number from one to four? Would the difference between the responses of one and two necessarily reflect the same difference in satisfaction as the difference between the responses two and three? The answer is No. Changing the response format to numbers does not change the meaning of the scale. We still are in no position to assert that the mental step from 1 to 2 (for example) is the same as the mental step from 3 to 4.

Activity 14.2

Take a moment to reflect on ordinal scale. State what differentiate it from nominal scale?

Activity 14.2 Feedback: Ordinal scale has rank, while nominal scale does not have. E.g. in ordinal scale 12 is greater than 10; but in nominal neither is greater.

Interval scales

Interval scales are numerical scales in which intervals have the same interpretation throughout. As an example, consider the Fahrenheit scale of temperature. The difference between 30 degrees and 40 degrees represents the same temperature difference as the difference between 80 degrees and 90 degrees. This is because each 10-degree interval has the same physical meaning (in terms of the kinetic energy of molecules).

Interval scales are not perfect, however. In particular, they do not have a true zero point even if one of the scaled values happens to carry the name "zero." The Fahrenheit scale illustrates the issue. Zero degrees Fahrenheit does not represent the complete absence of temperature (the absence of any molecular kinetic energy). In reality, the label "zero" is applied to its temperature for quite accidental reasons connected to the history of temperature measurement. Since an interval scale has no true zero point, it does not make sense to compute ratios of temperatures. For example, there is no sense in which the ratio of 40 to 20 degrees Fahrenheit is the same as the ratio of 100 to 50 degrees; no interesting physical property is preserved across the two ratios. After all, if the "zero" label were applied at the temperature that Fahrenheit happens to label as 10 degrees, the two ratios would instead be 30 to 10 and 90 to 40, no longer the same! For this reason, it does not make sense to say that 80 degrees is "twice as hot" as 40 degrees. Such a claim would depend on an arbitrary decision about where to "start" the temperature scale, namely, what temperature to call zero (whereas the claim is intended to make a more fundamental assertion about the underlying physical reality).

Activity 14.3

Take a moment to reflect on interval scale. Identify what is lacking in this scale?

Activity 14.3 Feedback: It does not have true zero point and does not follow multiplication principle.

Ratio scales

The ratio scale of measurement is the most informative scale. It is an interval scale with the additional property that its zero position indicates the absence of the quantity being measured. You can think of a ratio scale as the three earlier scales rolled up in one. Like a nominal scale, it provides a name or category for each object (the numbers serve as labels). Like an ordinal scale, the objects are ordered (in terms of the ordering of the numbers). Like an interval scale, the same difference at two places on the scale has the same meaning. And in addition, the same ratio at two places on the scale also carries the same meaning.

The Fahrenheit scale for temperature has an arbitrary zero point and is therefore not a ratio scale. However, zero on the Kelvin scale is absolute zero. This makes the Kelvin scale a ratio scale. For example, if one temperature is twice as high as another as measured on the Kelvin scale, then it has twice the kinetic energy of the other temperature.

Another example of a ratio scale is the amount of money you have in your pocket right now (25 cents, 55 cents, etc.). Money is measured on a ratio scale because, in addition to having the properties of an interval scale, it has a true zero point: if you have zero money, this implies the absence of money. Since money has a true zero point, it makes sense to say that someone with 50 cents has twice as much money as someone with 25 cents (or that Bill Gates has a million times more money than you do).

What level of measurement is used for psychological variables?

Rating scales are used frequently in psychological research. For example, experimental subjects may be asked to rate their level of pain, how much they like a consumer product, their attitudes about capital punishment, their confidence in an answer to a test question. Typically these ratings are made on a 5-point or a 7-point scale. These scales are ordinal scales since there is no assurance that a given difference represents the same thing across the range of the scale. For example, there is no way to be sure that a treatment that reduces pain from a rated pain level of 3 to a rated pain level of 2 represents the same level of relief as a treatment that reduces pain from a rated pain level of 7 to a rated pain level of 6.

In memory experiments, the dependent variable is often the number of items correctly recalled. What scale of measurement is this? You could reasonably argue that it is a ratio scale. First, there is a true zero point: some subjects may get no items correct at all. Moreover, a difference of one represents a difference of one item recalled across the entire scale. It is certainly valid to say that someone who recalled 12 items recalled twice as many items as someone who recalled only 6 items.

But number-of-items recalled is a more complicated case than it appears at first. Consider the following example in which subjects are asked to remember as many items as possible from a list of 10. Assume that (a) there are 5 easy items and 5 difficult items, (b) half of the subjects are able to recall all the easy items and different numbers of difficult items, while (c) the other half of the subjects are unable to recall any of the difficult items but they do remember different numbers of easy items. Some sample data are shown below.

Subject	**Easy Items**					**Difficult Items**					**Score**
A	0	0	1	1	0	0	0	0	0	0	**2**
B	1	0	1	1	0	0	0	0	0	0	**3**
C	1	1	1	1	1	1	1	0	0	0	**7**
D	1	1	1	1	1	0	1	1	0	1	**8**

Let's compare (1) the difference between Subject A's score of 2 and Subject B's score of 3 with

(2) the difference between Subject C's score of 7 and Subject D's score of 8. The former difference is a difference of one easy item; the latter difference is a difference of one difficult item. Do these two differences necessarily signify the same difference in memory? We are inclined to respond "No" to this question since only a little more memory may be needed to retain the additional easy item whereas a lot more memory may be needed to retain the additional hard item. The general point is that it is often inappropriate to consider psychological measurement scales as either interval or ratio.

Consequences of level of measurement

Why are we so interested in the type of scale that measures a dependent variable? The crux of the matter is the relationship between the variable's level of measurement and the statistics that can be meaningfully computed with that variable. For example, consider a hypothetical study in which 5 children are asked to choose their favourite colour from blue, red, yellow, green, and purple. The researcher codes the results as follows:

Colour	**Code**
Blue	1
Red	2
Yellow	3
Green	4
Purple	5

This means that if a child said her favourite colour was "Red," then the choice was coded as "2," if the child said her favourite colour was "Purple," then the response was coded as 5, and so forth. Consider the following hypothetical data:

Subject	**Colour**	**Code**
1	Blue	1
2	Blue	1
3	Green	4
4	Green	4
5	Purple	5

Each code is a number, so nothing prevents us from computing the average code assigned to the children. The average happens to be 3, but you can see that it would be senseless to conclude that the average favourite colour is yellow (the colour with a code of 3). Such nonsense arises because favourite colour is a nominal scale, and taking the average of its numerical labels is like counting the number of letters in the name of a snake to see how long the beast is.

Does it make sense to compute the mean of numbers measured on an ordinal scale? This is a difficult question, one that statisticians have debated for decades. You will be able to explore this issue yourself in a simulation shown in the next section and reach your own conclusion. The prevailing (but by no means unanimous) opinion of statisticians is that for almost all practical situations, the mean of an ordinally-measured variable is a meaningful statistic. However, as you will see in the simulation, there are extreme situations in which computing the mean of an ordinally-measured variable can be very misleading.

- o Help your friend that cannot recall the scales of measurement?

- Your answer should be: nominal, ordinal, interval and ratio.

Box 14.1: The Four Levels of Measurement

It is important to note the following:

- Nominal scale is for identification and categorization.
- Ordinal scale is used in ranking, but does not have any arithmetic value.
- Interval scale have arithmetic vale of addition and subtraction, besides the characteristics possessed by nominal and ordinal scales. It does not have true zero.
- Ratio scale has the characteristics of the other scales, as well as having true zero. It also has all arithmetic values.

Summary of Study Session 14

In Session 14, you have learned that:

- Measurement is the process of mapping or assigning numbers to objects or observations.
- Four of the most general scales were discussed - the nominal, ordinal, interval, and ratio scales.
- Nominal and ordinal measurements are the most common types achieved in the behavioural sciences.
- The interval and ratio measurement are the most common types achieved in the physical sciences.

Self-Assessment Questions (SAQs) for Session 14

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 14.1 (tests learning outcome 14.1)

Numbers on football jerseys and motor vehicle license plates are examples of (a) plate numbers (b) numbering (c) nominal scaling (d) interval scale (e) ordinal scaling

SAQ 14.2 (tests learning outcome 14.1)

A good example of ordinal scaling is (a) residential areas (b) number of tarred roads (c) socio economic status (d) housing units (e) all the above.

SAQ 14.3 (tests learning outcome 14.1)

The interval scale possesses the two attributes of (a) absolute zero point (b) equal intervals (c) magnitude (d) both b and c, (e) none of the above

SAQ 14.4 (tests learning outcome 14.1)

The ratio type of scaling possesses all the three attributes of scaling, namely, magnitude, equal intervals and (a) distance (b) class boundaries (c) variance (d) standard deviation (e) absolute zero point.

SAQ 14.5 (tests learning outcome 14.2)

Most attributes in psychology, education, sociology, etc, are not in (a) absolute zero points (b) ratio scales (c) interval scales (d) ordinal scales (e) none of the above.

Notes on the Self-Assessment Question

SAQ **14.1: C-** nominal scaling
SAQ **14.2: C-** socio-economic status
SAQ **14.3: D-** equal interval and magnitude
SAQ **14.4: E-** absolute zero point
SAQ **14.5: B-** ratio scales

References

McCall 13. Robert (1975), *Fundamental Statistics for Psychology* New York: Harcourt Brace Jovanovich, Inc.

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Study Session 15: Parametric and Non-parametric Statistics

Expected duration: 1 week or 2 contact hours

Introduction

This last session is going to give you the idea of parametric and nonparametric statistics. Examples of statistical techniques that have made use of some population characteristics are the indices of central values - the mean, mode, median; the measures of dispersion, the variance and standard deviations; and the correlation coefficients. These parameters are collectively called parametric statistical tests. On the other hand, methods that do not test hypotheses about specific parameters and require different (and sometimes fewer) assumptions are known as non-parametric tests.

Learning Outcomes for Session 15

When you have studied this session, you should be able to:

15.1. State the assumptions underlying the use of parametric statistical tests. (SAQ 15.1)
15.2. State the assumptions underlying the use of parametric statistical tests. (SAQ 15.2)

15.1 Parametric and Non parametric Statistics

15.1.1Parametric and nonparametric methods

Hopefully, after this somewhat lengthy introduction, the need is evident for statistical procedures that enable us to process data of "low quality," from small samples, on variables about which nothing is known (concerning their distribution). Specifically, nonparametric methods were developed to be used in cases when the researcher knows nothing about the parameters of the variable of interest in the population (hence the name *nonparametric*). In more technical terms, nonparametric methods do not rely on the estimation of parameters (such as the mean or the standard deviation) describing the distribution of the variable of interest in the population. Therefore, these methods are also sometimes (and more appropriately) called *parameter-free* methods or *distribution-free* methods.

Activity 15.1

Take a moment to reflect on what you have read so far. Differentiate between parametric and non-parametric statistics?

Activity 15.1 Feedback: Parametric statistics consider the characteristics of the population, while non-parametric does not.

Brief Overview of Nonparametric Methods

Basically, there is at least one nonparametric equivalent for each parametric general type of test. In general, these tests fall into the following categories:

- Tests of differences between groups (independent samples);
- Tests of differences between variables (dependent samples);
- Tests of relationships between variables.

Differences between independent groups. Usually, when we have two samples that we want to compare concerning their mean value for some variable of interest, we would use the *t*-test for independent samples); nonparametric alternatives for this test are the Wald-Wolfowitz runs test, the Mann-Whitney U test, and the Kolmogorov-Smirnov two-sample test. If we have multiple groups, we would use analysis of variance (see ANOVA/MANOVA; the nonparametric equivalents to this method are the Kruskal-Wallis analysis of ranks and the Median test.

Differences between dependent groups. If we want to compare two variables measured in the same sample we would customarily use the t-test for dependent samples (in Basic Statistics for example, if we wanted to compare students' math skills at the beginning of the semester with their skills at the end of the semester). Nonparametric alternatives to this test are the *Sign* test and *Wilcoxon's matched pairs* test. If the variables of interest are dichotomous in nature (i.e., "pass" vs. "no pass") then McNemar's Chi-square test is appropriate. If there are more than two variables that were measured in the same sample, then we would customarily use repeated measures ANOVA. Nonparametric alternatives to this method are Friedman's two-way analysis of variance and Cochran Q test (if the variable was measured in terms of categories, e.g., "passed" vs. "failed"). Cochran Q is particularly useful for measuring changes in frequencies (proportions) across time.

- o Tell your friend a parametric statistics and non-parametric statistics that can help in finding differences between dependent groups?

- Parametric (*Sign* test and *Wilcoxon's matched pairs* test; McNemar's Chi-square test; and repeated measures ANOVA) Non-parametric (Friedman's two-way ANOVA; Cochran Q test)

Relationships between variables. To express a relationship between two variables one usually computes the correlation coefficient. Nonparametric equivalents to the standard correlation coefficient are Spearman R, Kendall Tau, and coefficient Gamma (see Nonparametric correlations). If the two variables of interest are categorical in nature (e.g., "passed" vs. "failed" by "male" vs. "female") appropriate nonparametric statistics for testing the relationship between the two variables are the Chi-square test, the Phi coefficient, and the Fisher exact test. In addition, a simultaneous test for relationships between multiple cases is available: Kendall coefficient of concordance. This test is often used for expressing inter-rater agreement among independent judges who are rating (ranking) the same stimuli.

Descriptive statistics. When one's data are not normally distributed, and the measurements at best contain rank order information, then computing the standard descriptive statistics (e.g., mean, standard deviation) is sometimes not the most informative way to summarize the data. For example, in the area of psychometrics it is well known that the rated intensity of a stimulus (e.g., perceived brightness of a light) is often a logarithmic function of the actual intensity of the stimulus (brightness as measured in objective units of Lux). In this example, the simple mean rating (sum of ratings divided by the number of stimuli) is not an adequate summary of the average actual intensity of the stimuli. (In this example, one would probably rather compute the geometric mean.) Nonparametrics and Distributions will compute a wide variety of measures of location (mean, median, mode, etc.) and dispersion (variance, average deviation, quartile range, etc.) to provide the "complete picture" of one's data.

When to Use Which Method

It is not easy to give simple advice concerning the use of nonparametric procedures. Each nonparametric procedure has its peculiar sensitivities and blind spots. For example, the Kolmogorov-Smirnov two-sample test is not only sensitive to differences in the location of distributions (for example, differences in means) but is also greatly affected by differences in their shapes. The Wilcoxon matched pairs test assumes that one can rank order the magnitude of differences in matched observations in a meaningful manner. If this is not the case, one should rather use the Sign test. In general, if the result of a study is important (e.g., does a very expensive and painful drug therapy help people get better?), then it is always advisable to run different nonparametric tests; should discrepancies in the results occur contingent upon which test is used, one should try to understand why some tests give different results. On the other hand, nonparametric statistics are less statistically powerful (sensitive) than their parametric counterparts, and if it is important to detect even small effects (e.g., is this food additive harmful to people?) one should be very careful in the choice of a test statistic.

- Tell your friend your view about when to use Kolmogorov-Smirnov two-sample test?

- Your answer may be: differences in location and shapes.

Large data sets and nonparametric methods. Nonparametric methods are most appropriate when the sample sizes are small. When the data set is large (e.g., n > 100) it often makes little sense to use nonparametric statistics at all. Elementary Concepts briefly discusses the idea of the central limit theorem. In a nutshell, when the samples become very large, then the sample means will follow the normal distribution even if the respective variable is not normally distributed in the population, or is not measured very well. Thus, parametric methods, which are usually much more sensitive (i.e., have more statistical power) are in most cases appropriate for large samples. However, the tests of significance of many of the nonparametric statistics described here are based on asymptotic (large sample) theory; therefore, meaningful tests can often not be performed if the sample sizes become too small. Please refer to the descriptions of the specific tests to learn more about their power and efficiency.

Activity 15.3

Take a moment to reflect on what you have read. What is the most appropriate statistical method to employ when dealing with small sample?

Activity 15.3 Feedback: Non-parametric method

Nonparametric Correlations

The following are three types of commonly used nonparametric correlation coefficients (Spearman_R, Kendall Tau, and Gamma coefficients). Note that the chi-square statistic computed for two-way frequency tables, also provides a careful measure of a relation between the two (tabulated) variables, and unlike the correlation measures listed below, it can be used for variables that are measured on a simple nominal scale.

Spearman R. Spearman R (Siegel & Castellan, 1988) assumes that the variables under consideration were measured on at least an ordinal (rank order) scale, that is, that the individual observations can be ranked into two ordered series. Spearman R can be thought of as the regular Pearson product moment correlation coefficient, that is, in terms of proportion of variability accounted for, except that Spearman R is computed from ranks.

Kendall tau. Kendall tau is equivalent to Spearman R with regard to the underlying assumptions. It is also comparable in terms of its statistical power. However, Spearman R and Kendall tau are usually not identical in magnitude because their underlying logic as well as their computational formulas are very different. Siegel and Castellan (1988) express the relationship of the two measures in terms of the inequality: More importantly, Kendall tau and Spearman R imply different interpretations: Spearman R can be thought of as the regular Pearson product moment correlation coefficient, that is, in terms of proportion of variability accounted for, except that Spearman R is computed from ranks. Kendall tau, on the other hand, represents a probability, that is, it is the difference between the probabilities that in the observed data the two variables are in the same order versus the probability that the two variables are in different orders.

Gamma. The Gamma statistic (Siegel & Castellan, 1988) is preferable to Spearman R or Kendall tau when the data contain many tied observations. In terms of the underlying assumptions, Gamma is equivalent to Spearman R or Kendall tau; in terms of its interpretation and computation it is more similar to Kendall tau than Spearman R. In short, Gamma is also a probability; specifically, it is computed as the difference between the probabilities that the rank ordering of the two variables agree minus the probability that they disagree, divided by 1 minus the probability of ties. Thus, Gamma is basically equivalent to Kendall tau, except that ties are explicitly taken into account.

It’s safe to say that most people who use statistics are more familiar with parametric analyses than nonparametric analyses. Nonparametric tests are also called distribution-free tests because they don’t assume that your data follow a specific distribution.

You may have heard that you should use nonparametric tests when your data don’t meet the assumptions of the parametric test, especially the assumption about normally distributed data. That sounds like a nice and straightforward way to choose, but there are additional considerations.

In this chapter, you are going to be helped when you should use a:

- Parametric analysis to test group means.
- Nonparametric analysis to test group medians.

In particular, I'll focus on an important reason to use nonparametric tests that I don’t think gets mentioned often enough!

Activity 15.4

Take a moment to think about when to use data contain many tied observations.

Activity 15.4 Feedback: Gamma

Hypothesis Tests of the Mean and Median

Nonparametric tests are like a parallel universe to parametric tests. The table shows related pairs of hypothesis tests that Minitab statistical software offers.

Parametric tests (means)	**Nonparametric tests (medians)**
1-sample t test	1-sample Sign, 1-sample Wilcoxon
2-sample t test	Mann-Whitney test
One-Way ANOVA	Kruskal-Wallis, Mood’s median test
Factorial DOE with one factor and one blocking variable	Friedman test

Scenario 15.3

- Your friend could not give examples of parametric and non-parametric tests of mean and median, what examples would you suggest?
- Your answer may be: check the note above.

Reasons to Use Parametric Tests

Reason 1: Parametric tests can perform well with skewed and non-normal distributions

This may be a surprise but parametric tests can perform well with continuous data that are nonnormal if you satisfy these sample size guidelines.

Parametric analyses	Sample size guidelines for nonnormal data
1-sample t test	Greater than 20
2-sample t test	Each group should be greater than 15
One-Way ANOVA	• If you have 2-9 groups, each group should be greater than 15. • If you have 10-12 groups, each group should be greater than 20.

Reason 2: Parametric tests can perform well when the spread of each group is different

While nonparametric tests don't assume that your data follow a normal distribution, they do have other assumptions that can be hard to meet. For nonparametric tests that compare groups, a common assumption is that the data for all groups must have the same spread (dispersion). If your groups have a different spread, the nonparametric tests might not provide valid results.

On the other hand, if you use the 2-sample t test or One-Way ANOVA, you can simply go to the **Options** sub-dialog and uncheck *Assume equal variances*. Voilà, you're good to go even when the groups have different spreads!

Reason 3: Statistical power

Parametric tests usually have more statistical power than nonparametric tests. Thus, you are more likely to detect a significant effect when one truly exists.

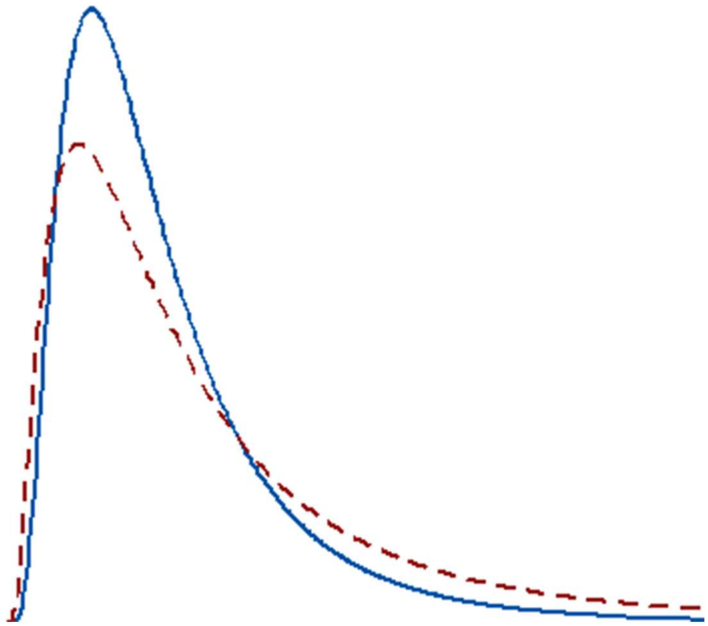
Activity 15.4

Take a moment to recall some reason for adopting parametric tests.

Activity 15.4 Feedback: High statistical power of test, etc.

Reasons to Use Nonparametric Tests

Reason 1: Your area of study is better represented by the median



This is my favourite reason to use a nonparametric test and the one that isn't mentioned often enough! The fact that you *can* perform a parametric test with nonnormal data doesn't imply that the mean is the best measure of the central tendency for your data.

For example, the center of a skewed distribution, like income, can be better measured by the median where 50% are above the median and 50% are below. If you add a few billionaires to a sample, the mathematical mean increases greatly even though the income for the typical person doesn't change.

When your distribution is skewed enough, the mean is strongly affected by changes far out in the distribution's tail whereas the median continues to more closely reflect the center of the distribution. For these two distributions, a random sample of 100 from each distribution produces means that are significantly different, but medians that are not significantly different.

Two of my colleagues have written excellent blog posts that illustrate this point:

- Michelle Paret: Using the Mean in Data Analysis: It's Not Always a Slam-Dunk
- Redouane Kouiden: The Non-parametric Economy: What Does Average Actually Mean?

Reason 2: You have a very small sample size

If you don't meet the sample size guidelines for the parametric tests and you are not confident that you have normally distributed data, you should use a nonparametric test. When you have a really small sample, you might not even be able to ascertain the distribution of your data because the distribution tests will lack sufficient power to provide meaningful results.

In this scenario, you're in a tough spot with no valid alternative. Nonparametric tests have less power to begin with and it's a double whammy when you add a small sample size on top of that!

Reason 3: You have ordinal data, ranked data, or outliers that you can't remove

Typical parametric tests can only assess continuous data and the results can be significantly affected by outliers. Conversely, some nonparametric tests can handle ordinal data, ranked data, and not be seriously affected by outliers. Be sure to check the assumptions for the nonparametric test because each one has its own data requirements.

If you have Likert data and want to compare two groups, read my post Best Way to Analyze Likert Item Data: Two Sample T-Test versus Mann-Whitney.

Activity 15.5

Take a moment to write some sentences on why you should adopt non-parametric test.

Activity 15.5 Feedback: When the sample is small; when you have ordinal data; etc.

It's commonly thought that the need to choose between a parametric and nonparametric test occurs when your data fail to meet an assumption of the parametric test. This can be the case when you have both a small sample size and non-normal data. However, other considerations often play a role because parametric tests can often handle non-normal data. Conversely, nonparametric tests have strict assumptions that you can't disregard.

The decision often depends on whether the mean or median more accurately represents the center of your data's distribution.

- If the mean accurately represents the center of your distribution and your sample size is large enough, consider a parametric test because they are more powerful.
- If the median better represents the center of your distribution, consider the nonparametric test even when you have a large sample.

Finally, if you have a very small sample size, you might be stuck using a nonparametric test. Please, collect more data next time if it is at all possible! As you can see, the sample size guidelines aren't really that large. Your chance of detecting a significant effect when one exists can be very small when you have both a small sample size and you need to use a less efficient nonparametric test!

Parametric and Nonparametric Tests

A parametric statistical test is a test whose model specifies certain conditions about the parameters of the population from which the research sample was drawn. These conditions are as follows:

(a) the observations must be independent. (b) The observations must be drawn from normally distributed populations (c) These populations must have the same variance that is, homogeneity of variance and (d) The variables involved scale, so that it is possible to use the operations of arithmetic on the scores.

A nonparametric statistical test is a test whose model does not specify conditions about the parameters of the population from which the sample was drawn. Certain assumptions are associated with most nonparametric tests, that is, that the observations are independent and that the variable under study has underlying continuity. Nonparametric tests also do not require measurements so strong that is required for the parametric tests. Most nonparametric tests apply to data in an ordinal and nominal scale of measurement.

Advantages

1. These techniques are computationally simple.
2. Nonparametric tests are very useful with small samples.
3. Nonparametric tests are available to treat data which are inherently in ranks as well as data whose seemingly numerical scores have the strength of ranks.
4. Non-parametric methods are available to treat which are simply classificatory, that is, are measured in a nominal scale.
5. Nonparametric statistical tests are typically much easier to learn and to apply than are parametric tests.

Disadvantages

1. Because nonparametric tests have very low *power-efficiency,* they tend to waste data than do the parametric tests.
2. There are as yet no nonparametric methods for testing interactions in the analysis of variance model, unless special assumptions are made about additives.

- o You and your friend should write on the advantages and disadvantages of parametric and non-parametric tests respectively.

- Compare your attempts with the above session.

Box 15.1: Parametric and Non parametric Statistics

It is important to note the following:

- Parametric test consider the characteristics of data obtained from the population.
- Non-parametric test does not consider the estimation of characteristics or peculiarity of data obtained from the population.
- They both have advantages and disadvantages; and should be employed when certain conditions are well considered.

Summary of Study Session 15

In Session 15, you have learned that:

- Parametric tests possess the advantages of being somewhat robust with respect to violations of assumptions, having relatively more information about a phenomenon (e.g. interactions in the analysis of variance).
- However, when departures from normality or homogeneity of variance are terribly severe, or when the data are nominal or ordinal a nonparametric method may be more appropriate.

Self-Assessment Questions (SAQs) for Session 15

Now that you have completed this study session, you can assess how well you have achieved its Learning Outcomes by answering these questions. You can check your answers with the Notes on the Self-Assessment Questions at the end of this Module.

SAQ 15.1 (tests learning outcome 15.1)

One of the assumptions underlying the use of parametric tests is………. (a) population must be decreased (b) population must be varied (c) the observations must be independent (d) the observations must be dependent (e) all the above.

SAQ 15.2 (tests learning outcome 15.2)

Another assumption underlying the use of the parametric test is (a) abnormal distribution (b) frequent distribution (c) distribution free (d) normal distribution (e) all the above.

SAQ 15.3 (tests learning outcome 15.2)

Nonparametric tests does not specify ___________ about the population. (a) distribution (b) conditions (c) population (d) parameters (e) both (b) and (d).

SAQ 15.4 (tests learning outcome 15.2)

The major advantage of the nonparametric test is that it could be applied to (a) large samples (b) human population (c) animal population (d) small samples (e) school samples

SAQ 15.5 (tests learning outcome 15.2)

The nonparametric test is more useful when………… (a) the data are nominal (b) the data are ordinal (c) both (a) and (b), (d) all the above, (e) none of the above.

Notes on the Self-Assessment Question

SAQ 15.1: C

SAQ 15.1: D
SAQ 15.1: D
SAQ 15.1: D
SAQ 15.1: C

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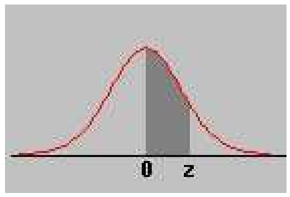
Distribution Tables

Area between 0 and z

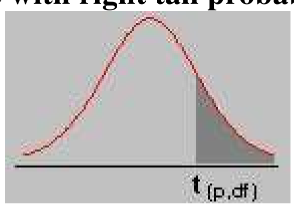


	0.00	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916

2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

Student's t Table

t table with right tail probabilities

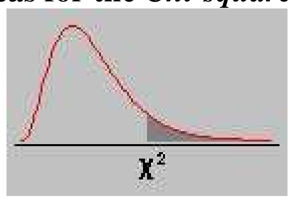


df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405

15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905

Chi-Square Table

Right tail areas for the *Chi-square* Distribution

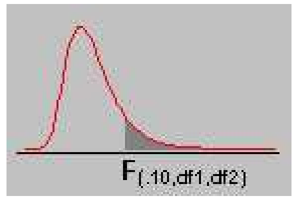


df\area	**.995**	**.990**	**.975**	**.950**	**.900**	**.750**	**.500**	**.250**	**.100**	**.050**	**.025**	**.010**	**.005**
1	0.00004	0.00016	0.00098	0.00393	0.01579	0.10153	0.45494	1.32330	2.70554	3.84146	5.02389	6.63490	7.87944
2	0.01003	0.02010	0.05064	0.10259	0.21072	0.57536	1.38629	2.77259	4.60517	5.99146	7.37776	9.21034	10.59663
3	0.07172	0.11483	0.21580	0.35185	0.58437	1.21253	2.36597	4.10834	6.25139	7.81473	9.34840	11.34487	12.83816
4	0.20699	0.29711	0.48442	0.71072	1.06362	1.92256	3.35669	5.38527	7.77944	9.48773	11.14329	13.27670	14.86026
5	0.41174	0.55430	0.83121	1.14548	1.61031	2.67460	4.35146	6.62568	9.23636	11.07050	12.83250	15.08627	16.74960
6	0.67573	0.87209	1.23734	1.63538	2.20413	3.45460	5.34812	7.84080	10.64464	12.59159	14.44938	16.81189	18.54758
7	0.98926	1.23904	1.68987	2.16735	2.83311	4.25485	6.34581	9.03715	12.01704	14.06714	16.01276	18.47531	20.27774
8	1.34441	1.64650	2.17973	2.73264	3.48954	5.07064	7.34412	10.21885	13.36157	15.50731	17.53455	20.09024	21.95495
9	1.73493	2.08790	2.70039	3.32511	4.16816	5.89883	8.34283	11.38875	14.68366	16.91898	19.02277	21.66599	23.58935
10	2.15586	2.55821	3.24697	3.94030	4.86518	6.73720	9.34182	12.54886	15.98718	18.30704	20.48318	23.20925	25.18818
11	2.60322	3.05348	3.81575	4.57481	5.57778	7.58414	10.34100	13.70069	17.27501	19.67514	21.92005	24.72497	26.75685
12	3.07382	3.57057	4.40379	5.22603	6.30380	8.43842	11.34032	14.84540	18.54935	21.02607	23.33666	26.21697	28.29952
13	3.56503	4.10692	5.00875	5.89186	7.04150	9.29907	12.33976	15.98391	19.81193	22.36203	24.73560	27.68825	29.81947
14	4.07467	4.66043	5.62873	6.57063	7.78953	10.16531	13.33927	17.11693	21.06414	23.68479	26.11895	29.14124	31.31935
15	4.60092	5.22935	6.26214	7.26094	8.54676	11.03654	14.33886	18.24509	22.30713	24.99579	27.48839	30.57791	32.80132

16	5.14221	5.81221	6.90766	7.96165	9.31224	11.91222	15.33850	19.36886	23.54183	26.29623	28.84535	31.99993	34.26719
17	5.69722	6.40776	7.56419	8.67176	10.08519	12.79193	16.33818	20.48868	24.76904	27.58711	30.19101	33.40866	35.71847
18	6.26480	7.01491	8.23075	9.39046	10.86494	13.67529	17.33790	21.60489	25.98942	28.86930	31.52638	34.80531	37.15645
19	6.84397	7.63273	8.90652	10.11701	11.65091	14.56200	18.33765	22.71781	27.20357	30.14353	32.85233	36.19087	38.58226
20	7.43384	8.26040	9.59078	10.85081	12.44261	15.45177	19.33743	23.82769	28.41198	31.41043	34.16961	37.56623	39.99685
21	8.03365	8.89720	10.28290	11.59131	13.23960	16.34438	20.33723	24.93478	29.61509	32.67057	35.47888	38.93217	41.40106
22	8.64272	9.54249	10.98232	12.33801	14.04149	17.23962	21.33704	26.03927	30.81328	33.92444	36.78071	40.28936	42.79565
23	9.26042	10.19572	11.68855	13.09051	14.84796	18.13730	22.33688	27.14134	32.00690	35.17246	38.07563	41.63840	44.18128
24	9.88623	10.85636	12.40115	13.84843	15.65868	19.03725	23.33673	28.24115	33.19624	36.41503	39.36408	42.97982	45.55851
25	10.51965	11.52398	13.11972	14.61141	16.47341	19.93934	24.33659	29.33885	34.38159	37.65248	40.64647	44.31410	46.92789
26	11.16024	12.19815	13.84390	15.37916	17.29188	20.84343	25.33646	30.43457	35.56317	38.88514	41.92317	45.64168	48.28988
27	11.80759	12.87850	14.57338	16.15140	18.11390	21.74940	26.33634	31.52841	36.74122	40.11327	43.19451	46.96294	49.64492
28	12.46134	13.56471	15.30786	16.92788	18.93924	22.65716	27.33623	32.62049	37.91592	41.33714	44.46079	48.27824	50.99338
29	13.12115	14.25645	16.04707	17.70837	19.76774	23.56659	28.33613	33.71091	39.08747	42.55697	45.72229	49.58788	52.33562
30	13.78672	14.95346	16.79077	18.49266	20.59923	24.47761	29.33603	34.79974	40.25602	43.77297	46.97924	50.89218	53.67196

F Distribution Tables

F Table for alpha=.10



df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
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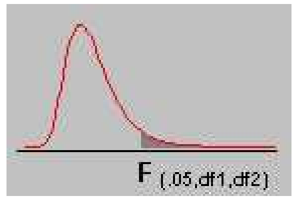
2	8.52632	9.00000	9.16179	9.24342	9.29263	9.32553	9.34908	9.36677	9.38054	9.39157	9.40813	9.42471	9.44131	9.44962	9.45793	9.46624	9.47456	9.48289	9.49122
3	5.53832	5.46238	5.39077	5.34264	5.30916	5.28473	5.26619	5.25167	5.24000	5.23041	5.21562	5.20031	5.18448	5.17636	5.16811	5.15972	5.15119	5.14251	5.13370
4	4.54477	4.32456	4.19086	4.10725	4.05058	4.00975	3.97897	3.95494	3.93567	3.91988	3.89553	3.87036	3.84434	3.83099	3.81742	3.80361	3.78957	3.77527	3.76073
5	4.06042	3.77972	3.61948	3.52020	3.45298	3.40451	3.36790	3.33928	3.31628	3.29740	3.26824	3.23801	3.20665	3.19052	3.17408	3.15732	3.14023	3.12279	3.10500
6	3.77595	3.46330	3.28876	3.18076	3.10751	3.05455	3.01446	2.98304	2.95774	2.93693	2.90472	2.87122	2.83634	2.81834	2.79996	2.78117	2.76195	2.74229	2.72216
7	3.58943	3.25744	3.07407	2.96053	2.88334	2.82739	2.78493	2.75158	2.72468	2.70251	2.66811	2.63223	2.59473	2.57533	2.55546	2.53510	2.51422	2.49279	2.47079
8	3.45792	3.11312	2.92380	2.80643	2.72645	2.66833	2.62413	2.58935	2.56124	2.53804	2.50196	2.46422	2.42464	2.40410	2.38302	2.36136	2.33910	2.31618	2.29257
9	3.36030	3.00645	2.81286	2.69268	2.61061	2.55086	2.50531	2.46941	2.44034	2.41632	2.37888	2.33962	2.29832	2.27683	2.25472	2.23196	2.20849	2.18427	2.15923
10	3.28502	2.92447	2.72767	2.60534	2.52164	2.46058	2.41397	2.37715	2.34731	2.32260	2.28405	2.24351	2.20074	2.17843	2.15543	2.13169	2.10716	2.08176	2.05542
11	3.22520	2.85951	2.66023	2.53619	2.45118	2.38907	2.34157	2.30400	2.27350	2.24823	2.20873	2.16709	2.12305	2.10001	2.07621	2.05161	2.02612	1.99965	1.97211
12	3.17655	2.80680	2.60552	2.48010	2.39402	2.33102	2.28278	2.24457	2.21352	2.18776	2.14744	2.10485	2.05968	2.03599	2.01149	1.98610	1.95973	1.93228	1.90361
13	3.13621	2.76317	2.56027	2.43371	2.34672	2.28298	2.23410	2.19535	2.16382	2.13763	2.09659	2.05316	2.00698	1.98272	1.95757	1.93147	1.90429	1.87591	1.84620
14	3.10221	2.72647	2.52222	2.39469	2.30694	2.24256	2.19313	2.15390	2.12195	2.09540	2.05371	2.00953	1.96245	1.93766	1.91193	1.88516	1.85723	1.82800	1.79728
15	3.07	2.69	2.48	2.36	2.27	2.20	2.15	2.11	2.08	2.05	2.01	1.97	1.92	1.89	1.87	1.84	1.81	1.78	1.75

	319	517	979	143	302	808	818	853	621	932	707	222	431	904	277	539	676	672	505
16	3.04811	2.66817	2.46181	2.33274	2.24376	2.17833	2.12800	2.08798	2.05533	2.02815	1.98539	1.93992	1.89127	1.86556	1.83879	1.81084	1.78156	1.75075	1.71817
17	3.02623	2.64464	2.43743	2.30775	2.21825	2.15239	2.10169	2.06134	2.02839	2.00094	1.95772	1.91169	1.86236	1.83624	1.80901	1.78053	1.75063	1.71909	1.68564
18	3.00698	2.62395	2.41601	2.28577	2.19583	2.12958	2.07854	2.03789	2.00467	1.97698	1.93334	1.88681	1.83685	1.81035	1.78269	1.75371	1.72322	1.69099	1.65671
19	2.98990	2.60561	2.39702	2.26630	2.17596	2.10936	2.05802	2.01710	1.98364	1.95573	1.91170	1.86471	1.81416	1.78731	1.75924	1.72979	1.69876	1.66587	1.63077
20	2.97465	2.58925	2.38009	2.24893	2.15823	2.09132	2.03970	1.99853	1.96485	1.93674	1.89236	1.84494	1.79384	1.76667	1.73822	1.70833	1.67678	1.64326	1.60738
21	2.96096	2.57457	2.36489	2.23334	2.14231	2.07512	2.02325	1.98186	1.94797	1.91967	1.87497	1.82715	1.77555	1.74807	1.71927	1.68896	1.65691	1.62278	1.58615
22	2.94858	2.56131	2.35117	2.21927	2.12794	2.06050	2.00840	1.96680	1.93273	1.90425	1.85925	1.81106	1.75899	1.73122	1.70208	1.67138	1.63885	1.60415	1.56678
23	2.93736	2.54929	2.33873	2.20651	2.11491	2.04723	1.99492	1.95312	1.91888	1.89025	1.84497	1.79643	1.74392	1.71588	1.68643	1.65535	1.62237	1.58711	1.54903
24	2.92712	2.53833	2.32739	2.19488	2.10303	2.03513	1.98263	1.94066	1.90625	1.87748	1.83194	1.78308	1.73015	1.70185	1.67210	1.64067	1.60726	1.57146	1.53270
25	2.91774	2.52831	2.31702	2.18424	2.09216	2.02406	1.97138	1.92925	1.89469	1.86578	1.82000	1.77083	1.71752	1.68898	1.65895	1.62718	1.59335	1.55703	1.51760
26	2.90913	2.51910	2.30749	2.17447	2.08218	2.01389	1.96104	1.91876	1.88407	1.85503	1.80902	1.75957	1.70589	1.67712	1.64682	1.61472	1.58050	1.54368	1.50360
27	2.90119	2.51061	2.29871	2.16546	2.07298	2.00452	1.95151	1.90909	1.87427	1.84511	1.79889	1.74917	1.69514	1.66616	1.63560	1.60320	1.56859	1.53129	1.49057
28	2.89385	2.50276	2.29060	2.15714	2.06447	1.99585	1.94270	1.90014	1.86520	1.83593	1.78951	1.73954	1.68519	1.65600	1.62519	1.59250	1.55753	1.51976	1.47841
29	2.88703	2.49548	2.28307	2.14941	2.05658	1.98781	1.93452	1.89184	1.85679	1.82741	1.78081	1.73060	1.67593	1.64655	1.61551	1.58253	1.54721	1.50899	1.46704
30	2.88069	2.48872	2.27607	2.14223	2.04925	1.98033	1.92692	1.88412	1.84896	1.81949	1.77270	1.72227	1.66731	1.63774	1.60648	1.57323	1.53757	1.49891	1.45636
40	2.83535	2.44037	2.22609	2.09095	1.99682	1.92688	1.87252	1.82886	1.79290	1.76269	1.71456	1.66241	1.60515	1.57411	1.54108	1.50562	1.46716	1.42476	1.37691
60	2.79107	2.39325	2.17741	2.04099	1.94571	1.87472	1.81939	1.77483	1.73802	1.70701	1.65743	1.60337	1.54349	1.51072	1.47554	1.43734	1.39520	1.34757	1.29146
120	2.74781	2.34734	2.12999	1.99230	1.89587	1.82381	1.76748	1.72196	1.68425	1.65238	1.60120	1.54500	1.48207	1.44723	1.40938	1.36760	1.32034	1.26457	1.19256
Inf	2.70554	2.30259	2.08380	1.94486	1.84727	1.77411	1.71672	1.67020	1.63152	1.59872	1.54578	1.48714	1.42060	1.38318	1.34187	1.29513	1.23995	1.16860	1.00000

F Table for alpha=.05



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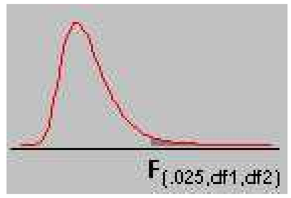
2	18.5128	19.0000	19.1643	19.2468	19.2964	19.3295	19.3532	19.3710	19.3848	19.3959	19.4125	19.4291	19.4458	19.4541	19.4624	19.4707	19.4791	19.4874	19.4957
3	10.1280	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5720	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.7170	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.3650
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.9130	2.8450	2.7740	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536	2.7876	2.7186	2.6464	2.6090	2.5705	2.5309	2.4901	2.4480	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.3410	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.4630	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658

16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.1040	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.3080	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.8780
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660	2.3210	2.2504	2.1757	2.0960	2.0540	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.9380	1.8894	1.8380	1.7831
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.0050	1.9605	1.9139	1.8648	1.8128	1.7570
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.9390	1.8920	1.8424	1.7896	1.7330
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.7110
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.9010	1.8533	1.8027	1.7488	1.6906
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360	2.1900	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401	1.9926	1.9174	1.8364	1.7480	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.4290	1.3519	1.2539
Inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.3940	1.3180	1.2214	1.0000

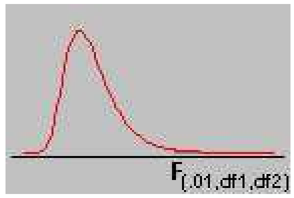
F Table for alpha=.025



df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
1	647.7890	799.5000	864.1630	899.5833	921.8479	937.1111	948.2169	956.6562	963.2846	968.6274	976.7079	984.8668	993.1028	997.2492	1001.414	1005.598	1009.800	1014.020	1018.258
2	38.5063	39.0000	39.1655	39.2484	39.2982	39.3315	39.3552	39.3730	39.3869	39.3980	39.4146	39.4313	39.4479	39.4562	39.465	39.473	39.481	39.490	39.498
3	17.4434	16.0441	15.4392	15.1010	14.8848	14.7347	14.6244	14.5399	14.4731	14.4189	14.3366	14.2527	14.1674	14.1241	14.081	14.037	13.992	13.947	13.902
4	12.2179	10.6491	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047	8.8439	8.7512	8.6565	8.5599	8.5109	8.461	8.411	8.360	8.309	8.257
5	10.0070	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6811	6.6192	6.5245	6.4277	6.3286	6.2780	6.227	6.175	6.123	6.069	6.015
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.6955	5.5996	5.5234	5.4613	5.3662	5.2687	5.1684	5.1172	5.065	5.012	4.959	4.904	4.849
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8993	4.8232	4.7611	4.6658	4.5678	4.4667	4.4150	4.362	4.309	4.254	4.199	4.142
8	7.5709	6.0595	5.4160	5.0526	4.8173	4.6517	4.5286	4.4333	4.3572	4.2951	4.1997	4.1012	3.9995	3.9472	3.894	3.840	3.784	3.728	3.670
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.1970	4.1020	4.0260	3.9639	3.8682	3.7694	3.6669	3.6142	3.560	3.505	3.449	3.392	3.333
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.7790	3.7168	3.6209	3.5217	3.4185	3.3654	3.311	3.255	3.198	3.140	3.080
11	6.7241	5.2559	4.6300	4.2751	4.0440	3.8807	3.7586	3.6638	3.5879	3.5257	3.4296	3.3299	3.2261	3.1725	3.118	3.061	3.004	2.944	2.883
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.6065	3.5118	3.4358	3.3736	3.2773	3.1772	3.0728	3.0187	2.963	2.906	2.848	2.787	2.725
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.3880	3.3120	3.2497	3.1532	3.0527	2.9477	2.8932	2.837	2.780	2.720	2.659	2.595
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093	3.1469	3.0502	2.9493	2.8437	2.7888	2.732	2.674	2.614	2.552	2.487
15	6.1995	4.7650	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227	3.0602	2.9633	2.8621	2.7559	2.7006	2.644	2.585	2.524	2.461	2.395

16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488	2.9862	2.8890	2.7875	2.6808	2.6252	2.568	2.509	2.447	2.383	2.316
17	6.0420	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.0610	2.9849	2.9222	2.8249	2.7230	2.6158	2.5598	2.502	2.442	2.380	2.315	2.247
18	5.9781	4.5597	3.9539	3.6083	3.3820	3.2209	3.0999	3.0053	2.9291	2.8664	2.7689	2.6667	2.5590	2.5027	2.445	2.384	2.321	2.256	2.187
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8801	2.8172	2.7196	2.6171	2.5089	2.4523	2.394	2.333	2.270	2.203	2.133
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365	2.7737	2.6758	2.5731	2.4645	2.4076	2.349	2.287	2.223	2.156	2.085
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.8740	2.7977	2.7348	2.6368	2.5338	2.4247	2.3675	2.308	2.246	2.182	2.114	2.042
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628	2.6998	2.6017	2.4984	2.3890	2.3315	2.272	2.210	2.145	2.076	2.003
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9023	2.8077	2.7313	2.6682	2.5699	2.4665	2.3567	2.2989	2.239	2.176	2.111	2.041	1.968
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027	2.6396	2.5411	2.4374	2.3273	2.2693	2.209	2.146	2.080	2.010	1.935
25	5.6864	4.2909	3.6943	3.3530	3.1287	2.9685	2.8478	2.7531	2.6766	2.6135	2.5149	2.4110	2.3005	2.2422	2.182	2.118	2.052	1.981	1.906
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.8240	2.7293	2.6528	2.5896	2.4908	2.3867	2.2759	2.2174	2.157	2.093	2.026	1.954	1.878
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.8021	2.7074	2.6309	2.5676	2.4688	2.3644	2.2533	2.1946	2.133	2.069	2.002	1.930	1.853
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.7820	2.6872	2.6106	2.5473	2.4484	2.3438	2.2324	2.1735	2.112	2.048	1.980	1.907	1.829
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.8840	2.7633	2.6686	2.5919	2.5286	2.4295	2.3248	2.2131	2.1540	2.092	2.028	1.959	1.886	1.807
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.7460	2.6513	2.5746	2.5112	2.4120	2.3072	2.1952	2.1359	2.074	2.009	1.940	1.866	1.787
40	5.4239	4.0510	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519	2.3882	2.2882	2.1819	2.0677	2.0069	1.943	1.875	1.803	1.724	1.637
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.3344	2.2702	2.1692	2.0613	1.9445	1.8817	1.815	1.744	1.667	1.581	1.482
120	5.1523	3.8046	3.2269	2.8943	2.6740	2.5154	2.3948	2.2994	2.2217	2.1570	2.0548	1.9450	1.8249	1.7597	1.690	1.614	1.530	1.433	1.310
Inf	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136	2.0483	1.9447	1.8326	1.7085	1.6402	1.566	1.484	1.388	1.268	1.000

F Table for alpha=.01



df2/df1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	INF
1	4052.181	4999.500	5403.352	5624.583	5763.650	5858.986	5928.356	5981.070	6022.473	6055.847	6106.321	6157.285	6208.730	6234.631	6260.649	6286.782	6313.030	6339.391	6365.864
2	98.503	99.000	99.166	99.249	99.299	99.333	99.356	99.374	99.388	99.399	99.416	99.433	99.449	99.458	99.466	99.474	99.482	99.491	99.499
3	34.116	30.817	29.457	28.710	28.237	27.911	27.672	27.489	27.345	27.229	27.052	26.872	26.690	26.598	26.505	26.411	26.316	26.221	26.125
4	21.198	18.000	16.694	15.977	15.522	15.207	14.976	14.799	14.659	14.546	14.374	14.198	14.020	13.929	13.838	13.745	13.652	13.558	13.463
5	16.258	13.274	12.060	11.392	10.967	10.672	10.456	10.289	10.158	10.051	9.888	9.722	9.553	9.466	9.379	9.291	9.202	9.112	9.020
6	13.745	10.925	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.718	7.559	7.396	7.313	7.229	7.143	7.057	6.969	6.880
7	12.246	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.469	6.314	6.155	6.074	5.992	5.908	5.824	5.737	5.650
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.667	5.515	5.359	5.279	5.198	5.116	5.032	4.946	4.859
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.111	4.962	4.808	4.729	4.649	4.567	4.483	4.398	4.311
10	10.044	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.706	4.558	4.405	4.327	4.247	4.165	4.082	3.996	3.909
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.397	4.251	4.099	4.021	3.941	3.860	3.776	3.690	3.602
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.155	4.010	3.858	3.780	3.701	3.619	3.535	3.449	3.361
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	3.960	3.815	3.665	3.587	3.507	3.425	3.341	3.255	3.165
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.800	3.656	3.505	3.427	3.348	3.266	3.181	3.094	3.004
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.666	3.522	3.372	3.294	3.214	3.132	3.047	2.959	2.868

16	8.531	6.226	5.292	4.773	4.437	4.202	4.026	3.890	3.780	3.691	3.553	3.409	3.259	3.181	3.101	3.018	2.933	2.845	2.753
17	8.400	6.112	5.185	4.669	4.336	4.102	3.927	3.791	3.682	3.593	3.455	3.312	3.162	3.084	3.003	2.920	2.835	2.746	2.653
18	8.285	6.013	5.092	4.579	4.248	4.015	3.841	3.705	3.597	3.508	3.371	3.227	3.077	2.999	2.919	2.835	2.749	2.660	2.566
19	8.185	5.926	5.010	4.500	4.171	3.939	3.765	3.631	3.523	3.434	3.297	3.153	3.003	2.925	2.844	2.761	2.674	2.584	2.489
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.231	3.088	2.938	2.859	2.778	2.695	2.608	2.517	2.421
21	8.017	5.780	4.874	4.369	4.042	3.812	3.640	3.506	3.398	3.310	3.173	3.030	2.880	2.801	2.720	2.636	2.548	2.457	2.360
22	7.945	5.719	4.817	4.313	3.988	3.758	3.587	3.453	3.346	3.258	3.121	2.978	2.827	2.749	2.667	2.583	2.495	2.403	2.305
23	7.881	5.664	4.765	4.264	3.939	3.710	3.539	3.406	3.299	3.211	3.074	2.931	2.781	2.702	2.620	2.535	2.447	2.354	2.256
24	7.823	5.614	4.718	4.218	3.895	3.667	3.496	3.363	3.256	3.168	3.032	2.889	2.738	2.659	2.577	2.492	2.403	2.310	2.211
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	2.993	2.850	2.699	2.620	2.538	2.453	2.364	2.270	2.169
26	7.721	5.526	4.637	4.140	3.818	3.591	3.421	3.288	3.182	3.094	2.958	2.815	2.664	2.585	2.503	2.417	2.327	2.233	2.131
27	7.677	5.488	4.601	4.106	3.785	3.558	3.388	3.256	3.149	3.062	2.926	2.783	2.632	2.552	2.470	2.384	2.294	2.198	2.097
28	7.636	5.453	4.568	4.074	3.754	3.528	3.358	3.226	3.120	3.032	2.896	2.753	2.602	2.522	2.440	2.354	2.263	2.167	2.064
29	7.598	5.420	4.538	4.045	3.725	3.499	3.330	3.198	3.092	3.005	2.868	2.726	2.574	2.495	2.412	2.325	2.234	2.138	2.034
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979	2.843	2.700	2.549	2.469	2.386	2.299	2.208	2.111	2.006
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.665	2.522	2.369	2.288	2.203	2.114	2.019	1.917	1.805
60	7.077	4.977	4.126	3.649	3.339	3.119	2.953	2.823	2.718	2.632	2.496	2.352	2.198	2.115	2.028	1.936	1.836	1.726	1.601
120	6.851	4.787	3.949	3.480	3.174	2.956	2.792	2.663	2.559	2.472	2.336	2.192	2.035	1.950	1.860	1.763	1.656	1.533	1.381